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Brief answers to the examination problems in ECON 3120/4120, 9 December 2005

Problem 1

(a) $x_0 = \ln \dfrac{b}{a}\,, \quad x_1 = \ln \dfrac{b}{2a}\,, \quad x_0 - x_1 = \ln 2\,.$

(b) $f(x) > 0 \iff x > x_0\,, \quad f \searrow$ over $(-\infty, x_1]$, $\quad f \nearrow$ over $[x_1, \infty)$.

(c) $\lim_{x\to-\infty} f(x) = 0\,, \quad \lim_{x\to\infty} f(x) = \infty\,.$

Problem 2

(a) $f_1' = 4x(x^2 - y) + 3x^2 - 6x\,, \quad f_2' = -2(x^2 - y)\,,$
$f_{11}'' = 12x^2 - 4y + 6x - 6\,, \quad f_{12}'' = -4x\,, \quad f_{22}'' = 2\,.$

(b) $(0,0)$ is a saddle point, $(2,4)$ is a local minimum point.

(c) No global extreme points. (Note that along the curve $y = x^2$ the value of f is $f(x, x^2) = x^3 - 3x^2 = x^2(x-3)$, which tends to ∞ as x tends to ∞ and tends to $-\infty$ as x tends to $-\infty$.)

Problem 3

(a) $\displaystyle\int_0^{\sqrt{15}} 3x\sqrt{1+x^2}\,dx = 63\,.$

(b) $x \equiv 0$ and $x = \dfrac{-1}{(1+e^x)\ln(1+e^x) - e^x + C}\,.$

Problem 4

(a) $|\mathbf{A}_a| = 2 - a\,.$ $\mathbf{A}_a$ has an inverse $\iff a \neq 2\,.$

(b) $\mathbf{A}_a\mathbf{B}_b = \begin{pmatrix} b+10 & 0 & 0 \\ b+2a-3 & a-2 & 15-3a \\ b+7 & 0 & 3 \end{pmatrix}$, so $\mathbf{A}_a\mathbf{B}_b = 3\mathbf{I}_3$ if $a = 5$ and $b = 7\,.$

$\mathbf{A}_5^{-1} = \frac{1}{3}\mathbf{B}_7 = \frac{1}{3}\begin{pmatrix} -7 & -2 & 12 \\ 2 & 1 & -3 \\ 3 & 0 & -3 \end{pmatrix}.$

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