University of Oslo
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ECON3120/4120 Mathematics 2, spring 2007
Problems for Seminar 3, 14 February 2007

1 MA I: 10.6.1 (d). (EMEA: 9.5.1 (d).)

2 MA I: 10.7.2 (c). (EMEA: 9.6.2 (c).)

3 Problem 63 in the exam problem booklet:

(a) The equation
$$3xe^{xy^2} - 2y = 3x^2 + y^2$$

defines y as a differentiable function of x around the point $(x^*, y^*) = (1, 0)$. Find the slope of the graph at this point by implicit differentiation. What is the linear approximation to y around $x^* = 1$?

(b) In an equilibrium model the following system of equations is studied:

$$pF'(L) - r = 0$$
$$pF(L) - rL - B = 0 \tag{$*$}$$

where F is a twice differentiable function with $F'(L) > 0$ and $F''(L) < 0$. All the variables are positive. Consider r and B as exogenous and p and L as endogenous variables, so that p and L are functions of r and B. Find expressions for $\partial p/\partial r$, $\partial p/\partial B$, $\partial L/\partial r$, and $\partial L/\partial B$ by implicit differentiation.

(c) Determine, if possible, the signs of these partial derivatives. Show, in particular, that $\partial L/\partial r < 0$.

4 Consider the function f defined by $f(x) = x(\ln x)^2$ for all $x > 0$.

(a) Compute $f'(x)$ and $f''(x)$.

(b) Decide where f is increasing and where f is decreasing. Does f have global extreme points?

(c) Find $\displaystyle\int x(\ln x)^2\, dx$.

5 Show that $\displaystyle\int \sqrt{x^2 + 3}\, dx = \frac{1}{2}x\sqrt{x^2 + 3} + \frac{3}{2}\ln\left(x + \sqrt{x^2 + 3}\right) + C$.

Comments: 1, 3(a) and 4 should be relatively straightforward. In 2 you should be bold in choosing a new variable. In the last question in 3(a) you are actually asked to find the tangent to the curve at $(1, 0)$. 3(b) is important. Start by taking total differentials of each equation. The last part of 3(c) may be tricky. In problem 5, think carefully about what you are asked to do.