## UNIVERSITY OF OSLO DEPARTMENT OF ECONOMICS

## Exam: ECON4160 – Econometrics – Modeling and Systems Estimation

Date of exam: Friday, May 22, 2009

Grades are given: Tuesday, June 16

Time for exam: 09:00 a.m. - 12:00 noon

The problem set covers 6 pages

Resources allowed:

• All written and printed resources, in addition to calculator, are allowed.

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

Exam in: ECON 4160: Econometric Modelling and System Estimation

**Day of exam:** 25 May, 2009

Time of day: 9:00-12:00

This is a 3 hour school exam.

## Guidelines:

In the grading, each of the 5 questions will count 20 %. For reference, some relevant critical values in the F distribution are given at the end of the question set.

- 1. Assume that  $y_1, y_2, \dots, y_n$  are *n* stochastic variables. Assume that, based on economic theory, a hypothesis is formulated saying that "y depends on x".
  - (a) Explain how this hypothesis can be tested with the use of equation

(1) 
$$y_i = \beta_1 + \beta_2 x_i + \varepsilon_i, \ i = 1, 2, 3, ..., n,$$

and ordinary least squares (OLS) estimation.

- (b) Does it matter for your answer to 1(a) whether the explanatory variable  $x_i$  is deterministic or stochastic? Explain briefly.
- (c) Assume that a fellow student remarks critically that inference based on OLS estimation of (1) is misleading, because, as she says: "the disturbances in equation (1) may be heteroscedastic". How would you respond to this critique?
- 2. Assume that we have a sample of observations of total income (not total consumption expenditure) for n households. The income variable is denoted  $x_i$ (i = 1, 2, ..., n). We also have observations of J commodity expenditures for the n households. We denote the expenditures by  $y_{ji}$  (j = 1, 2, ..., J; i = 1, 2, ..., n). We want to estimate the parameters  $(\alpha_{j0}, \alpha_{j1})$  in the J equations:

(2)  

$$y_{1i} = \alpha_{10} + \alpha_{11}x_i + \varepsilon_{1i}, i = 1, 2, ..n.$$

$$y_{2i} = \alpha_{20} + \alpha_{21}x_i + \varepsilon_{2i}, i = 1, 2, ..n.$$

$$\vdots$$

$$y_{Ji} = \alpha_{J0} + \alpha_{J1}x_i + \varepsilon_{Ji}, i = 1, 2, ..n.$$

where  $\varepsilon_{1i}, \ldots, \varepsilon_{Ji}$  denote the disturbance terms.

- (a) Assume that the disturbances in each equation of (2) are independent and identically distributed, but that there are correlations between the disturbances in the different equations. Give one or more reasons for why this kind of correlation is relevant for the "expenditure system" in (2).
- (b) Would you choose the SURE or the OLS estimator to estimate the parameters (α<sub>j0</sub>, α<sub>j1</sub>)? Explain your answer.

(c) Assume that  $x_i$  is an unobservable variable. Instead of  $x_i$ , we observe  $x_i^*$  which is defined by the equation

$$(3) x_i^* = x_i + v_i,$$

where  $v_i$  is a stochastic variable that satisfies the classical disturbance assumptions, and is independent of both  $\varepsilon_{ji}$  and  $x_i$ . How would this change in model specification affect your choice of estimation method?

3. Explain how you would estimate the parameters *in each equation* of the following model.

(4) 
$$y_{1t} + \beta_{12}y_{2t} + \gamma_{11} + \gamma_{12}x_{2t} = \varepsilon_{1t}$$
$$y_{2t} + \gamma_{21} + \gamma_{23}x_{3t} = \varepsilon_{2t}$$
$$\beta_{32}y_{2t} + y_{3t} + \gamma_{31} + \gamma_{33}x_{3t} = \varepsilon_{3t}$$

where  $y_{1t}$ ,  $y_{2t}$  and  $y_{3t}$  are endogenous variables.

HINT: Take care to base your answer on a complete econometric specification of the model.

4. In this question we discuss the empirical relationship between the money market interest rate, denoted  $R_t$ , and the banks' loan interest rate in the Norwegian Central Bank, denoted  $CBR_t$ . The subscript t denotes time period. The initial hypothesis is that in an inflation targeting regime,  $R_t$  is a linear function of  $CBR_t$ , as in

(5) 
$$R_t = \beta_1 + \beta_2 C B R_t + \varepsilon_t$$

where  $\beta_1$  and  $\beta_2$  are parameters, and  $\varepsilon_t$  is a disturbance term. Note that both  $R_t$  and  $CBR_t$  are measured as percentages, so their range of variation can be taken to be between 0 % and 100 %.

Formally, the mandate of the Norwegian Central Bank changed to inflation targeting on 29 March 2001, but the general understanding is that in practice inflation targeting was in operation long before. In order to test that belief, we estimate (5) on three different samples of quarterly data. The first sample is from 1997, first quarter, to 2001, first quarter (i.e., 1997q1-2001q1). The second sample is 2001q2-2008q4. Finally we estimate (5) on the full sample; 1997q1-2008q4. Below we have listed sample sizes (denoted  $T_1, T_2$  and T), and the corresponding sum of squared OLS residuals, denoted RSS.

(a) Use this information to calculate a Chow-test statistic which is relevant for the hypothesis that the parameters of (5) are constant. What is your conclusion about the stability of (5)?

A colleague proposes to extend equation (5) with a variable  $\Delta RW_t$  which is the change in the foreign money market interest rate,  $RW_t$ . (This interest rate is also measured in percentages). Her theory is that if the foreign interest rate is growing ( $\Delta RW_t > 0$ ), the difference between  $R_t$ and  $CBR_t$  will increase, and that the difference will become smaller if  $\Delta RW_t$  is negative. Another colleague points out that a relationship like (5) cannot be expected to hold during the financial crisis that hit the Norwegian money market in the fourth quarter of 2008. Based on this insight, an augmented model is estimated with the results reported in equation (6).  $d2008q4_t$  is a dummy which is one in 2008q4, and zero otherwise. The numbers in round brackets below the parameter estimates are the corresponding standard errors.

(6)  

$$R_{t} = \begin{array}{c} 0.2155 \\ (0.0683) \end{array} + \begin{array}{c} 1.024 \\ (0.0137) \end{array} \begin{array}{c} CBR_{t} + \begin{array}{c} 0.4748 \\ (0.082) \end{array} \Delta RW_{t} \\ + \begin{array}{c} 1.606 \\ (0.185) \end{array} \\ OLS, T = 48 \ \text{(Sample is 1997q1-2008q4)} \\ RSS = 1.31497. \end{array}$$

- (b) Based on the evidence above, test the joint hypothesis that the coefficients of  $\Delta RW_t$  and  $d2008q_t$  are zero. You can base you answer on the assumption that the disturbances have the classical properties known from the textbooks.
- (c) Test also the hypothesis that a unit increase in  $CBR_t$  leads to a unit increase in  $R_t$ .
- 5. The empirical relationship between the central bank's policy interest rate  $CBR_t$  and macroeconomic variables is also of interest. In an inflation targeting regime, the single most important variable is probably the annual inflation rate. We denote this variable by  $INF_t$ , and measure it as the 4 quarter percentage change in the consumer price index (adjusted for taxes and energy). The Norwegian Central Bank has communicated that (crisis situations aside) it will change the policy interest rate gradually and in small steps. This suggests that  $CBR_t$  can be modelled by an equation that includes  $CBR_{t-1}$  and  $INF_t$  as explanatory variables, and with  $d2008q4_t$  to take account of the credit crises. Equation (7) gives the results from OLS estimation:

(7)  

$$CBR_{t} = 0.08755 + 0.6939 CBR_{t-1} + 0.7752 INF_{t}$$

$$- 1.98 d2008q4_{t}$$

$$OLS, T = 48 (Sample is 1997q1-2008q4)$$

$$R^{2} = 0.93, \quad \hat{\sigma}_{OLS} = 0.54$$

(a) Do you find the coefficient estimates in (7) statistically significant, and theoretically interpretable? (You can take as a given thing that the disturbances are homoscedastic and that there is no autocorrelation.  $\mathbb{R}^2$  is the multiple correlation coefficient.  $\hat{\sigma}_{OLS}$  denotes the estimated residual standard error.

- (b) One of your colleagues claims that  $CBR_{t-1}$  is not an exogenous variable in (7). What would his argument be?
- (c) A senior colleague says that in practice, you can regard  $CBR_{t-1}$  as exogenous, since the bias is probably of no great importance. She says that it may be more important to look into the possibility that  $INF_t$  is an endogenous variable, but she expects that you can spell out the argument in more detail. What would your answer to this challenge be?
- (d) Together with your colleagues, you decide that  $U_t$ , the unemployment percentage, its lagged value,  $U_{t-1}$ , and lagged inflation  $INF_{t-1}$  are valid instruments for  $INF_t$ , and you therefore re-estimate the equation by 2SLS. The results are given in (8).

$$CBR_{t} = 0.05501 + 0.9042 INF_{t} + 0.652 CBR_{t-1}$$

$$(8) \qquad -2.079 \ d2008q4_{t}$$

$$2SLS, T = 48 (Sample is 1997q1-2008q4)$$
 $\hat{\sigma}_{2SLS} = 0.5, \ \mathsf{R}_{first-step}^{2} = 0.900$ 

Do you agree that this result is logically consistent with the policy communicated by the Norwegian Central Bank: namely that it primarily looks at the underlying inflation *tendency*, rather than at the actual rate of inflation, when the interest rate is decided? Explain. ( $\hat{\sigma}_{2SLS}$  is the estimated residual standard error in (8), and  $R_{first-step}^2$  is the  $R^2$  of the  $INF_t$  equation in the first step of the 2SLS procedure).

(e) Finally, you want to investigate whether the central bank is "forward-looking" in its interest rate setting policy. Explain why one way of doing this is to replace  $INF_t$  by  $INF_{t+1}$ , and estimate with  $U_t$ ,  $U_{t-1}$ , and  $INF_{t-1}$  as instruments for  $INF_{t+1}$ .

The results of this final equation is:

$$CBR_{t} = 0.8686 INF_{t+1} + 0.7007 CBR_{t-1} - 0.1275_{(0.163)}$$

$$- 2.03 d2008q4_{t}$$

$$2SLS, T = 48 (Sample is 1997q1-2008q4)$$

$$\hat{\sigma}_{2SLS} = 0.53, R_{first-step}^{2} = 0.810$$

On the basis of the above evidence, what is your conclusion about how forward-looking monetary policy has been?

(f) The  $CBR_t$  interest rate used above is the average interest rate in each quarter. Suppose instead that the dependent variable is *qualitative*, with 0 representing a decision (by the central bank) to lower the interest rate, 1 representing a decision to keep the policy interest rate constant, and 2 representing an increased central bank interest rate. Endogeneity issues aside, would you choose a linear regression model in this case?

Reference: 5% Critical values of  $F(v_1, v_2)$ 

			$v_1$	
	$v_2$	1	2	
:	:			
	44	4.05	3.15	
	:			