## UNIVERSITY OF OSLO DEPARTMENT OF ECONOMICS

Exam: ECON4160 - Econometrics - Modeling and systems estimation
Date of exam: Thursday, May 24, 2007
Grades are given: Thursday, June 14
Time for exam: 02:30 p.m. - 05:30 p.m.
The problem set covers 10 pages (incl. cover sheet)
Resources allowed:

- All written and printed resources, as well as calculator

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

# ECON 4160: ECONOMETRICS - <br> MODELLING AND SYSTEMS ESTIMATION 

PROBLEM SET, EXAM SPRING 2007

PROBLEM 1 (weight: $40 \%$ )
A relationship is often assumed to exist between finished goods inventories (in Norwegian: lager av ferdigvarer) and sales of manufactured commodities, but economists do not agree on how they are related. Ideally, for any such good, the difference between production and sales should equal the increase in the inventories. To explore this issue we have collected a data set which contains these two variables and a few others. The data set consists of seasonally adjusted quarterly data from the US, in billions of 2000 dollars, from 1982:1 to 2001:4 ( $T=80$ observations) for the following four variables and their one-quarter differences:

```
HRAW = Raw material inventories for the manufacturing sector
HWIP = Work in progress inventories for the manufacturing sector
HFIN = Finished goods inventories for the manufacturing sector
SMAN = Real manufacturing sales
DHRAW = One-quarter difference in HRAW
DHWIP = One-quarter difference in HWIP
DHFIN = One-quarter difference in HFIN
DSMAN = One-quarter difference in SMAN
```

Estimation results, from PcGive, is given at the end of the problem.
(1A):
(a) Consider first the two Ordinary Least Squares (OLS) regressions in EQ(1.1) and $\mathrm{EQ}(1.2)$. Could you explain why they have the same $R^{2}$ ?
(b) The estimate of the coefficient of HFIN in EQ(1.2) is not very far from the inverse of the estimate of the coefficient of SMAN in $\mathrm{EQ}(1.1)$. On the other hand, the gap between the corresponding coefficient estimates in the differenced equations $\mathrm{EQ}(1.3)$ and $\mathrm{EQ}(1.5)$ is much larger. Give a brief explanation of this.
(1B):
It has been suggested, as a way of examining whether SMAN or HFIN should be treated as exogenous, to compute (i) the correlation coefficient between SMAN and the residuals from EQ(1.1) and (ii) the empirical correlation coefficient between HFIN and the residuals from $\mathrm{EQ}(1.2)$ to see how they relate to the zero correlation assumption between disturbances and regressors in a well-specified classical OLS regression equation with stochastic regressors. Comment briefly on this suggestion.

## (1C):

By comparing $\mathrm{EQ}(1.1)$ with $\mathrm{EQ}(1.3)$ and $\mathrm{EQ}(1.4)$, we note that when transforming the equation between HFIN and SMAN from levels to differences, the Durbin-Watson statistic (DW) is substantially increased. Explain this, perform the Durbin-Watson tests and state your conclusion. An extract from a table for the Durbin-Watson critical values is given below.

> DW $\mathbf{5 \%}$ Critical Values (dL,dU).
> $T=$ No. of obs.; $K=$ No. of coef. (incl. intercept)

| T | K | dL | dU | T | K | dL | dU |
| ---: | :---: | :---: | :---: | ---: | ---: | ---: | :---: |
| 73 | 2 | 1.59243 | 1.64788 | 77 | 2 | 1.60361 | 1.65614 |
| 73 | 3 | 1.56446 | 1.67681 | 77 | 3 | 1.57710 | 1.68348 |
| 73 | 4 | 1.53599 | 1.70667 | 77 | 4 | 1.55015 | 1.71166 |
| 74 | 2 | 1.59530 | 1.65001 | 78 | 2 | 1.60626 | 1.65812 |
| 74 | 3 | 1.56772 | 1.67852 | 78 | 3 | 1.58010 | 1.68509 |
| 74 | 4 | 1.53966 | 1.70793 | 78 | 4 | 1.55351 | 1.712877 |
| 75 | 2 | 1.59813 | 1.65209 | 79 | 2 | 1.60887 | 1.66006 |
| 75 | 3 | 1.57091 | 1.68020 | 79 | 3 | 1.58304 | 1.68667 |
| 75 | 4 | 1.54323 | 1.70920 | 79 | 4 | 1.55679 | 1.71407 |
| 76 | 2 | 1.60090 | 1.65413 | 80 | 2 | 1.61143 | 1.66197 |
| 76 | 3 | 1.57404 | 1.68185 | 80 | 3 | 1.58592 | 1.68823 |
| 76 | 4 | 1.54673 | 1.71043 | 80 | 4 | 1.56001 | 1.71526 |

(1D):
(a) There is reason to claim that neither SMAN nor HFIN is exogenous, but determined jointly with other variables in a multi-equation model. If this is true, what would you say about the properties of the estimates in $\mathrm{EQ}(1.1)-\mathrm{EQ}(1.6)$ ? Explain, with this in mind, how you would interpret the printouts in $\mathrm{EQ}(1.7)$ and $\mathrm{EQ}(1.8)$.
(b) Can you from the printout in $\mathrm{EQ}(1.7)-\mathrm{EQ}(1.10)$ draw conclusions about the quality of DHRAW and DHWIP as instruments for DHFIN and DSMAN.
(c) When OLS in $\mathrm{EQ}(1.5)-\mathrm{EQ}(1.6)$ is replaced with IVE in $\mathrm{EQ}(1.7)-\mathrm{EQ}(1.8)$, the estimated coefficients of DHFIN increase. The values of sigma and RSS also increase. Do you find this reasonable?
(d) Would you, when performing IVE estimation, recommend that the equation between DSMAN and DHFIN is specified with the latter as left-hand side variable and the former as right-hand side variable, rather than the opposite, as in $\mathrm{EQ}(1.8)$ ? State briefly the reasons for your answers.

## PCGIVE PRINTOUTS FOR PROBLEM 1

EQ(1.1) Modelling HFIN by OLS. The estimation sample is: 1982(1) to 2001(4)

|  | Coefficient | Std.Error | t-value | t-prob Part.R~2 |  |
| :--- | ---: | :--- | ---: | ---: | ---: |
| Constant | 13.7841 | 3.493 | 3.95 | 0.000 | 0.1665 |
| SMAN | 0.428138 | 0.01247 | 34.3 | 0.000 | 0.9379 |
|  |  |  |  |  |  |
| sigma | 4.88443 | RSS |  |  |  |
| R^2 | 0.937929 | F $(1,78)=$ | 1179 | $[0.000] * *$ |  |
| log-likelihood | -239.387 | DW |  | 0.171 |  |
| no. of observations | 80 | no. of parameters | 2 |  |  |
| mean(HFIN) | 132.215 | var(HFIN) | 374.755 |  |  |

EQ(1.2) Modelling SMAN by OLS. The estimation sample is: 1982(1) to 2001(4)

|  | Coefficient | Std.Error | t-value | t-prob Part.R~2 |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Constant | -13.0273 | 8.527 | -1.53 | 0.131 | 0.0291 |
| HFIN | 2.19072 | 0.06381 | 34.3 | 0.000 | 0.9379 |
|  |  |  |  |  |  |
| sigma | 11.0488 | RSS |  |  |  |
| R^2 | 0.937929 | F $(1,78)=$ | 1179 | $[0.000] * *$ |  |
| log-likelihood | -304.688 | DW |  | 0.172 |  |
| no. of observations | 80 | no. of parameters | 2 |  |  |
| mean(SMAN) | 276.619 | var (SMAN) | 1917.56 |  |  |

$* * * * * * * * * * * * * * * * * * * * * * * * * * * * *$

EQ(1.3) Modelling DHFIN by OLS. The estimation sample is: 1982(2) to 2001(4)

|  | Coefficient | Std.Error | t-value | t-prob Part.R~2 |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Constant | 0.615119 | 0.1797 | 3.42 | 0.001 | 0.1321 |
| DSMAN | 0.0534312 | 0.04514 | 1.18 | 0.240 | 0.0179 |
|  |  |  |  |  |  |
| sigma | 1.47532 | RSS |  | 167.595971 |  |
| R^2 | 0.0178742 | F $(1,77)=$ | 1.401 | $[0.240]$ |  |
| log-likelihood | -141.804 | DW | 1.39 |  |  |
| no. of observations | 79 | no. of parameters | 2 |  |  |
| mean(DHFIN) | 0.696646 | var (DHFIN) | 2.16008 |  |  |

EQ(1.4) Modelling DHFIN by OLS. The estimation sample is: 1982(2) to 2001(4)

|  | Coefficient | Std.Error | t-value | t-prob Part.R~2 |  |
| :--- | ---: | :--- | ---: | ---: | ---: |
| DSMAN | 0.112638 | 0.04446 | 2.53 | 0.013 | 0.0760 |
|  |  |  |  |  |  |
| sigma | 1.57341 | RSS |  | 193.097317 |  |
| log-likelihood | -147.399 | DW | 1.31 |  |  |
| no. of observations | 79 | no. of parameters | 1 |  |  |
| mean(DHFIN) | 0.696646 | var(DHFIN) | 2.16008 |  |  |

EQ(1.5) Modelling DSMAN by OLS. The estimation sample is: 1982(2) to 2001(4)

|  | Coefficient | Std.Error | t-value | t-prob Part. $\mathrm{R}^{\wedge} 2$ |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Constant | 1.29278 | 0.4596 | 2.81 | 0.006 | 0.0932 |
| DHFIN | 0.334528 | 0.2826 | 1.18 | 0.240 | 0.0179 |
|  |  |  |  |  |  |
| sigma | 3.69152 | RSS |  | 1049.30278 |  |
| R^2 | 0.0178742 | F $(1,77)=$ | 1.401 | [0.240] |  |
| log-likelihood | -214.26 | DW |  | 1.27 |  |
| no. of observations | 79 | no. of parameters | 2 |  |  |
| mean(DSMAN) | 1.52582 | var(DSMAN) | 13.524 |  |  |
| $* * * * * * * * * * * * * * * * * * * * * * * * * * * * *$ |  |  |  |  |  |

EQ(1.6) Modelling DSMAN by OLS. The estimation sample is: 1982(2) to 2001(4)

|  | Coefficient | Std.Error | t-value | t-prob Part.R~2 |  |
| :--- | ---: | :--- | ---: | ---: | ---: |
| DHFIN | 0.674971 | 0.2664 | 2.53 | 0.013 | 0.0760 |
|  |  |  |  |  |  |
| sigma | 3.85159 | RSS |  | 1157.11119 |  |
| log-likelihood | -218.123 | DW | 1.26 |  |  |
| no. of observations | 79 | no. of parameters | 1 |  |  |
| mean(DSMAN) | 1.52582 | var (DSMAN) | 13.524 |  |  |

EQ(1.7) Modelling DSMAN by IVE. The estimation sample is: 1982(2) to 2001(4)


EQ(1.8) Modelling DSMAN by IVE. The estimation sample is: 1982(2) to 2001(4)


EQ(1.9) Modelling DHFIN by OLS. The estimation sample is: 1982(2) to 2001(4)

|  | Coefficient | Std.Error | t-value | t-prob Part.R~2 |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Constant | 0.366541 | 0.1414 | 2.59 | 0.011 | 0.0812 |
| DHRAW | 0.505126 | 0.09277 | 5.44 | 0.000 | 0.2806 |
| DHWIP | 0.196277 | 0.08548 | 2.30 | 0.024 | 0.0649 |
|  |  |  |  |  |  |
| sigma | 1.17377 | RSS |  |  |  |
| R^2 | .386403 | F $(2,76)=$ | 23.93 | $[0.000] * *$ |  |
| log-likelihood | -123.224 | DW |  | 2.07 |  |
| no. of observations | 79 | no. of parameters | 3 |  |  |
| mean(DHFIN) | 0.696646 | var(DHFIN) | 2.16008 |  |  |

EQ(1.10) Modelling DSMAN by OLS. The estimation sample is: 1982(2) to 2001(4)

|  | Coefficient | Std.Error | t-value | t-prob Part.R~2 |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Constant | 1.26427 | 0.4325 | 2.92 | 0.005 | 0.1011 |
| DHRAW | 0.270277 | 0.2837 | 0.953 | 0.344 | 0.0118 |
| DHWIP | 0.526503 | 0.2614 | 2.01 | 0.048 | 0.0507 |
|  |  |  |  |  |  |
| sigma | 3.58988 | RSS |  | 979.429648 |  |
| R^2 | 0.083274 | F $(2,76)=$ | $3.452[0.037] *$ |  |  |
| log-likelihood | -211.538 | DW |  | 1.38 |  |
| no. of observations | 79 | no. of parameters | 3 |  |  |
| mean(DSMAN) | 1.52582 | var (DSMAN) | 13.524 |  |  |

END OF PRINTOUTS FOR PROBLEM 1

## PROBLEM 2 (weight: $30 \%$ )

Consider an econometric two-equation model with equations of the form:

$$
\begin{equation*}
y=a+b x \quad+u \tag{1}
\end{equation*}
$$

$$
\begin{equation*}
x=c+d y+e z+v \tag{2}
\end{equation*}
$$

where $(y, x, z)$ are variables, $(a, b, c, d, e)$ are constants, $(u, v)$ are disturbances with (unknown) variances $\sigma_{u u}>0, \sigma_{v v}>0$ and covariance $\sigma_{u v}$. We are in particular interested in estimating $b$ consistently.

Specify which variables are exogenous and endogenous, and explain whether $b$ is identified. If $b$ is identified, how you would estimate it in the following six cases. Answer very briefly, in only one or two sentences in each case.
(2A): $(y, x, z)$ are all observable;
( $a, b, c, d, e$ ) are unknown;
$\operatorname{cov}(z, u)=\operatorname{cov}(z, v)=0$;
$\sigma_{u v}$ is unknown.
(2B): $(y, x, z)$ are all observable; $e=0,(a, b, c, d)$ are unknown; $\operatorname{cov}(z, u)=\operatorname{cov}(z, v)=0$; $\sigma_{u v}$ is unknown.
(2C): $(y, x, z)$ are all observable; $d=0,(a, b, c, e)$ are unknown; $\operatorname{cov}(z, u)=\operatorname{cov}(z, v)=0$; $\sigma_{u v}=0$.
(2D): $(y, z)$ are observable, $x$ is not observable; $d=0,(a, b, c, e)$ are unknown; $\operatorname{cov}(z, u)=\operatorname{cov}(z, v)=0$; $\sigma_{u v}=0$,
$(2 \mathbf{E}):(y, z)$ are observable, $x$ is not observable; $c=d=0, e=1,(a, b)$ are unknown;
$\operatorname{cov}(z, u)=\operatorname{cov}(z, v)=0$;
$\sigma_{u v}=0$
(2F): $(y, z)$ are observable, $x$ is not observable;
$c=d=0, e=1,(a, b)$ are unknown;
$\operatorname{cov}(x, u)=\operatorname{cov}(x, v)=0 ;$
$\sigma_{u v}=0$

PROBLEM 3 (weight: $30 \%$ )
We are interested in examining how females' decisions work or not depends on age, education, work experience and some other socioeconomic variables. The data set - from 1975 for $n=753$ females in the US - contains the following 8 variables:

```
DUMW = Dummy variable = 1 if female worked in 1975, else 0
AGE = Female's age, in years
AGESQ = Female's age squared
EDU = Female's educational attainment, in years
WEXP = Female's previous labor market experience, in years
FAEDU = Father's educational attainment, in years
MOEDU = Mother's educational attainment, in years
CIT = Dummy variable = 1 if female lives in a large city, else 0
```

The vector $\boldsymbol{x}=[$ AGE, AGESQ, EDU, WEXP, FAEDU, MOEDU, CIT] $]$, contains the variables to be treated as exogenous in the analysis below.

## (3A):

Estimation results from OLS regression of DUMW on $\boldsymbol{x}$ is given in EQ(3.1) in the printout. Explain what you can conclude about the effects of a one year longer education period and a one year longer working experience on the females' propensity to work.
(3B):
(a) Logit and Probit models are used more frequently than linear regression models in analyzing individuals' discrete choice. Logit and Probit estimation results relating to female labour market responses are given in $\mathrm{CS}(3.2)$ and $\mathrm{CS}(3.3)$ in the printouts. Interpret these results, and in particular explain what you conclude about the effect on the females' propensity to work of
(i) a one year increase in the education period,
(ii) a one year increase in the working experience.
(b) Could you explain why the Logit estimates are substantially higher (in absolute value) than the corresponding Probit estimates, even if the underlying problem is the same?
(3C):
About $57 \%$ of the females in the sample are working. Take $\bar{P}=0.5$ as a rough estimate of the probability of being employed. Can you - from the Logit results - estimate how the probability that a female will be working is affected by a one year increase in
(i) her education period,
(ii) her period of past working experience,
(iii) her father's education period and
(iv) her mother's education period?

## PCGIVE PRINTOUTS FOR PROBLEM 3

EQ(3.1) Modelling DUMW by OLS-CS. The estimation sample is: 1 to 753

|  | Coefficient | Std.Error | t-value | t-prob Part.R~2 |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Constant | -0.903595 | 0.4827 | -1.87 | 0.062 | 0.0047 |
| AGE | 0.0525896 | 0.02231 | 2.36 | 0.019 | 0.0074 |
| AGESQ | -0.000745911 | 0.0002573 | -2.90 | 0.004 | 0.0112 |
| EDU | 0.0289180 | 0.008401 | 3.44 | 0.001 | 0.0157 |
| WEXP | 0.0247053 | 0.002176 | 11.4 | 0.000 | 0.1475 |
| FAEDU | 0.00448683 | 0.006211 | 0.722 | 0.470 | 0.0007 |
| MOEDU | -0.00161627 | 0.005842 | -0.277 | 0.782 | 0.0001 |
| CIT | -0.0187213 | 0.03509 | -0.534 | 0.594 | 0.0004 |
|  |  |  |  |  |  |
| sigma | 0.448639 | RSS |  | 149.951028 |  |
| R^2 | 0.188259 | F(7,745) | 24.68 | $[0.000] * *$ |  |
| log-likelihood | -460.881 | DW |  | 0.351 |  |
| no. of observations | 753 | no. of parameters |  |  |  |
| mean(DUMW) | 0.568393 | var (DUMW) |  | 0.245322 |  |

CS(3.2) Modelling DUMW by Logit. The estimation sample is: 1 to 753

|  | Coefficient | Std.Error | t-value | t-prob |
| :--- | ---: | ---: | ---: | ---: |
| Constant | -7.60072 | 2.497 | -3.04 | 0.002 |
| AGE | 0.287136 | 0.1163 | 2.47 | 0.014 |
| AGESQ | -0.00399504 | 0.001362 | -2.93 | 0.003 |
| EDU | 0.148748 | 0.04359 | 3.41 | 0.001 |
| WEXP | 0.125311 | 0.01290 | 9.72 | 0.000 |
| FAEDU | 0.0208852 | 0.03126 | 0.668 | 0.504 |
| MOEDU | -0.00911546 | 0.02897 | -0.315 | 0.753 |
| CIT | -0.0947049 | 0.1773 | -0.534 | 0.593 |


| log-likelihood | -436.246857 | no. of states | 2 |
| :--- | ---: | :--- | ---: |
| no. of observations | 753 | no. of parameters | 8 |
| mean(DUMW) | 0.568393 | var(DUMW) | 0.245322 |

BFGS estimation (eps1=0.0001; eps2=0.005) : Strong convergence

|  | Count | Frequency Probability | loglik |  |
| :--- | ---: | ---: | ---: | ---: |
| State 0 | 325 | 0.43161 | 0.43160 | -229.5 |
| State 1 | 428 | 0.56839 | 0.56840 | -206.8 |
| Total | 753 | 1.00000 | 1.00000 | -436.2 |

CS(3.3) Modelling DUMW by Probit. The estimation sample is: 1 to 753

|  | Coefficient | Std.Error | t-value | t-prob |
| :--- | ---: | ---: | ---: | ---: |
| Constant | -4.55365 | 1.485 | -3.07 | 0.002 |
| AGE | 0.172773 | 0.06895 | 2.51 | 0.012 |
| AGESQ | -0.00240777 | 0.0008037 | -3.00 | 0.003 |
| EDU | 0.0888289 | 0.02588 | 3.43 | 0.001 |
| WEXP | 0.0740782 | 0.007302 | 10.1 | 0.000 |
| FAEDU | 0.0131296 | 0.01876 | 0.700 | 0.484 |
| MOEDU | -0.00610996 | 0.01744 | -0.350 | 0.726 |
| CIT | -0.0440375 | 0.1061 | -0.415 | 0.678 |


| log-likelihood | -436.702481 | no. of states | 2 |
| :--- | ---: | :--- | ---: |
| no. of observations | 753 | no. of parameters | 8 |
| mean(DUMW) | 0.568393 | var(DUMW) | 0.245322 |
| BFGS estimation (eps1=0.0001; | eps2=0.005) : Strong convergence |  |  |


|  | Count | Frequency | Probability | loglik |
| :--- | ---: | ---: | ---: | ---: |
| State 0 | 325 | 0.43161 | 0.43167 | -229.6 |
| State 1 | 428 | 0.56839 | 0.56833 | -207.1 |
| Total | 753 | 1.00000 | 1.00000 | -436.7 |

END OF PRINTOUTS FOR PROBLEM 3

