UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: ECON4160 – Econometrics – Modeling and systems estimation

Date of exam: Friday, May 20, 2005 **Grades will be given: Monday, June 13**

Time for exam: 2:30 p.m. – 5:30 p.m.

The problem set covers 5 pages

Resources allowed:

- All printed and written resources, as well as calculator

Please answer both part I) and part II) of the problem set. Both parts will be given equal weight in the evaluation.

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

Part I)

Enclosed are some results from estimations of the relative demand for different types of labour in the economy. The labour force is classified into three types of labour, according to the level of education. We have observations of relative employment and relative wages from 12 different European countries from the years 1974 to 2001. We do not have observations from each country each year, and the total number of observations of country *x* year is 131.

Model 1 of the appendix reports estimation results from a seemingly unrelated regression model with two equations, explaining the relative demand for two different types of workers:
relempl1= log of relative employment for group 1 relative to group 3,
relempl2= log of relative employment for group 2 relative to group 3.
The unit of observation is country per year. The equation for relempl1 includes a measure of relative wages for group 1:
relwage1=the log of wages for group 1 relative to the wages of group 3,
and similarly the equation for relemp2 includes a measure of relative wages for group 2:
relwage2=the log of wages for group 2 relative to the wage of group 3.

Both relations include in addition the same set of 11 different country dummies (*of which only ctryechp1,ctryechp2 and ctryechp3 are shown in the table*) and 12 country specific time trends (*of which only tnor, tspa and tswe are shown in the table*), as well as a common time trend squared (*t2*). The table also reports the correlation matrix of residuals between the two equations.

Question 1.

Assume that relative wages are exogenous in this model. Set up the stochastic model and describe the assumptions underlying this estimation strategy (the Seemingly Unrelated Regression model). What is the advantage of using SUR rather than separate ordinary least square models (OLS) under these assumptions? Explain. Discuss the usefulness of the Breusch-Pagan test of independence of the residuals, whose test statistic is reported at the bottom of the page, for the choice of modeling strategy.

Question 2.

Model II provides estimation results for a different, but related SUR model. This time the relative wage of *both* groups are included as regressors in both equations. Consider now running separate OLS regressions on the two equations in this model. Based on the information on this page (Model II), indicate what the OLS estimate would be for each of the two relative wage measures, *relwage1* and *relwage2*, in the OLS regression of relempl2. Explain how you arrived at these numbers. In this model, all parameters are allowed to vary freely. What type of parameter restrictions would lead you to prefer SUR rather than OLS?

Question 3.

Assume now that the correlation between the disturbance terms in the two equations is zero. Assume furthermore that the disturbance term, u_2, of the second equation is homoskedastic, [$var(u_{2i})=\sigma_2^2$], but that the variance of the disturbance term, u_1, of the first equation, is proportional to the inverse of the size of the population in the country [$var(u_{1i})=\sigma_1^2/POP_i$]. Suggest an estimation method in this case and compare it to using two separate OLS regressions.

Part II)

Question 4.

Consider the following model:

1) $y_{1i} = a_1 + b_{11}y_{2i} + \gamma_{11}x_{1i} + \gamma_{12}x_{2i} + \gamma_{13}x_{3i} + \gamma_{14}x_{4i} + v_{1i}$, $i=1,...,n$
2) $y_{2i} = a_2 + b_{21}y_{1i} + \gamma_{21}x_{1i} + \gamma_{22}x_{2i} + \gamma_{23}x_{3i} + \gamma_{24}x_{4i} + v_{2i}$, $i=1,...,n$

Discuss assumptions required for the x's to be exogenous in this model. Derive expressions for the expectation and variance, both conditional on the vector x, of the endogenous variables y_1 and y_2 in the model. Suggest exclusion restrictions that would make both equations exactly identifiable. Derive the indirect least square estimators (ILS) of b_{11} and b_{21} in that case.

Question 5.

Consider the following simpler model:

1) $y_{1i} = a_1 + b_{11}y_{2i} + \gamma_{11}x_{1i} + \gamma_{12}x_{2i} + \gamma_{13}x_{3i} + \gamma_{14}x_{4i} + v_{1i}$, $i=1,...,n$
2) $y_{2i} = a_2 + b_{21}y_{1i} + v_{2i}$, $i=1,...,n$

where again the x's are assumed to be exogenous, while y_1 and y_2 are endogenous.

Discuss identification of equation (2). Two researchers suggest the following instrumental variable estimators for b_{21} each:

3) $\beta^{IV1} = M(y_2,x_1)/ M(y_1,x_1)$
4) $\beta^{IV2} = M(y_2,x_2)/ M(y_1,x_2)$

where M(y,x) is the empirical covariance between y and x. Show that both of these estimators are consistent. Still, these two estimators generally provide us with two different estimates. Suggest a statistics that can be used to evaluate if one of the two estimators, (3) or (4), is better than the other. Describe the optimal instrument for the estimation of b_{21} in equation 2. Under what restrictions would the IV-estimator given in equation (3) be the optimal one? Compare this estimator to the ILS estimator in that case.

MODEL I)

```
Seemingly unrelated regression
----------------------------------------------------------------------
Equation          Obs  Parms        RMSE    "R-sq"       chi2        P
----------------------------------------------------------------------
relempl1          131     32    .0390764    0.9975   54832.97   0.0000
relempl2          131     32    .0577976    0.9952   27647.58   0.0000
----------------------------------------------------------------------
------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
relempl1     |
   ctryechp1 |  -1.073731    .480049    -2.24   0.025     -2.01461    -.132852
   ctryechp2 |  -.8686135   .6285343    -1.38   0.167    -2.100518    .3632911
   ctryechp3 |  -3.251651   .4527519    -7.18   0.000    -4.139028   -2.364274
.................
        tnor |   .2425992    .011017    22.02   0.000     .2210062    .2641923
        tspa |   .2614227   .0184553    14.17   0.000      .225251    .2975944
        tswe |   .2352761    .011219    20.97   0.000     .2132873    .2572649
          t2 |  -.0042338   .0002966   -14.27   0.000    -.0048151   -.0036524
    relwage1 |  -1.842812   .2979266    -6.19   0.000    -2.426737   -1.258887
       _cons |  -2.660008    .262904   -10.12   0.000     -3.17529   -2.144725
-------------+----------------------------------------------------------------

relempl2     |
   ctryechp1 |    .056858    .718363     0.08   0.937    -1.351108    1.464824
   ctryechp2 |  -2.946798   .9518112    -3.10   0.002    -4.812314   -1.081282
   ctryechp3 |  -4.111755   .7272126    -5.65   0.000    -5.537065   -2.686444
................
        tnor |   .1385451   .0160861     8.61   0.000     .1070169    .1700733
        tspa |   .1508126   .0276108     5.46   0.000     .0966965    .2049288
        tswe |   .1237072   .0164715     7.51   0.000     .0914237    .1559907
          t2 |  -.0020179   .0004366    -4.62   0.000    -.0028737   -.0011621
    relwage2 |  -5.113312   1.043669    -4.90   0.000    -7.158865   -3.067759
       _cons |   .1531386    .410832     0.37   0.709    -.6520773    .9583545
------------------------------------------------------------------------------
Correlation matrix of residuals:
          relempl1  relempl2
relempl1    1.0000
relempl2    0.7421    1.0000
Breusch-Pagan test of independence: chi2(1) =    72.134, Pr = 0.0000
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MODEL II)

Seemingly unrelated regression

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
relempl1	131	33	.0382953	0.9976	54854.26	0.0000
relempl2	131	33	.0549356	0.9957	30164.95	0.0000

	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
relempl1						
ctryechp1	-.8623758	.5687173	-1.52	0.129	-1.977041	.2522897
ctryechp2	-1.256032	.6450218	-1.95	0.052	-2.520251	.0081874
ctryechp3	-3.857581	.5194508	-7.43	0.000	-4.875686	-2.839476
....						
tnor	.2495834	.0114287	21.84	0.000	.2271836	.2719833
tspa	.2589843	.0185685	13.95	0.000	.2225907	.2953779
tswe	.2414091	.0116127	20.79	0.000	.2186487	.2641696
t2	-.0043987	.0003027	-14.53	0.000	-.0049919	-.0038055
relwage1	-2.631374	.588853	-4.47	0.000	-3.785505	-1.477243
relwage2	-.1363279	1.377138	-0.10	0.921	-2.835469	2.562813
_cons	-2.114946	.31238	-6.77	0.000	-2.7272	-1.502693
relempl2						
ctryechp1	1.327777	.8158387	1.63	0.104	-.2712376	2.926791
ctryechp2	-3.409934	.9252992	-3.69	0.000	-5.223487	-1.59638
ctryechp3	-4.73703	.7451646	-6.36	0.000	-6.197526	-3.276534
....						
tnor	.1596685	.0163948	9.74	0.000	.1275353	.1918016
tspa	.1403827	.026637	5.27	0.000	.0881752	.1925903
tswe	.1433452	.0166587	8.60	0.000	.1106948	.1759957
t2	-.0024761	.0004342	-5.70	0.000	-.0033271	-.0016251
relwage2	-1.504857	1.975538	-0.76	0.446	-5.376839	2.367126
relwage1	-2.90679	.8447238	-3.44	0.001	-4.562418	-1.251162
_cons	.9463941	.4481166	2.11	0.035	.0681017	1.824686

Correlation matrix of residuals:

	relempl1	relempl2
relempl1	1.0000	
relempl2	0.7770	1.0000

Breusch-Pagan test of independence: chi2(1) = 79.079, Pr = 0.0000