Mathematics 2

Introduction :

→ what you will learn

→ what you should know already

→ practicalities

* Books:

- English <u>or</u> Norwegian.
  - ↳ M2 : EMEA
  - ↳ M3 : FMEA (+ bits...)
  - ↳ M2 : MA1 + LA + MA2 for 1 kap
  - ↳ M3 : LA + MA2

* Teaching language

- English...

Topics (non-exhaustive list!)

→ Optimization (max/min)

→ under eq./ineq. constraints

- You should already know

→ 1^{st} & 2^{nd} order cond's in one variable (and have seen it in two, with Lagrange)

→ concavity/convexity, 1 var.

... but will be covered from scratch

M2

→ Integrals: You know $f'(x)$, find $f(x)$

- Should know: differentiation

→ Differential equations: e.g. find the <u>functions</u> $x(t)$ s.t.

$$x'(t) = r\,x(t).$$

[Solution: $C e^{rt}$]

M2

- Linear algebra
  - Vector & matrix notation
  - Vector & matrix algebra
  - Linear eq. systems
  - Applications to <u>linearized</u> eq. systems: e.g.

$$\begin{cases} F(x,y,u(x,y),v(x,y)) = C \\ G(x,y,u(x,y),v(x,y)) = D \end{cases}$$

Find $u'_x(x,y)$

Some notation / terminology

$\mathbb{R}$: the real numbers

$\mathbb{N}$: the natural numbers : $\{1, 2, 3, 4 \ldots\}$

(... except in French, which obeys ISO 31-11 : $\{0, 1, 2, 3 \ldots\}$)

There are also $\mathbb{Z}$ = integers, $\mathbb{Q}$ = rationals

C^n : The functions whose derivatives up to & including order n, exists and are continuous.

($C^n \supseteq C^{n+1}$!)

Intervals: (a, b) does not include a or b

$[a, b)$ includes a, not b

note: $[a, +\infty)$, (a, ∞) . Infinity is not a number

Closed / open set: includes <u>every</u> / resp: <u>no</u> boundary point

Vectors

(M2: 3200/4200 will use this ; M3: M3 will use this)

"Vectors", • to mathematicians:

"objects you can scale and add"

• to M2/M3: ordered n-tuples

(pairs, triplets, quadruples ...)

e.g. $(1, 3, -4, 2.1, -0.4)$ decimal point

separators

or $(\frac{1}{2}, -\frac{1}{3})$ etc.

("ordered": $(1,2) \neq (2,1)$)

Ex: <u>price list</u>

Vector notation, distinguishing them from numbers

$\vec{x}$ common in physics

$\bar{x}$ . . . . non-physics

$\underset{\frown}{x}$ statistics

$\mathbf{x}$ (boldfaced): textbooks

$\mathbb{x}$ ("blackboard bold") mimicking textbooks

"n copies of the real line"

$x \in \mathbb{R}^n$: mathematicians often do not use any particular font (rather, they specify)

Matrices: same, but upper-case.

You are free to choose among these!

(But you are better off being consistent!)

$\bar{x} = (x_1, \ldots, x_n)$

vector　　numbers

Book: $\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}$ are two vectors

Function f of several variables: $f(\bar{x})$
(M3 will <u>use</u> this!)

So: the following are the same:

$f(x,y,z)$　　$f(x_1, x_2, x_3)$　　$f(\bar{x}), \ \bar{x} \in \mathbb{R}^3$

We gather the <u>partial derivatives</u> in one vector:

∇f denotes $(f_1', \ldots, f_n')$.

(the "gradient" of f).

There are row vectors: $(1, 0, -4)$

and column vectors: $\begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix}$

As long as one has not introduced matrices, the choice of row notation vs column notation is just a matter of convenience.

M2: rows until matrices are introduced

M2 from then on, and M3: columns unless otherwise specified

$\to$ ∇f always a row!

3200/4200 etc: follow that course's convention.

Thus far, vectors are <u>bookkeeping</u> tools

We can do calculations too.

"dot"

$$\bar{p} \cdot \bar{q} = \underline{p_1 q_1 + p_2 q_2 + \dots + p_n q_n}$$

price list   quantities bought   = total cost

E.g. $\left(\$1/\text{newspaper},\ \$2/\ell \text{ gasoline},\ -\$1/\text{bag of waste}\right)$

$\cdot \left(4 \text{ newspapers},\ \frac{15}{4}\ \ell \text{ gasoline},\ -2 \text{ bags of waste}\right)$

$= \$4 + \$\frac{7}{2} + \$2 = \underline{\underline{\$9.50}}$

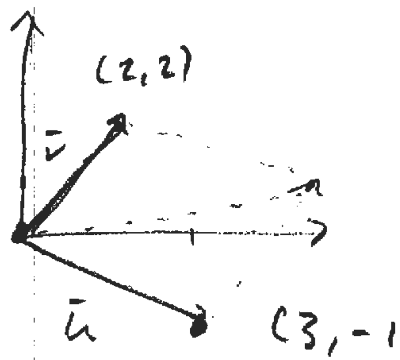
Vector addition: $\bar{u}$ $\bar{v}$

$$\bar{u} = (u_1, \ldots, u_n)$$

$$\bar{v} = (v_1, \ldots, v_n)$$

$$\bar{u} + \bar{v} = (u_1 + v_1, \ldots, u_n + v_n)$$

Geometric



$\bar{u} + \bar{v} = (5,1)$

Length: $\|\bar{v}\| = \sqrt{v_1^2 + \ldots + v_n^2}$ (Pythagoras)

Matrices: stack numbers into an array, e.g.

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \end{pmatrix}$$ two rows, three columns.

Example: covariance matrix

equal, "symmetric" matrix.

$$\begin{pmatrix} \mathrm{var}(X,Y) & \mathrm{covar}(X,Y) \\ \mathrm{covar}(Y,X) & \mathrm{var}(Y) \end{pmatrix}$$

Example: For $f = f(x,y)$ of two variables, we can gather its 2^{nd} derivatives in the so-called Hessian matrix

$$\bar{H} = \bar{H}(x,y) = \begin{pmatrix} f''_{xx}(x,y) & f''_{xy}(x,y) \\ f''_{yx}(x,y) & f''_{yy}(x,y) \end{pmatrix}$$

equal

$$\left(= \begin{pmatrix} \bar{\nabla} \frac{\partial f}{\partial x} \\ \bar{\nabla} \frac{\partial f}{\partial y} \end{pmatrix} \right)$$

We shall use the **determinant** of a square matrix.

For a 2×2, $\bar{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the determinant

is $ad - bc$ (ie: a TIMES d minus c TIMES b)

Example: the Hessian **determinant** is

$$\det \bar{H} = f''_{xx} f''_{yy} - f''_{xy} f''_{yx}$$

Looks familiar?

Put $A = a = f''_{xx}$

$B = b = c = f''_{xy} = f''_{yx}$

$C = d = f''_{yy}$

The second-order cond'n has a connection to statistics:

$\bar{H}$ is a covariance $\longleftrightarrow$ loc. min.

at a stationary point