

UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON4160 – Econometrics – modeling and systems estimation**

Date of exam: Tuesday, November 30, 2010 **Grades will be given: December 21**

Time for exam: 2:30 p.m. – 5:30 p.m.

The problem set covers 4 pages

Resources allowed:

- Open book exam. All written and printed resources, as well as calculator, are allowed.

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

Please do not forget the periodic course evaluation for ECON4160, which you will find on the website for the course. The deadline is December 19!

**ECON 4160: ECONOMETRICS –
MODELLING AND SYSTEMS ESTIMATION**

PROBLEM SET, EXAM AUTUMN 2010

PROBLEM 1 (weight: 1/3)

In this problem we consider the two-equation model:

$$\begin{aligned} (1) \quad & y_{1t} = \beta_2 y_{2t} + \alpha_1 + u_{1t}, \\ (2) \quad & y_{2t} = \beta_1 y_{1t} + \gamma_1 x_{1t} + \gamma_2 x_{2t} + \alpha_2 + u_{2t}, \end{aligned}$$

where (y_{1t}, y_{2t}) are endogenous, (x_{1t}, x_{2t}) are exogenous and (u_{1t}, u_{2t}) are disturbances.

1A. Complete the model description, and show that equation (1) is identified. Explain briefly why satisfaction of the order condition for identification is equivalent to requiring that the number of instruments for the variables which need instruments should be sufficiently large.

1B. Explain briefly why the Ordinary Least Squares (OLS) estimator of β_2 obtained from (1) is inconsistent. How would you estimate β_2 consistently by using x_{1t} as an instrument for y_{2t} ?

1C. Two-stage least squares (2SLS) is another possible estimation method for β_2 . Would you prefer it to the method proposed in **1C**? State the reason for your answer.

1D. Assume now that observations on x_{2t} are, for some reason, unavailable, although our theory claims that it should be included in equation (2). Assume also that x_{2t} is correlated with x_{1t} . In this situation it has been proposed to use as instrument for y_{2t} in equation (1), the variable \hat{y}_{2t} obtained as the fitted values in a regression of y_{2t} on x_{1t} . This is equivalent to use the fitted values from an OLS estimation of the reduced form equation for y_{2t} after having omitted x_{2t} from this reduced form equation.

Economists **a** and **b** discuss this method and state:

- a:** “This method is quite OK and will produce a consistent estimator of β_2 even if x_{2t} is missing and omitted.”
- b:** “Since x_{2t} is omitted from the reduced form and is correlated with x_{1t} , the reduced form coefficient estimator will be inconsistent (omitted variables bias). This will bias the values of \hat{y}_{2t} obtained and therefore lead to inconsistent estimation of β_2 in equation (1).”

Do you agree with **a** or with **b**? State the reason for your answer.

PROBLEM 2 (weight: 1/3)

We are concerned with the dynamic modelling of an aggregate consumption function, specified as a linear relation between the logarithm of private consumption, $y=\ln(C)$, the logarithm of disposable income, $x=\ln(R)$, and the real interest rate, rint . We have quarterly data from the U.S. and use observations from the period 1975.1–2000.4 (104 quarters) in the estimation. C and R are measured at constant prices, the real interest rate is measured as a decimal number (0.05 represents a 5% pro anno rate, etc.). (The actual data series are longer than 104 quarters, up to three additional observations are used to construct lags.) Consider, for simplicity, x and rint as exogenous relative to y . Edited printouts from PcGive for three dynamic specifications estimated by OLS are given below. In the second specification is used a smoothed log-income variable, xs , calculated from x as follows:

$$xs = x + 0.75*x(-1) + 0.50*x(-2) + 0.25*x(-3)$$

2A. Interpret the lag distributions involved in these three estimations.

2B. Calculate from the printouts: (a) estimates of the short-run and the long-run elasticity of consumption with respect to income, (b) estimates of the short-run and the long-run effect of the interest rate on log-consumption.

2C. The estimated short-run effects differ considerably across the specifications, also the t -values are markedly different. The estimated long-run effects are more similar. Can you give an interpretation of this finding? Supplementary statistics (for simplicity suppressed) indicate that the disturbances in all the three cases are autocorrelated.

EQ(1) Modelling y by OLS. Estimation sample: 1975.1–2000.4 (104 observations)

	Coefficient	Std. Error	t-value	t-prob	Part. R ²
Constant	-1.01331	0.05666	-17.9	0.0000	0.7673
x	1.03887	0.1636	6.35	0.0000	0.2936
x_1	0.210678	0.2090	1.01	0.3160	0.0104
x_2	-0.0593267	0.2094	-0.283	0.7775	0.0008
x_3	-0.0823840	0.1606	-0.513	0.6091	0.0027
rint	-0.139708	0.05732	-2.44	0.0166	0.0577
rint_1	-0.181896	0.05766	-3.15	0.0021	0.0930

sigma	0.0142624	RSS		0.0197314186	
R ²	0.996607	F(6,97) =	4749	[0.000]**	

EQ(2) Modelling y by OLS. Estimation sample: 1975.1–2000.4 (104 observations)

	Coefficient	Std. Error	t-value	t-prob	Part. R ²
Constant	-0.981466	0.06062	-16.2	0.0000	0.7238
xs	0.442028	0.002894	153.	0.0000	0.9957
rint	-0.134583	0.06171	-2.18	0.0315	0.0454
rint_1	-0.148841	0.06155	-2.42	0.0174	0.0552

sigma	0.0153862	RSS		0.023673647	
R ²	0.995929	F(3,100) =	8156	[0.000]**	

EQ(3) Modelling y by OLS. Estimation sample: 1975.1–2000.4 (104 observations)

	Coefficient	Std. Error	t-value	t-prob	Part. R ²
Constant	-0.0454084	0.05228	-0.869	0.3871	0.0075
y_1	0.948131	0.04590	20.7	0.0000	0.8101
x	0.0575368	0.05080	1.13	0.2601	0.0127
rint	-0.00207814	0.02479	-0.0838	0.9334	0.0001

sigma	0.00645669	RSS		0.00416888716	
R ²	0.999283	F(3,100) =	4.647e+004	[0.000]**	

PROBLEM 3 (weight: 1/3)

Consider the following three-equation model explaining two observable variables Y_1 and Y_2 , for example the consumption of two commodities, by the same unobservable explanatory variable X^* , for example normal income, and an equation describing the relationship between the measured and the unobserved value of the explanatory variable:

$$\begin{aligned} (1) \quad & Y_1 = a_1 + b_1 X^* + v_1, \\ (2) \quad & Y_2 = a_2 + b_2 X^* + v_2, \\ (3) \quad & X = X^* + \varepsilon, \end{aligned}$$

where X is the observed counterpart to X^* and ε is its measurement error. Assume that (v_1, v_2, ε) are (i) mutually uncorrelated, (ii) uncorrelated with X^* and (iii) have zero expectations and variances $(\sigma_1^2, \sigma_2^2, \sigma_\varepsilon^2)$, respectively. We have n observations of (Y_1, Y_2, X) .

3A. Make the other assumptions you may find necessary and complete the model description. Would you say that (1)–(3) represent a system of regression equations? Explain briefly.

3B. Derive from equations (1) and (3) a relationship between Y_1 and X . Next derive from this an expression for the asymptotic bias (inconsistency) of the OLS estimator of b_1 .

3C. Show that Y_2 satisfies the requirements for being an instrument for X in the equation obtained under **3B**, and write the expression for the estimator this motivates. Derive its probability limit to show that it is consistent.

3D. In econometrics one often looks for ‘good’ instruments. Could you propose one or more criteria by means of which you can decide whether a proposed instrument, say Y_2 for X in question **3C**, is ‘good’ or ‘bad’?

3E. It has been argued that the disturbances v_1 and v_2 should be allowed to be correlated. If this is the case, would you then modify your conclusion under **3C**? Explain briefly.