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ECON3120/4120 Mathematics 2, spring 2007

Problems for Seminar 1, 31 January 2007

1 Consider the function f defined by

$$f(x) = \frac{3-x}{3x-3}$$

(a) Where is $f(x)$ defined? Calculate $f(x)$ for $x = -3$, $x = -1/2$, $x = 1/4$, $x = 3/2$, $x = 3$, and $x = 9$.

(b) Where is $f(x) \leq 0$? Where is $f(x) \leq 1$?

(c) Draw the graph of f and see if your answers to (b) are confirmed.

(d) Define $g(x) = \ln[f(x)]$. Where is $g(x)$ defined? Where is $g(x) > 0$?

2 Use l'Hôpital's rule (or other methods) to find the limits:

(a) $\lim_{x\to 3} \dfrac{3x^2-27}{x-3}$ (b) $\lim_{x\to 0} \dfrac{e^{-3x}-e^{-2x}+x}{x^2}$ (c) $\lim_{x\to\infty} \left(\sqrt{x^2+\frac{1}{2}x}-x\right)$

3 (a) The equation $e^L + KL = Ke^K$ defines L as a differentiable function of K. Find an expression for dL/dK.

(b) If $z = F(u,v,w)$ and $u = f(x,y)$, $v = e^{-x}$, and $w = \ln y$, find an expression for $\partial z/\partial x$ and $\partial z/\partial y$.

4 Find the differential of z expressed in terms of the differentials of u and v:

(a) $z = uv^2$ (b) $z = u^2/v^3$ (c) $z = F(u^2, v^3)$ (d) $z = u^2 - f(u+v)$

5 The following system defines u and v as C^1 functions of x and y around the point $P = (x,y,u,v) = (1,2,1,1)$:

$$u^2 + v^2 = xy$$
$$xu^2 + yv^2 = x + y$$

Differentiate the system. Then find the values of $\partial u/\partial x$, $\partial u/\partial y$, $\partial v/\partial x$ and $\partial v/\partial y$ at the point P.