University of Oslo
Department of Economics

Compulsory term paper 2 in ECON3120/4120 Mathematics 2

Handed out: Thursday 12 April 2007.

To be handed in on Thursday 26 April 2007 before 2:00 pm.

Hand in at the department reception, 12th floor.

Further instructions:

- This term paper is **compulsory**.
- This paper will NOT be given a grade that counts towards your final grade for this course. A possible grade is meant only for your guidance.
- You must use a preprinted front page, which you will find at *http://www.oekonomi.uio.no/info/EMNER/Forside_obl_eng.doc*
- It is important that the term paper is delivered by the deadline (see above). Term papers delivered after the deadline **will not be read or marked.**[*)]
- All term papers must be delivered at the place given above. You must not deliver your term paper to the course teacher or send it by e-mail. If you want to hand in your term paper **before** the deadline, please contact the department reception on the 12th floor.
- If your term paper is not accepted as satisfactory, you will be allowed a new attempt with a very short deadline. If you still do not succeed, you will not be permitted to take the exam in this course. You will then be withdrawn from the exam, so that it will not count as an attempt.

[*)] If you believe that you have good a reason for not meeting the deadline (e.g. illness), you should discuss the matter with your course teacher and seek a formal extension. Normally, an extension will be granted only when there is a good reason backed by supporting evidence (e.g. a medical certificate).

Problem 1

Let $f(x) = \frac{1}{2}x - \frac{1}{4}x^2 + 5\ln(x+2)$.

(a) What is the domain of definition of f? Compute $f'(x)$ and $f''(x)$.

(b) Find the extreme points of f, if any.

(c) How many solutions does $f(x) = 0$ have? Sketch the graph of f.

(d) Compute the integral $\int_0^4 \left(\frac{1}{2}x - \frac{1}{4}x^2 + 5\ln(x+2)\right) dx$.

(Cont.)

Problem 2

(a) Use Lagrange's method to solve the problem

$$\text{maximize} \quad 24x - x^2 + 16y - 2y^2 \quad \text{subject to} \quad x^2 + 2y^2 = 44\,.$$

(b) Suppose we replace the constraint in part (a) by $x^2 + 2y^2 \leq 44$. Write down the necessary Kuhn–Tucker conditions for a point (x, y) to solve this new problem, and find all solutions of these conditions.

Problem 3

For each real number t, define the matrix $\mathbf{A}_t$ as $\mathbf{A}_t = \begin{pmatrix} 1 & 1 & 2 \\ -t & 3 & 2 \\ t & 1 & 2 \end{pmatrix}$.

(a) Compute $|\mathbf{A}_t|$ and $|(\mathbf{A}_2)^3|$.

(b) Show that for a suitable value of s the matrix $\mathbf{B} = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 1/2 & s \\ 2 & -1/4 & -5/4 \end{pmatrix}$ is the inverse of $\mathbf{A}_2$. (It is not a good idea to use the formula for the inverse.)

Problem 4

The equation system

$$\begin{aligned} tx^2 + y &= 2t - \tfrac{1}{2} \\ 2\ln x + 3y &= x + \ln(2y) + t - \tfrac{1}{2} \end{aligned}$$

defines x and y as differentiable functions of t around the point $x = 1$, $y = \frac{1}{2}$, $t = 1$. Find dx/dt and dy/dt at this point.