UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Postponed exam: **ECON4160 – Econometrics – Modeling and systems estimation**

Date of exam: Monday, January 7, 2013

Time for exam: 09:00 a.m. – 12:00 noon

The problem set covers 14 pages (incl. cover sheet)

Resources allowed:

- All written and printed resources, as well as calculator, is allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

ECON 4160: ECONOMETRICS –
MODELLING AND SYSTEMS ESTIMATION

PROBLEM SET, POSTPONED EXAM AUTUMN 2012

PROBLEM 1 *(weight: 1/3)*

In analyzing econometrically relationships between consumption and income from data from individual households, it is often a problem that these variables are improperly measured. We may, however, in addition to error-ridden measures of consumption and income, have observations on other variables which, according to our theory, are (theoretically) correlated with the true values of these improperly measured variables. We therefore consider the following model, i indexing household:

$$(1.1) \qquad \eta_i = \alpha + \beta\xi_i,$$
$$(1.2) \qquad y_i = \eta_i + \varepsilon_i,$$
$$(1.3) \qquad x_i = \xi_i + \delta_i,$$
$$(1.4) \qquad \xi_i = \lambda_x + \gamma_x q_i + u_i, \qquad i = 1, \dots, n,$$

where (η_i, ξ_i) denote true, unobserved consumption and income, respectively, (y_i, x_i) are their observed counterparts, $(\varepsilon_i, \delta_i)$ are measurement errors, q_i is an exogenous variable which determines true consumption, say age, wealth, or education, and u_i is a disturbance. We assume [the general notation $\mathsf{IID}(\mu, \theta^2)$ means: identically, independently distributed with expectation μ and variance θ^2]

$$(1.5) \qquad \begin{array}{ll} \varepsilon_i|q_i \sim \mathsf{IID}(0, \sigma_\varepsilon^2), \quad \delta_i|q_i \sim \mathsf{IID}(0, \sigma_\delta^2), \quad u_i|q_i \sim \mathsf{IID}(0, \sigma_u^2), & \\ \varepsilon_i,\ \delta_i,\ u_i \text{ are uncorrelated}, & i = 1, \dots, n. \end{array}$$

1A. Express $\text{var}(y_i)$, $\text{var}(x_i)$, $\text{cov}(y_i, x_i)$, $\text{cov}(y_i, q_i)$, and $\text{cov}(x_i, q_i)$ by means of α, β, $\sigma_\varepsilon^2, \sigma_\delta^2, \sigma_u^2$, and $\sigma_q^2 = \text{var}(q_i)$.

1B. Derive from (1.1)–(1.3), by eliminating η_i and ξ_i, an equation between y_i and x_i, and explain why q_i satisfies the requirements for being a valid instrumental variable for either of y_i and x_i in this equation.

To estimate the marginal propensity to consume of latent income, β, it has been proposed to use an estimator of the form

$$(1.6) \qquad \widehat{\beta} = \frac{M[y, z]}{M[x, z]},$$

where $M[y, z]$ and $M[x, z]$ denote the empirical covariance of, respectively, y and z and of x and z, and z is a so far unspecified, observable, variable.

1C. Consider the following choices of z_i:

(i) $z_i = \widehat{x}_i = \widehat{\lambda}_x + \widehat{\gamma}_x q_i$, where $(\widehat{\lambda}_x, \widehat{\gamma}_x)$ are obtained by OLS regression of x_i on q_i.

(ii) $z_i = \widehat{y}_i = \widehat{\lambda}_y + \widehat{\gamma}_y q_i$, where $(\widehat{\lambda}_y, \widehat{\gamma}_y)$ are obtained by OLS regression of y_i on q_i.

How would you characterize the estimators $\widehat{\gamma}_x$, $\widehat{\gamma}_y$ and $\widehat{\beta}$ obtained?

Assume now that equations (1.1)–(1.3) are part of a simultaneous household model containing a total of K observable, exogenous variables whose values for household i are: $q_{1i}, \dots, q_{Ki}$. These K variables may include age, net wealth, the number of household members, their education, etc. The reduced-form equation for true income ξ_i, replacing (1.4), therefore has the form:

$$\xi_i = \Pi_0 + \sum_{j=1}^{K} \Pi_j q_{ji} + u_i,$$

where the Π_j's denote reduced-form coefficients and u_i is a disturbance. Let the OLS estimators of the Π_j's obtained by regressing x_i on $q_{1i}, \dots, q_{Ki}$ be denoted by a tilde ($\tilde{}$).

1D. It has been suggested to estimate β by using $\widehat{\beta}$, given by (1.6), with

(iii) $$z_i = \widetilde{\Pi}_0 + \sum_{j=1}^{K} \widetilde{\Pi}_j q_{ji}$$

Comment on this choice of instrument z_i. Would you prefer it to (i) and (ii)? Explain briefly.

PROBLEM 2 *(weight: 1/3)*

Consider a two-equation system of the form:

(2.1) $$y = a + bx \qquad + u,$$
(2.2) $$x = c + dy + ez + v,$$

where (y, x, z) are variables, (a, b, c, d, e) are constants, (u, v) are disturbances with variances σ_{uu} and σ_{vv} and covariance σ_{uv}. We are in particular interested in estimating b consistently.

2A. Specify which variables are exogenous and endogenous, and explain whether b is identified. If b is identified, how you would estimate it in the following three models? Answer briefly, in only one or two sentences in each case.

Model a: As (2.1)–(2.2) **with:**
(y, x, z) are all observable;
(a, b, c, d, e) are unknown;
$\text{cov}(z, u) = \text{cov}(z, v) = 0$;
σ_{uv} is unknown.

Model b: As Model a with $e = 0$ **imposed:**
(y, x, z) are all observable;
$e = 0$, (a, b, c, d) are unknown;
$\text{cov}(z, u) = \text{cov}(z, v) = 0$;
σ_{uv} is unknown.

Model c: As Model a with $d = 0$ **and** $\sigma_{uv} = 0$ **imposed:**
(y, x, z) are all observable;
$d = 0$, (a, b, c, e) are unknown;
$\text{cov}(z, u) = \text{cov}(z, v) = 0$;
$\sigma_{uv} = 0$.

2B. Assume next that only (y, z) can be observed, while x is unobservable. Consider two such models:

Model d: As Model c with x unobservable = Model a modified by:
(y, z) are observable, x is not observable;
$d = 0$, (a, b, c, e) are unknown;
$\text{cov}(z, u) = \text{cov}(z, v) = 0$;
$\sigma_{uv} = 0$.

Model e: As Model d with $c = 0, e = 1$ imposed = Model a modified by:
(y, z) are observable, x is not observable;
$c = d = 0, e = 1$, (a, b) are unknown;
$\text{cov}(z, u) = \text{cov}(z, v) = 0$;
$\sigma_{uv} = 0$

Can the coefficient b be identified in **Model d**? If so, propose an estimator for it. *Hint:* Eliminate x by using (2.2).

Can the coefficient b be identified in **Model e**? If so, propose an estimator for it.

PROBLEM 3 *(weight: 1/3)*

The simple Keynesian business cycle model suggests that investments depend on the activity level in the economy. To analyze whether this is true, the following relationship has been estimated by OLS for US data on the sample 1948q1–2008q1:

$$\log(I_t) = \alpha + \beta \log(Y_t) + \varepsilon_{I,t} \tag{3.1}$$

where $\log(I_t)$ represents the log of gross investments, while $\log(Y_t)$ is the log of GDP. The results are reported in **Printout 1**.

3A. How would you interpret the estimated coefficient on $\log(Y_t)$? Does it seem like investments depend on GDP? It has been suggested that GDP is endogenous to investments, which will lead to a bias in the OLS estimate of β. Explain what is meant by the simultaneity bias and derive an expression for the bias in the case when Y_t is in fact an endogenous variable.

Due to the endogneity of GDP, it has been suggested that it is better to estimate (3.1) by Instrumental variable estimation. As an instrumental variable, it has been suggested to use government expenditures. The results from the IVE are reported in **Printout 2**.

3B.

- Discuss one or more methods that could be used to evaluate the appropriateness/strength of the proposed instrument.
- The OLS and the IV estimates are almost equal. A colleague has said that it then does not matter whether you rely on the OLS or the IV estimates. Do you agree with this statement? Explain what arguments your conclusions rests on.

Another colleague has suggested that in addition to GDP, investments also depend on the interest rate. She has therefore suggested that it is better to consider a modified version of (3.1):

$$\log(I_t) = a + b\log(Y_t) + cr_t + \epsilon_{I,t} \tag{3.2}$$

where r_t denotes the interest rate. In the following, you can assume that r_t is exogenous. This relationship has been estimated for two different time periods: 1948q1–2008q1 and 1948q1–2012q3. The results are reported in **Printout 3** and **Printout 4**. The coefficient of $\log(Y_t)$ is almost unchanged when the sample is extended to include four more years of data, while the coefficient on the interest rate goes from negative to positive (though insignificant). Your colleague has suggested that this has to do with the financial crisis, and have therefore estimated the model with a dummy variable, fincrisis, that is equal to one from 2008q2 to 2012q3. The estimates from this model are reported in **Printout 5**.

3C.

- How would you interpret the estimated coefficient on the interest rate?
- Use the reported results in the printouts to calculate the Chow-test for parameter stability. The residual sum of squares (RSS/SSE/SSR) when (3.2) is estimated on the sample 2008q2–2012q3 is 0.02295. Critical values for the F-distribution are tabulated at the end of this document. What is your conclusion? Has there been a structural break? Does the "extra" regression with the dummy included (**Printout 5**) give further evidence to your conclusion? Discuss briefly.

  Hint: *If you do not find the exact critical value, you can use the number of degrees of freedom that is closest when you round down*

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Printout 1: Modelling Log(I) by OLS
The estimation sample is: 1948(1) - 2008(1)

                  Coefficient  Std.Error  t-value  t-prob
Constant             -4.08544    0.08970    -45.5  0.0000
Log(Y)                1.23532    0.01044     118.  0.0000

sigma                0.0934679  RSS              2.08796273
R^2                   0.983202  F(1,239) =   1.399e+004 [0.000]**
Adj.R^2               0.983132  log-likelihood        230.243
no. of observations        241  no. of parameters           2

AR 1-5 test:      F(5,234)  =   189.41 [0.0000]**
Normality test:   Chi^2(2)  =   6.8201 [0.0330]*
Hetero test:      F(2,238)  =   9.6656 [0.0001]**
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Printout 2: Modelling Log(I) by IVE
       The estimation sample is: 1948(1) - 2008(1)

                  Coefficient  Std.Error  t-value
Log(Y)       Y       1.23362    0.01063     116.
Constant            -4.07081    0.09129    -44.6

sigma               0.0934731  RSS                 2.08819593
Reduced-form sigma    0.16649
no. endogenous variables    2  no. of instruments           2
no. of observations       241  no. of parameters            2
Additional instruments:  [0] = Gov

AR 1-5 test:      F(5,234)  =   189.67 [0.0000]**
Normality test:   Chi^2(2)  =   6.2518 [0.0439]*
Hetero test:      F(2,238)  =   9.6835 [0.0001]**
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Printout 3: Modelling Log(I) by IVE
       The estimation sample is: 1948(1) - 2008(1)

                  Coefficient  Std.Error  t-value
Constant            -4.09117   0.09155    -44.7
Log(Y)      Y        1.23913   0.01100     113.
r                   -0.567773  0.222 0    -2.56

sigma               0.0922815  RSS                 2.02678001
Reduced-form sigma    0.14559
no. endogenous variables    2  no. of instruments           3
no. of observations       241  no. of parameters            3
Additional instruments: [0] = Gov

AR 1-5 test:      F(5,233)  =   174.05 [0.0000]**
Normality test:   Chi^2(2)  =   3.6360 [0.1624]
Hetero test:      F(4,236)  =   4.1983 [0.0027]**
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Printout 4: Modelling Log(I) by IVE
       The estimation sample is: 1948(1) - 2012(3)

                  Coefficient  Std.Error  t-value
Constant            -3.76015    0.09266    -40.6
Log(Y)       Y       1.19470    0.01079     111.
r                    0.279433   0.2179      1.28

sigma                0.103005  RSS                 2.71616241
Reduced-form sigma    0.16258
no. endogenous variables    2  no. of instruments           3
no. of observations       259  no. of parameters            3
Additional instruments:  [0] = Gov

AR 1-5 test:      F(5,251)  =   260.82 [0.0000]**
Normality test:   Chi^2(2)  = 0.30858 [0.8570]
Hetero test:      F(4,254)  =   12.831 [0.0000]**
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Printout 5: Modelling Log(I) by IVE
       The estimation sample is: 1948(1) - 2012(3)

                  Coefficient    Std.Error   t-value
Constant            -4.08507      0.09270    -44.1
Log(Y)       Y       1.23823      0.01115     111.
r                   -0.530921     0.2238     -2.37
fincrisis           -0.204937     0.02752    -7.45

sigma               0.0927312  RSS                 2.19276362
Reduced-form sigma    0.14716
no. endogenous variables    2  no. of instruments           4
no. of observations       259  no. of parameters            4
Additional instruments:  [0] = Gov

AR 1-5 test:      F(5,250)  =   179.23 [0.0000]**
Normality test:   Chi^2(2)  =   2.8559 [0.2398]
Hetero test:      F(5,253)  =   3.7602 [0.0027]**
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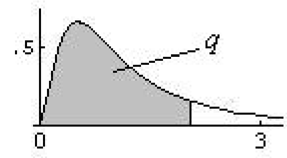
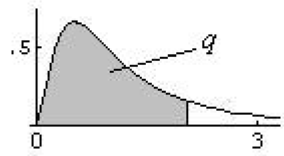


Quantiles of the F-Distribution

Tabulated values are q-th quantiles for $q \in \{.5, .75, .9, .95, .975, .99, .995\}$

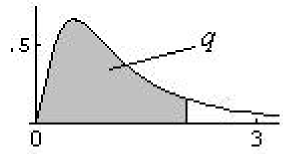
denom-	numerator degrees of freedom													
inator	1							2						
df	.5	.75	.9	.95	.975	.99	.995	.5	.75	.9	.95	.975	.99	.995
1	1.00	5.83	39.9	161	648	4052	16E3	1.50	7.50	49.5	199	800	5000	20E3
2	0.67	2.57	8.53	18.5	38.5	98.5	199	1.00	3.00	9.00	19.0	39.0	99.0	199
3	0.59	2.02	5.54	10.1	17.4	34.1	55.6	0.88	2.28	5.46	9.55	16.0	30.8	49.8
4	0.55	1.81	4.54	7.71	12.2	21.2	31.3	0.83	2.00	4.32	6.94	10.6	18.0	26.3
5	0.53	1.69	4.06	6.61	10.0	16.3	22.8	0.80	1.85	3.78	5.79	8.43	13.3	18.3
6	0.51	1.62	3.78	5.99	8.81	13.7	18.6	0.78	1.76	3.46	5.14	7.26	10.9	14.5
7	0.51	1.57	3.59	5.59	8.07	12.2	16.2	0.77	1.70	3.26	4.74	6.54	9.55	12.4
8	0.50	1.54	3.46	5.32	7.57	11.3	14.7	0.76	1.66	3.11	4.46	6.06	8.65	11.0
9	0.49	1.51	3.36	5.12	7.21	10.6	13.6	0.75	1.62	3.01	4.26	5.71	8.02	10.1
10	0.49	1.49	3.29	4.96	6.94	10.0	12.8	0.74	1.60	2.92	4.10	5.46	7.56	9.43
11	0.49	1.47	3.23	4.84	6.72	9.65	12.2	0.74	1.58	2.86	3.98	5.26	7.21	8.91
12	0.48	1.46	3.18	4.75	6.55	9.33	11.8	0.73	1.56	2.81	3.89	5.10	6.93	8.51
13	0.48	1.45	3.14	4.67	6.41	9.07	11.4	0.73	1.55	2.76	3.81	4.97	6.70	8.19
14	0.48	1.44	3.10	4.60	6.30	8.86	11.1	0.73	1.53	2.73	3.74	4.86	6.51	7.92
15	0.48	1.43	3.07	4.54	6.20	8.68	10.8	0.73	1.52	2.70	3.68	4.77	6.36	7.70
16	0.48	1.42	3.05	4.49	6.12	8.53	10.6	0.72	1.51	2.67	3.63	4.69	6.23	7.51
17	0.47	1.42	3.03	4.45	6.04	8.40	10.4	0.72	1.51	2.64	3.59	4.62	6.11	7.35
18	0.47	1.41	3.01	4.41	5.98	8.29	10.2	0.72	1.50	2.62	3.55	4.56	6.01	7.21
19	0.47	1.41	2.99	4.38	5.92	8.18	10.1	0.72	1.49	2.61	3.52	4.51	5.93	7.09
20	0.47	1.40	2.97	4.35	5.87	8.10	9.94	0.72	1.49	2.59	3.49	4.46	5.85	6.99
21	0.47	1.40	2.96	4.32	5.83	8.02	9.83	0.72	1.48	2.57	3.47	4.42	5.78	6.89
22	0.47	1.40	2.95	4.30	5.79	7.95	9.73	0.72	1.48	2.56	3.44	4.38	5.72	6.81
23	0.47	1.39	2.94	4.28	5.75	7.88	9.63	0.71	1.47	2.55	3.42	4.35	5.66	6.73
24	0.47	1.39	2.93	4.26	5.72	7.82	9.55	0.71	1.47	2.54	3.40	4.32	5.61	6.66
25	0.47	1.39	2.92	4.24	5.69	7.77	9.48	0.71	1.47	2.53	3.39	4.29	5.57	6.60
26	0.47	1.38	2.91	4.23	5.66	7.72	9.41	0.71	1.46	2.52	3.37	4.27	5.53	6.54
27	0.47	1.38	2.90	4.21	5.63	7.68	9.34	0.71	1.46	2.51	3.35	4.24	5.49	6.49
28	0.47	1.38	2.89	4.20	5.61	7.64	9.28	0.71	1.46	2.50	3.34	4.22	5.45	6.44
29	0.47	1.38	2.89	4.18	5.59	7.60	9.23	0.71	1.45	2.50	3.33	4.20	5.42	6.40
30	0.47	1.38	2.88	4.17	5.57	7.56	9.18	0.71	1.45	2.49	3.32	4.18	5.39	6.35
35	0.46	1.37	2.85	4.12	5.48	7.42	8.98	0.71	1.44	2.46	3.27	4.11	5.27	6.19
40	0.46	1.36	2.84	4.08	5.42	7.31	8.83	0.71	1.44	2.44	3.23	4.05	5.18	6.07
50	0.46	1.35	2.81	4.03	5.34	7.17	8.63	0.70	1.43	2.41	3.18	3.97	5.06	5.90
100	0.46	1.34	2.76	3.94	5.18	6.90	8.24	0.70	1.41	2.36	3.09	3.83	4.82	5.59
∞	0.45	1.32	2.71	3.84	5.02	6.63	7.88	0.69	1.39	2.30	3.00	3.69	4.61	5.30



Quantiles of the F-Distribution

Tabulated values are q-th quantiles for $q \in \{.5, .75, .9, .95, .975, .99, .995\}$

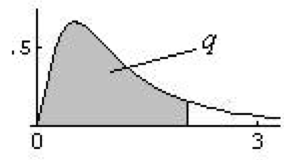
denom-inator	numerator degrees of freedom													
	3							4						
df	.5	.75	.9	.95	.975	.99	.995	.5	.75	.9	.95	.975	.99	.995
1	1.71	8.20	53.6	216	864	5403	22E3	1.82	8.58	55.8	225	900	5625	22E3
2	1.13	3.15	9.16	19.2	39.2	99.2	199	1.21	3.23	9.24	19.2	39.2	99.2	199
3	1.00	2.36	5.39	9.28	15.4	29.5	47.5	1.06	2.39	5.34	9.12	15.1	28.7	46.2
4	0.94	2.05	4.19	6.59	9.98	16.7	24.3	1.00	2.06	4.11	6.39	9.60	16.0	23.2
5	0.91	1.88	3.62	5.41	7.76	12.1	16.5	0.96	1.89	3.52	5.19	7.39	11.4	15.6
6	0.89	1.78	3.29	4.76	6.60	9.78	12.9	0.94	1.79	3.18	4.53	6.23	9.15	12.0
7	0.87	1.72	3.07	4.35	5.89	8.45	10.9	0.93	1.72	2.96	4.12	5.52	7.85	10.1
8	0.86	1.67	2.92	4.07	5.42	7.59	9.60	0.91	1.66	2.81	3.84	5.05	7.01	8.81
9	0.85	1.63	2.81	3.86	5.08	6.99	8.72	0.91	1.63	2.69	3.63	4.72	6.42	7.96
10	0.85	1.60	2.73	3.71	4.83	6.55	8.08	0.90	1.59	2.61	3.48	4.47	5.99	7.34
11	0.84	1.58	2.66	3.59	4.63	6.22	7.60	0.89	1.57	2.54	3.36	4.28	5.67	6.88
12	0.84	1.56	2.61	3.49	4.47	5.95	7.23	0.89	1.55	2.48	3.26	4.12	5.41	6.52
13	0.83	1.55	2.56	3.41	4.35	5.74	6.93	0.88	1.53	2.43	3.18	4.00	5.21	6.23
14	0.83	1.53	2.52	3.34	4.24	5.56	6.68	0.88	1.52	2.39	3.11	3.89	5.04	6.00
15	0.83	1.52	2.49	3.29	4.15	5.42	6.48	0.88	1.51	2.36	3.06	3.80	4.89	5.80
16	0.82	1.51	2.46	3.24	4.08	5.29	6.30	0.88	1.50	2.33	3.01	3.73	4.77	5.64
17	0.82	1.50	2.44	3.20	4.01	5.18	6.16	0.87	1.49	2.31	2.96	3.66	4.67	5.50
18	0.82	1.49	2.42	3.16	3.95	5.09	6.03	0.87	1.48	2.29	2.93	3.61	4.58	5.37
19	0.82	1.49	2.40	3.13	3.90	5.01	5.92	0.87	1.47	2.27	2.90	3.56	4.50	5.27
20	0.82	1.48	2.38	3.10	3.86	4.94	5.82	0.87	1.47	2.25	2.87	3.51	4.43	5.17
21	0.81	1.48	2.36	3.07	3.82	4.87	5.73	0.87	1.46	2.23	2.84	3.48	4.37	5.09
22	0.81	1.47	2.35	3.05	3.78	4.82	5.65	0.87	1.45	2.22	2.82	3.44	4.31	5.02
23	0.81	1.47	2.34	3.03	3.75	4.76	5.58	0.86	1.45	2.21	2.80	3.41	4.26	4.95
24	0.81	1.46	2.33	3.01	3.72	4.72	5.52	0.86	1.44	2.19	2.78	3.38	4.22	4.89
25	0.81	1.46	2.32	2.99	3.69	4.68	5.46	0.86	1.44	2.18	2.76	3.35	4.18	4.84
26	0.81	1.45	2.31	2.98	3.67	4.64	5.41	0.86	1.44	2.17	2.74	3.33	4.14	4.79
27	0.81	1.45	2.30	2.96	3.65	4.60	5.36	0.86	1.43	2.17	2.73	3.31	4.11	4.74
28	0.81	1.45	2.29	2.95	3.63	4.57	5.32	0.86	1.43	2.16	2.71	3.29	4.07	4.70
29	0.81	1.45	2.28	2.93	3.61	4.54	5.28	0.86	1.43	2.15	2.70	3.27	4.04	4.66
30	0.81	1.44	2.28	2.92	3.59	4.51	5.24	0.86	1.42	2.14	2.69	3.25	4.02	4.62
35	0.80	1.43	2.25	2.87	3.52	4.40	5.09	0.86	1.41	2.11	2.64	3.18	3.91	4.48
40	0.80	1.42	2.23	2.84	3.46	4.31	4.98	0.85	1.40	2.09	2.61	3.13	3.83	4.37
50	0.80	1.41	2.20	2.79	3.39	4.20	4.83	0.85	1.39	2.06	2.56	3.05	3.72	4.23
100	0.79	1.39	2.14	2.70	3.25	3.98	4.54	0.84	1.37	2.00	2.46	2.92	3.51	3.96
∞	0.79	1.37	2.08	2.60	3.12	3.78	4.28	0.84	1.35	1.94	2.37	2.79	3.32	3.72



Quantiles of the F-Distribution

Tabulated values are q-th quantiles for $q \in \{.5, .75, .9, .95, .975, .99, .995\}$

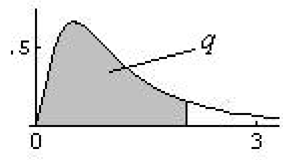
denom-inator df	numerator degrees of freedom													
	5							6						
	.5	.75	.9	.95	.975	.99	.995	.5	.75	.9	.95	.975	.99	.995
1	1.89	8.82	57.2	230	922	5764	23E3	1.94	8.98	58.2	234	937	5859	23E3
2	1.25	3.28	9.29	19.3	39.3	99.3	199	1.28	3.31	9.33	19.3	39.3	99.3	199
3	1.10	2.41	5.31	9.01	14.9	28.2	45.4	1.13	2.42	5.28	8.94	14.7	27.9	44.8
4	1.04	2.07	4.05	6.26	9.36	15.5	22.5	1.06	2.08	4.01	6.16	9.20	15.2	22.0
5	1.00	1.89	3.45	5.05	7.15	11.0	14.9	1.02	1.89	3.40	4.95	6.98	10.7	14.5
6	0.98	1.79	3.11	4.39	5.99	8.75	11.5	1.00	1.78	3.05	4.28	5.82	8.47	11.1
7	0.96	1.71	2.88	3.97	5.29	7.46	9.52	0.98	1.71	2.83	3.87	5.12	7.19	9.16
8	0.95	1.66	2.73	3.69	4.82	6.63	8.30	0.97	1.65	2.67	3.58	4.65	6.37	7.95
9	0.94	1.62	2.61	3.48	4.48	6.06	7.47	0.96	1.61	2.55	3.37	4.32	5.80	7.13
10	0.93	1.59	2.52	3.33	4.24	5.64	6.87	0.95	1.58	2.46	3.22	4.07	5.39	6.54
11	0.93	1.56	2.45	3.20	4.04	5.32	6.42	0.95	1.55	2.39	3.09	3.88	5.07	6.10
12	0.92	1.54	2.39	3.11	3.89	5.06	6.07	0.94	1.53	2.33	3.00	3.73	4.82	5.76
13	0.92	1.52	2.35	3.03	3.77	4.86	5.79	0.94	1.51	2.28	2.92	3.60	4.62	5.48
14	0.91	1.51	2.31	2.96	3.66	4.69	5.56	0.94	1.50	2.24	2.85	3.50	4.46	5.26
15	0.91	1.49	2.27	2.90	3.58	4.56	5.37	0.93	1.48	2.21	2.79	3.41	4.32	5.07
16	0.91	1.48	2.24	2.85	3.50	4.44	5.21	0.93	1.47	2.18	2.74	3.34	4.20	4.91
17	0.91	1.47	2.22	2.81	3.44	4.34	5.07	0.93	1.46	2.15	2.70	3.28	4.10	4.78
18	0.90	1.46	2.20	2.77	3.38	4.25	4.96	0.93	1.45	2.13	2.66	3.22	4.01	4.66
19	0.90	1.46	2.18	2.74	3.33	4.17	4.85	0.92	1.44	2.11	2.63	3.17	3.94	4.56
20	0.90	1.45	2.16	2.71	3.29	4.10	4.76	0.92	1.44	2.09	2.60	3.13	3.87	4.47
21	0.90	1.44	2.14	2.68	3.25	4.04	4.68	0.92	1.43	2.08	2.57	3.09	3.81	4.39
22	0.90	1.44	2.13	2.66	3.22	3.99	4.61	0.92	1.42	2.06	2.55	3.05	3.76	4.32
23	0.90	1.43	2.11	2.64	3.18	3.94	4.54	0.92	1.42	2.05	2.53	3.02	3.71	4.26
24	0.90	1.43	2.10	2.62	3.15	3.90	4.49	0.92	1.41	2.04	2.51	2.99	3.67	4.20
25	0.89	1.42	2.09	2.60	3.13	3.85	4.43	0.92	1.41	2.02	2.49	2.97	3.63	4.15
26	0.89	1.42	2.08	2.59	3.10	3.82	4.38	0.91	1.41	2.01	2.47	2.94	3.59	4.10
27	0.89	1.42	2.07	2.57	3.08	3.78	4.34	0.91	1.40	2.00	2.46	2.92	3.56	4.06
28	0.89	1.41	2.06	2.56	3.06	3.75	4.30	0.91	1.40	2.00	2.45	2.90	3.53	4.02
29	0.89	1.41	2.06	2.55	3.04	3.73	4.26	0.91	1.40	1.99	2.43	2.88	3.50	3.98
30	0.89	1.41	2.05	2.53	3.03	3.70	4.23	0.91	1.39	1.98	2.42	2.87	3.47	3.95
35	0.89	1.40	2.02	2.49	2.96	3.59	4.09	0.91	1.38	1.95	2.37	2.80	3.37	3.81
40	0.89	1.39	2.00	2.45	2.90	3.51	3.99	0.91	1.37	1.93	2.34	2.74	3.29	3.71
50	0.88	1.37	1.97	2.40	2.83	3.41	3.85	0.90	1.36	1.90	2.29	2.67	3.19	3.58
100	0.88	1.35	1.91	2.31	2.70	3.21	3.59	0.90	1.33	1.83	2.19	2.54	2.99	3.33
∞	0.87	1.33	1.85	2.21	2.57	3.02	3.35	0.89	1.31	1.77	2.10	2.41	2.80	3.09



Quantiles of the F-Distribution

Tabulated values are q-th quantiles for $q \in \{.5, .75, .9, .95, .975, .99, .995\}$

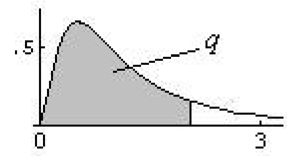
denom-inator	numerator degrees of freedom													
	7							8						
df	.5	.75	.9	.95	.975	.99	.995	.5	.75	.9	.95	.975	.99	.995
1	1.98	9.10	58.9	237	948	5928	24E3	2.00	9.19	59.4	239	957	5981	24E3
2	1.30	3.34	9.35	19.4	39.4	99.4	199	1.32	3.35	9.37	19.4	39.4	99.4	199
3	1.15	2.43	5.27	8.89	14.6	27.7	44.4	1.16	2.44	5.25	8.85	14.5	27.5	44.1
4	1.08	2.08	3.98	6.09	9.07	15.0	21.6	1.09	2.08	3.95	6.04	8.98	14.8	21.4
5	1.04	1.89	3.37	4.88	6.85	10.5	14.2	1.05	1.89	3.34	4.82	6.76	10.3	14.0
6	1.02	1.78	3.01	4.21	5.70	8.26	10.8	1.03	1.78	2.98	4.15	5.60	8.10	10.6
7	1.00	1.70	2.78	3.79	4.99	6.99	8.89	1.01	1.70	2.75	3.73	4.90	6.84	8.68
8	0.99	1.64	2.62	3.50	4.53	6.18	7.69	1.00	1.64	2.59	3.44	4.43	6.03	7.50
9	0.98	1.60	2.51	3.29	4.20	5.61	6.88	0.99	1.60	2.47	3.23	4.10	5.47	6.69
10	0.97	1.57	2.41	3.14	3.95	5.20	6.30	0.98	1.56	2.38	3.07	3.85	5.06	6.12
11	0.96	1.54	2.34	3.01	3.76	4.89	5.86	0.98	1.53	2.30	2.95	3.66	4.74	5.68
12	0.96	1.52	2.28	2.91	3.61	4.64	5.52	0.97	1.51	2.24	2.85	3.51	4.50	5.35
13	0.96	1.50	2.23	2.83	3.48	4.44	5.25	0.97	1.49	2.20	2.77	3.39	4.30	5.08
14	0.95	1.49	2.19	2.76	3.38	4.28	5.03	0.96	1.48	2.15	2.70	3.29	4.14	4.86
15	0.95	1.47	2.16	2.71	3.29	4.14	4.85	0.96	1.46	2.12	2.64	3.20	4.00	4.67
16	0.95	1.46	2.13	2.66	3.22	4.03	4.69	0.96	1.45	2.09	2.59	3.12	3.89	4.52
17	0.94	1.45	2.10	2.61	3.16	3.93	4.56	0.96	1.44	2.06	2.55	3.06	3.79	4.39
18	0.94	1.44	2.08	2.58	3.10	3.84	4.44	0.95	1.43	2.04	2.51	3.01	3.71	4.28
19	0.94	1.43	2.06	2.54	3.05	3.77	4.34	0.95	1.42	2.02	2.48	2.96	3.63	4.18
20	0.94	1.43	2.04	2.51	3.01	3.70	4.26	0.95	1.42	2.00	2.45	2.91	3.56	4.09
21	0.94	1.42	2.02	2.49	2.97	3.64	4.18	0.95	1.41	1.98	2.42	2.87	3.51	4.01
22	0.93	1.41	2.01	2.46	2.93	3.59	4.11	0.95	1.40	1.97	2.40	2.84	3.45	3.94
23	0.93	1.41	1.99	2.44	2.90	3.54	4.05	0.95	1.40	1.95	2.37	2.81	3.41	3.88
24	0.93	1.40	1.98	2.42	2.87	3.50	3.99	0.94	1.39	1.94	2.36	2.78	3.36	3.83
25	0.93	1.40	1.97	2.40	2.85	3.46	3.94	0.94	1.39	1.93	2.34	2.75	3.32	3.78
26	0.93	1.39	1.96	2.39	2.82	3.42	3.89	0.94	1.38	1.92	2.32	2.73	3.29	3.73
27	0.93	1.39	1.95	2.37	2.80	3.39	3.85	0.94	1.38	1.91	2.31	2.71	3.26	3.69
28	0.93	1.39	1.94	2.36	2.78	3.36	3.81	0.94	1.38	1.90	2.29	2.69	3.23	3.65
29	0.93	1.38	1.93	2.35	2.76	3.33	3.77	0.94	1.37	1.89	2.28	2.67	3.20	3.61
30	0.93	1.38	1.93	2.33	2.75	3.30	3.74	0.94	1.37	1.88	2.27	2.65	3.17	3.58
35	0.92	1.37	1.90	2.29	2.68	3.20	3.61	0.94	1.36	1.85	2.22	2.58	3.07	3.45
40	0.92	1.36	1.87	2.25	2.62	3.12	3.51	0.93	1.35	1.83	2.18	2.53	2.99	3.35
50	0.92	1.34	1.84	2.20	2.55	3.02	3.38	0.93	1.33	1.80	2.13	2.46	2.89	3.22
100	0.91	1.32	1.78	2.10	2.42	2.82	3.13	0.92	1.30	1.73	2.03	2.32	2.69	2.97
∞	0.91	1.29	1.72	2.01	2.29	2.64	2.90	0.92	1.28	1.67	1.94	2.19	2.51	2.74



Quantiles of the F-Distribution

Tabulated values are q-th quantiles for $q \in \{.5, .75, .9, .95, .975, .99, .995\}$

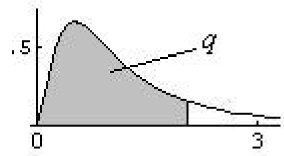
denom-inator	numerator degrees of freedom													
	9							10						
df	.5	.75	.9	.95	.975	.99	.995	.5	.75	.9	.95	.975	.99	.995
1	2.03	9.26	59.9	241	963	6022	24E3	2.04	9.32	60.2	242	969	6056	24E3
2	1.33	3.37	9.38	19.4	39.4	99.4	199	1.35	3.38	9.39	19.4	39.4	99.4	199
3	1.17	2.44	5.24	8.81	14.5	27.3	43.9	1.18	2.44	5.23	8.79	14.4	27.2	43.7
4	1.10	2.08	3.94	6.00	8.90	14.7	21.1	1.11	2.08	3.92	5.96	8.84	14.5	21.0
5	1.06	1.89	3.32	4.77	6.68	10.2	13.8	1.07	1.89	3.30	4.74	6.62	10.1	13.6
6	1.04	1.77	2.96	4.10	5.52	7.98	10.4	1.05	1.77	2.94	4.06	5.46	7.87	10.3
7	1.02	1.69	2.72	3.68	4.82	6.72	8.51	1.03	1.69	2.70	3.64	4.76	6.62	8.38
8	1.01	1.63	2.56	3.39	4.36	5.91	7.34	1.02	1.63	2.54	3.35	4.30	5.81	7.21
9	1.00	1.59	2.44	3.18	4.03	5.35	6.54	1.01	1.59	2.42	3.14	3.96	5.26	6.42
10	0.99	1.56	2.35	3.02	3.78	4.94	5.97	1.00	1.55	2.32	2.98	3.72	4.85	5.85
11	0.99	1.53	2.27	2.90	3.59	4.63	5.54	0.99	1.52	2.25	2.85	3.53	4.54	5.42
12	0.98	1.51	2.21	2.80	3.44	4.39	5.20	0.99	1.50	2.19	2.75	3.37	4.30	5.09
13	0.98	1.49	2.16	2.71	3.31	4.19	4.94	0.98	1.48	2.14	2.67	3.25	4.10	4.82
14	0.97	1.47	2.12	2.65	3.21	4.03	4.72	0.98	1.46	2.10	2.60	3.15	3.94	4.60
15	0.97	1.46	2.09	2.59	3.12	3.89	4.54	0.98	1.45	2.06	2.54	3.06	3.80	4.42
16	0.97	1.44	2.06	2.54	3.05	3.78	4.38	0.97	1.44	2.03	2.49	2.99	3.69	4.27
17	0.96	1.43	2.03	2.49	2.98	3.68	4.25	0.97	1.43	2.00	2.45	2.92	3.59	4.14
18	0.96	1.42	2.00	2.46	2.93	3.60	4.14	0.97	1.42	1.98	2.41	2.87	3.51	4.03
19	0.96	1.41	1.98	2.42	2.88	3.52	4.04	0.97	1.41	1.96	2.38	2.82	3.43	3.93
20	0.96	1.41	1.96	2.39	2.84	3.46	3.96	0.97	1.40	1.94	2.35	2.77	3.37	3.85
21	0.96	1.40	1.95	2.37	2.80	3.40	3.88	0.96	1.39	1.92	2.32	2.73	3.31	3.77
22	0.96	1.39	1.93	2.34	2.76	3.35	3.81	0.96	1.39	1.90	2.30	2.70	3.26	3.70
23	0.95	1.39	1.92	2.32	2.73	3.30	3.75	0.96	1.38	1.89	2.27	2.67	3.21	3.64
24	0.95	1.38	1.91	2.30	2.70	3.26	3.69	0.96	1.38	1.88	2.25	2.64	3.17	3.59
25	0.95	1.38	1.89	2.28	2.68	3.22	3.64	0.96	1.37	1.87	2.24	2.61	3.13	3.54
26	0.95	1.37	1.88	2.27	2.65	3.18	3.60	0.96	1.37	1.86	2.22	2.59	3.09	3.49
27	0.95	1.37	1.87	2.25	2.63	3.15	3.56	0.96	1.36	1.85	2.20	2.57	3.06	3.45
28	0.95	1.37	1.87	2.24	2.61	3.12	3.52	0.96	1.36	1.84	2.19	2.55	3.03	3.41
29	0.95	1.36	1.86	2.22	2.59	3.09	3.48	0.96	1.35	1.83	2.18	2.53	3.00	3.38
30	0.95	1.36	1.85	2.21	2.57	3.07	3.45	0.96	1.35	1.82	2.16	2.51	2.98	3.34
35	0.94	1.35	1.82	2.16	2.50	2.96	3.32	0.95	1.34	1.79	2.11	2.44	2.88	3.21
40	0.94	1.34	1.79	2.12	2.45	2.89	3.22	0.95	1.33	1.76	2.08	2.39	2.80	3.12
50	0.94	1.32	1.76	2.07	2.38	2.78	3.09	0.95	1.31	1.73	2.03	2.32	2.70	2.99
100	0.93	1.29	1.69	1.97	2.24	2.59	2.85	0.94	1.28	1.66	1.93	2.18	2.50	2.74
∞	0.93	1.27	1.63	1.88	2.11	2.41	2.62	0.93	1.25	1.60	1.83	2.05	2.32	2.52



Quantiles of the F-Distribution

Tabulated values are q-th quantiles for $q \in \{.5, .75, .9, .95, .975, .99, .995\}$

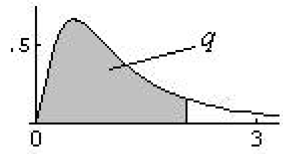
denom-	numerator degrees of freedom													
inator	11							12						
df	.5	.75	.9	.95	.975	.99	.995	.5	.75	.9	.95	.975	.99	.995
1	2.06	9.37	60.5	243	973	6083	24E3	2.07	9.41	60.7	244	977	6106	24E3
2	1.35	3.39	9.40	19.4	39.4	99.4	199	1.36	3.39	9.41	19.4	39.4	99.4	199
3	1.19	2.45	5.22	8.76	14.4	27.1	43.5	1.20	2.45	5.22	8.74	14.3	27.1	43.4
4	1.12	2.08	3.91	5.94	8.79	14.5	20.8	1.13	2.08	3.90	5.91	8.75	14.4	20.7
5	1.08	1.89	3.28	4.70	6.57	9.96	13.5	1.09	1.89	3.27	4.68	6.52	9.89	13.4
6	1.05	1.77	2.92	4.03	5.41	7.79	10.1	1.06	1.77	2.90	4.00	5.37	7.72	10.0
7	1.04	1.69	2.68	3.60	4.71	6.54	8.27	1.04	1.68	2.67	3.57	4.67	6.47	8.18
8	1.02	1.63	2.52	3.31	4.24	5.73	7.10	1.03	1.62	2.50	3.28	4.20	5.67	7.01
9	1.01	1.58	2.40	3.10	3.91	5.18	6.31	1.02	1.58	2.38	3.07	3.87	5.11	6.23
10	1.01	1.55	2.30	2.94	3.66	4.77	5.75	1.01	1.54	2.28	2.91	3.62	4.71	5.66
11	1.00	1.52	2.23	2.82	3.47	4.46	5.32	1.01	1.51	2.21	2.79	3.43	4.40	5.24
12	0.99	1.49	2.17	2.72	3.32	4.22	4.99	1.00	1.49	2.15	2.69	3.28	4.16	4.91
13	0.99	1.47	2.12	2.63	3.20	4.02	4.72	1.00	1.47	2.10	2.60	3.15	3.96	4.64
14	0.99	1.46	2.07	2.57	3.09	3.86	4.51	0.99	1.45	2.05	2.53	3.05	3.80	4.43
15	0.98	1.44	2.04	2.51	3.01	3.73	4.33	0.99	1.44	2.02	2.48	2.96	3.67	4.25
16	0.98	1.43	2.01	2.46	2.93	3.62	4.18	0.99	1.43	1.99	2.42	2.89	3.55	4.10
17	0.98	1.42	1.98	2.41	2.87	3.52	4.05	0.98	1.41	1.96	2.38	2.82	3.46	3.97
18	0.98	1.41	1.95	2.37	2.81	3.43	3.94	0.98	1.40	1.93	2.34	2.77	3.37	3.86
19	0.97	1.40	1.93	2.34	2.76	3.36	3.84	0.98	1.40	1.91	2.31	2.72	3.30	3.76
20	0.97	1.39	1.91	2.31	2.72	3.29	3.76	0.98	1.39	1.89	2.28	2.68	3.23	3.68
21	0.97	1.39	1.90	2.28	2.68	3.24	3.68	0.98	1.38	1.87	2.25	2.64	3.17	3.60
22	0.97	1.38	1.88	2.26	2.65	3.18	3.61	0.97	1.37	1.86	2.23	2.60	3.12	3.54
23	0.97	1.37	1.87	2.24	2.62	3.14	3.55	0.97	1.37	1.84	2.20	2.57	3.07	3.47
24	0.97	1.37	1.85	2.22	2.59	3.09	3.50	0.97	1.36	1.83	2.18	2.54	3.03	3.42
25	0.97	1.36	1.84	2.20	2.56	3.06	3.45	0.97	1.36	1.82	2.16	2.51	2.99	3.37
26	0.96	1.36	1.83	2.18	2.54	3.02	3.40	0.97	1.35	1.81	2.15	2.49	2.96	3.33
27	0.96	1.35	1.82	2.17	2.51	2.99	3.36	0.97	1.35	1.80	2.13	2.47	2.93	3.28
28	0.96	1.35	1.81	2.15	2.49	2.96	3.32	0.97	1.34	1.79	2.12	2.45	2.90	3.25
29	0.96	1.35	1.80	2.14	2.48	2.93	3.29	0.97	1.34	1.78	2.10	2.43	2.87	3.21
30	0.96	1.34	1.79	2.13	2.46	2.91	3.25	0.97	1.34	1.77	2.09	2.41	2.84	3.18
35	0.96	1.33	1.76	2.07	2.39	2.80	3.12	0.96	1.32	1.74	2.04	2.34	2.74	3.05
40	0.96	1.32	1.74	2.04	2.33	2.73	3.03	0.96	1.31	1.71	2.00	2.29	2.66	2.95
50	0.95	1.30	1.70	1.99	2.26	2.63	2.90	0.96	1.30	1.68	1.95	2.22	2.56	2.82
100	0.95	1.27	1.64	1.89	2.12	2.43	2.66	0.95	1.27	1.61	1.85	2.08	2.37	2.58
∞	0.94	1.25	1.57	1.79	1.99	2.25	2.43	0.95	1.24	1.55	1.75	1.94	2.18	2.36



Quantiles of the F-Distribution

Tabulated values are q-th quantiles for $q \in \{.5, .75, .9, .95, .975, .99, .995\}$

denom-	numerator degrees of freedom													
inator	15							20						
df	.5	.75	.9	.95	.975	.99	.995	.5	.75	.9	.95	.975	.99	.995
1	2.09	9.49	61.2	246	985	6157	25E3	2.12	9.58	61.7	248	993	6209	25E3
2	1.38	3.41	9.42	19.4	39.4	99.4	199	1.39	3.43	9.44	19.4	39.4	99.4	199
3	1.21	2.46	5.20	8.70	14.3	26.9	43.1	1.23	2.46	5.18	8.66	14.2	26.7	42.8
4	1.14	2.08	3.87	5.86	8.66	14.2	20.4	1.15	2.08	3.84	5.80	8.56	14.0	20.2
5	1.10	1.89	3.24	4.62	6.43	9.72	13.1	1.11	1.88	3.21	4.56	6.33	9.55	12.9
6	1.07	1.76	2.87	3.94	5.27	7.56	9.81	1.08	1.76	2.84	3.87	5.17	7.40	9.59
7	1.05	1.68	2.63	3.51	4.57	6.31	7.97	1.07	1.67	2.59	3.44	4.47	6.16	7.75
8	1.04	1.62	2.46	3.22	4.10	5.52	6.81	1.05	1.61	2.42	3.15	4.00	5.36	6.61
9	1.03	1.57	2.34	3.01	3.77	4.96	6.03	1.04	1.56	2.30	2.94	3.67	4.81	5.83
10	1.02	1.53	2.24	2.85	3.52	4.56	5.47	1.03	1.52	2.20	2.77	3.42	4.41	5.27
11	1.02	1.50	2.17	2.72	3.33	4.25	5.05	1.03	1.49	2.12	2.65	3.23	4.10	4.86
12	1.01	1.48	2.10	2.62	3.18	4.01	4.72	1.02	1.47	2.06	2.54	3.07	3.86	4.53
13	1.01	1.46	2.05	2.53	3.05	3.82	4.46	1.02	1.45	2.01	2.46	2.95	3.66	4.27
14	1.00	1.44	2.01	2.46	2.95	3.66	4.25	1.01	1.43	1.96	2.39	2.84	3.51	4.06
15	1.00	1.43	1.97	2.40	2.86	3.52	4.07	1.01	1.41	1.92	2.33	2.76	3.37	3.88
16	1.00	1.41	1.94	2.35	2.79	3.41	3.92	1.01	1.40	1.89	2.28	2.68	3.26	3.73
17	0.99	1.40	1.91	2.31	2.72	3.31	3.79	1.01	1.39	1.86	2.23	2.62	3.16	3.61
18	0.99	1.39	1.89	2.27	2.67	3.23	3.68	1.00	1.38	1.84	2.19	2.56	3.08	3.50
19	0.99	1.38	1.86	2.23	2.62	3.15	3.59	1.00	1.37	1.81	2.16	2.51	3.00	3.40
20	0.99	1.37	1.84	2.20	2.57	3.09	3.50	1.00	1.36	1.79	2.12	2.46	2.94	3.32
21	0.99	1.37	1.83	2.18	2.53	3.03	3.43	1.00	1.35	1.78	2.10	2.42	2.88	3.24
22	0.99	1.36	1.81	2.15	2.50	2.98	3.36	1.00	1.34	1.76	2.07	2.39	2.83	3.18
23	0.98	1.35	1.80	2.13	2.47	2.93	3.30	1.00	1.34	1.74	2.05	2.36	2.78	3.12
24	0.98	1.35	1.78	2.11	2.44	2.89	3.25	0.99	1.33	1.73	2.03	2.33	2.74	3.06
25	0.98	1.34	1.77	2.09	2.41	2.85	3.20	0.99	1.33	1.72	2.01	2.30	2.70	3.01
26	0.98	1.34	1.76	2.07	2.39	2.81	3.15	0.99	1.32	1.71	1.99	2.28	2.66	2.97
27	0.98	1.33	1.75	2.06	2.36	2.78	3.11	0.99	1.32	1.70	1.97	2.25	2.63	2.93
28	0.98	1.33	1.74	2.04	2.34	2.75	3.07	0.99	1.31	1.69	1.96	2.23	2.60	2.89
29	0.98	1.32	1.73	2.03	2.32	2.73	3.04	0.99	1.31	1.68	1.94	2.21	2.57	2.86
30	0.98	1.32	1.72	2.01	2.31	2.70	3.01	0.99	1.30	1.67	1.93	2.20	2.55	2.82
35	0.97	1.31	1.69	1.96	2.23	2.60	2.88	0.99	1.29	1.63	1.88	2.12	2.44	2.69
40	0.97	1.30	1.66	1.92	2.18	2.52	2.78	0.98	1.28	1.61	1.84	2.07	2.37	2.60
50	0.97	1.28	1.63	1.87	2.11	2.42	2.65	0.98	1.26	1.57	1.78	1.99	2.27	2.47
100	0.96	1.25	1.56	1.77	1.97	2.22	2.41	0.97	1.23	1.49	1.68	1.85	2.07	2.23
∞	0.96	1.22	1.49	1.67	1.83	2.04	2.19	0.97	1.19	1.42	1.57	1.71	1.88	2.00



Quantiles of the F-Distribution

Tabulated values are q-th quantiles for $q \in \{.5, .75, .9, .95, .975, .99, .995\}$

denom-inator	numerator degrees of freedom													
	30							∞						
df	.5	.75	.9	.95	.975	.99	.995	.5	.75	.9	.95	.975	.99	.995
1	2.15	9.67	62.3	250	1001	6261	25E3	2.20	9.85	63.3	254	1018	6366	25E3
2	1.41	3.44	9.46	19.5	39.5	99.5	199	1.44	3.48	9.49	19.5	39.5	99.5	199
3	1.24	2.47	5.17	8.62	14.1	26.5	42.5	1.27	2.47	5.13	8.53	13.9	26.1	41.8
4	1.16	2.08	3.82	5.75	8.46	13.8	19.9	1.19	2.08	3.76	5.63	8.26	13.5	19.3
5	1.12	1.88	3.17	4.50	6.23	9.38	12.7	1.15	1.87	3.10	4.36	6.02	9.02	12.1
6	1.10	1.75	2.80	3.81	5.07	7.23	9.36	1.12	1.74	2.72	3.67	4.85	6.88	8.88
7	1.08	1.66	2.56	3.38	4.36	5.99	7.53	1.10	1.65	2.47	3.23	4.14	5.65	7.08
8	1.07	1.60	2.38	3.08	3.89	5.20	6.40	1.09	1.58	2.29	2.93	3.67	4.86	5.95
9	1.05	1.55	2.25	2.86	3.56	4.65	5.62	1.08	1.53	2.16	2.71	3.33	4.31	5.19
10	1.05	1.51	2.16	2.70	3.31	4.25	5.07	1.07	1.48	2.06	2.54	3.08	3.91	4.64
11	1.04	1.48	2.08	2.57	3.12	3.94	4.65	1.06	1.45	1.97	2.40	2.88	3.60	4.23
12	1.03	1.45	2.01	2.47	2.96	3.70	4.33	1.06	1.42	1.90	2.30	2.72	3.36	3.90
13	1.03	1.43	1.96	2.38	2.84	3.51	4.07	1.05	1.40	1.85	2.21	2.60	3.17	3.65
14	1.03	1.41	1.91	2.31	2.73	3.35	3.86	1.05	1.38	1.80	2.13	2.49	3.00	3.44
15	1.02	1.40	1.87	2.25	2.64	3.21	3.69	1.05	1.36	1.76	2.07	2.40	2.87	3.26
16	1.02	1.38	1.84	2.19	2.57	3.10	3.54	1.04	1.34	1.72	2.01	2.32	2.75	3.11
17	1.02	1.37	1.81	2.15	2.50	3.00	3.41	1.04	1.33	1.69	1.96	2.25	2.65	2.98
18	1.02	1.36	1.78	2.11	2.44	2.92	3.30	1.04	1.32	1.66	1.92	2.19	2.57	2.87
19	1.01	1.35	1.76	2.07	2.39	2.84	3.21	1.04	1.30	1.63	1.88	2.13	2.49	2.78
20	1.01	1.34	1.74	2.04	2.35	2.78	3.12	1.03	1.29	1.61	1.84	2.09	2.42	2.69
21	1.01	1.33	1.72	2.01	2.31	2.72	3.05	1.03	1.28	1.59	1.81	2.04	2.36	2.61
22	1.01	1.32	1.70	1.98	2.27	2.67	2.98	1.03	1.28	1.57	1.78	2.00	2.31	2.55
23	1.01	1.32	1.69	1.96	2.24	2.62	2.92	1.03	1.27	1.55	1.76	1.97	2.26	2.48
24	1.01	1.31	1.67	1.94	2.21	2.58	2.87	1.03	1.26	1.53	1.73	1.94	2.21	2.43
25	1.00	1.31	1.66	1.92	2.18	2.54	2.82	1.03	1.25	1.52	1.71	1.91	2.17	2.38
26	1.00	1.30	1.65	1.90	2.16	2.50	2.77	1.03	1.25	1.50	1.69	1.88	2.13	2.33
27	1.00	1.30	1.64	1.88	2.13	2.47	2.73	1.03	1.24	1.49	1.67	1.85	2.10	2.29
28	1.00	1.29	1.63	1.87	2.11	2.44	2.69	1.02	1.24	1.48	1.65	1.83	2.06	2.25
29	1.00	1.29	1.62	1.85	2.09	2.41	2.66	1.02	1.23	1.47	1.64	1.81	2.03	2.21
30	1.00	1.28	1.61	1.84	2.07	2.39	2.63	1.02	1.23	1.46	1.62	1.79	2.01	2.18
35	1.00	1.27	1.57	1.79	2.00	2.28	2.50	1.02	1.20	1.41	1.56	1.70	1.89	2.04
40	0.99	1.25	1.54	1.74	1.94	2.20	2.40	1.02	1.19	1.38	1.51	1.64	1.80	1.93
50	0.99	1.23	1.50	1.69	1.87	2.10	2.27	1.01	1.16	1.33	1.44	1.55	1.68	1.79
100	0.98	1.20	1.42	1.57	1.71	1.89	2.02	1.01	1.11	1.21	1.28	1.35	1.43	1.49
∞	0.98	1.16	1.34	1.46	1.57	1.70	1.79	1.00	1.00	1.00	1.00	1.00	1.00	1.00