Supplement to Lecture 3: Confidence interval for the “natural rate of unemployment”. NLS and delta-Method

Ragnar Nymoen

University of Oslo

29 August 2014
The **natural rate of unemployment** is a central concept in macro economics.

In order to estimate the natural rate, we must specify a model where it is a parameter (explicitly or implicitly).

There are many such models, but we consider a **linear Phillips curve model** (PCM) for the purpose of inference.

\[
\pi_t = \beta_1 + \beta_2 U_t + \epsilon_t \tag{1}
\]

where \( \pi_t \) is Norwegian inflation in year \( t \) and \( U_t \) is the unemployment percentage.
PCM natural rate of unemployment II

The natural rate can defined in such a way that it becomes a parameter in (1). Re-write the PCM as

\[ \pi_t = \beta_2 (U_t - \frac{-\beta_1}{\beta_2}) + \varepsilon_t \]

\[ = \beta_2 (U_t - \mathcal{U}^{nat}) + \varepsilon_t \]  \hspace{1cm} (2)

and define

\[ \mathcal{U}^{nat} := \frac{-\beta_1}{\beta_2} \]

as the natural rate of unemployment.

\[ \mathcal{U}^{nat} \] is a parameter in both (1) and (2), though implicit in (1).
(2) is however NOT linear in parameters. To estimate $U^{nat}$ from (2) requires Non-linear Least Squares, (NLS).

However, with the use of the **delta method** we can make inference about $U^{nat}$ by estimating the linear-in parameter model (1)
With annual data from 1981 to 2010 ($T = 30$) we estimate:

$$\hat{\pi}_t = 8.37527 - 1.36632U_t$$

Nat-rate ($\hat{U}^{nat}$) \[\frac{8.37527}{1.36632} = 6.1298\]

IT-rate ($\hat{U}^{it}$) \[\frac{8.37527 - 2.5}{1.36632} = 4.3001\]

Note:

- $U^{it}$ is the “inflation target rate of unemployment”: the "natural rate for $\pi_t = 2.5$, instead of 0"
Inference about the “natural rate”

Use the **delta-method formula**:

\[
\text{var}(\hat{U_{nat}}) = \text{var}(\frac{-\hat{\beta}_1}{\hat{\beta}_2}) \approx \left(\frac{1}{\hat{\beta}_2}\right)^2 \left[ \text{var}(-\hat{\beta}_1) + (\hat{U_{nat}})^2 \text{var}(\hat{\beta}_2) - 2 (\hat{U_{nat}}) \text{cov}(-\hat{\beta}_1, \hat{\beta}_2) \right]
\]

From the estimation: \( \text{cov}(\hat{\beta}_1, \hat{\beta}_2) = -0.66876 \), \( \text{Var}(\hat{\beta}_1) = 2.4043 \), \( \text{Var}(\hat{\beta}_2) = 0.20915 \)

\[
\text{var}(\hat{U_{nat}}) = \sqrt{\left(\frac{1}{-1.36632}\right)^2 \cdot \left[ 2.4043 + (6.1298)^2 \cdot 0.20915 - 2 \cdot (5.73) \cdot 0.66876 \right]}
\]

\[
= 0.53567 \cdot 2.599 = 1.3922
\]

Approximate 95% confidence interval for \( U_{nat} \) is therefore

\[
6.1298 \pm 2 \cdot \sqrt{1.3922} = 5.73 - 2 \cdot 1.1799
\]

or

\[
[3.372\% ; 8.0898\%]
\]

Memo: Direct estimation using the Non Linear Least Squares (NLS) gives:

\[
\text{var}(\hat{U_{nat}}) = 1.052^2 = 1.1067
\]