UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON4160 – Econometrics – modelling and systems estimation**

Date of exam: Monday, November 26, 2012 **Grades will be given: 18. Dec. 2012**

Time for exam: 2:30 p.m. – 5:30 p.m.

The problem set covers 5 pages (incl. cover page)

Resources allowed:

- All printed and written resources, as well as calculator, is allowed.

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

**ECON 4160: ECONOMETRICS –
MODELLING AND SYSTEMS ESTIMATION**

PROBLEM SET, EXAM AUTUMN 2012

PROBLEM 1 *(weight: 40 %)*

A simple Keynesian macro model with a consumption function and a budget equation is:

$$C_t = \alpha + \beta Y_t + u_t, \tag{1.1}$$
$$Y_t = C_t + I_t + Z_t, \tag{1.2}$$

where t indicates year, Y_t (= Gross Domestic Product, GDP) and C_t (= Private Consumption) are endogenous, I_t (= Gross Investment) and Z_t (= Public Consumption and other final demand) are exogenous, and u_t is a disturbance. All variables are measured at constant prices.

1A. Make precise the exogeneity assumptions and complete the model description. Four estimators for the marginal propensity to consume, β, have been proposed:

$$\widehat{\beta}_1 = \frac{M[C,Y]}{M[Y,Y]},$$
$$\widehat{\beta}_2 = \frac{M[C,C]}{M[Y,C]},$$
$$\widetilde{\beta}_1 = \frac{M[C,I]}{M[Y,I]},$$
$$\widetilde{\beta}_2 = \frac{M[C,I]+M[C,Z]}{M[Y,I]+M[Y,Z]},$$

where, in general, $M[V,W]$ denotes the empirical covariance between arbitrary variables, V and W, based on T observations ($t = 1,\ldots,T$). Which of the estimators are consistent and which can be interpreted as Instrumental Variable estimators? Explain briefly, without giving proofs. Demonstrate consistency for one of the estimators which you think has this property. Which estimator would you prefer if you were to choose one? Do you need to see the dataset before you can give your advice? State briefly the reason for your answers.

1B. Show that $\widehat{\beta}_1/\widehat{\beta}_2 < 1$ always holds, and interpret this result in terms of Ordinary Least Squares (OLS).

1C. The above model is a kind of demand model which relies on the assumption that the economy runs below its capacity limit. Assume now instead that the economy runs at full capacity and that this is modeled by treating Y as exogenous, determined from the supply side of the economy, and that you instead let gross investment I be endogenous. Reformulate the stochastic assumptions for your model in agreement with this, and explain briefly which of the estimators in question **1A** you would then prefer.

1D. Have you a suggestion of how you could test statistically whether exogeneity of Y in relation to equation (1.1) is a valid assumption?

PROBLEM 2 *(weight: 40 %)*

In this problem, we consider a modified version of the Keynesian model (1.1)–(1.2) with private consumption C_t split into two components, C_{1t} and C_{2t}, and their price indices relative to the overall price index of consumption, P_{1t} and P_{2t}, respectively, have been added as explanatory variables. This gives the three-equation model

$$
\begin{aligned}
&(2.1) \qquad & C_{1t} &= \alpha_1 + \beta_1 Y_t + \gamma_1 P_{1t} + u_{1t}, \\
&(2.2) \qquad & C_{2t} &= \alpha_2 + \beta_2 Y_t + \gamma_2 P_{2t} + u_{2t}, \\
&(2.3) \qquad & Y_t &= C_{1t} + C_{2t} + Q_t,
\end{aligned}
$$

where Y_t, C_{1t} and C_{2t} are endogenous, $Q_t = I_t + Z_t$ (= Total final demand), P_{1t} and P_{2t} are exogenous, and u_{1t} and u_{2t} are non-autocorrelated disturbances with zero means, satisfying

$$\mathrm{var}(u_{1t}) = \sigma_{11}, \quad \mathrm{var}(u_{2t}) = \sigma_{22}, \quad \mathrm{cov}(u_{1t}, u_{2t}) = \sigma_{12} \quad \text{for all } t.$$

Eliminating Y_t in (2.1)–(2.2) by means of (2.3), we get a two-equation system

$$
\begin{aligned}
(1-\beta_1)C_{1t} &= \alpha_1 + \beta_1(C_{2t}+Q_t) + \gamma_1 P_{1t} + u_{1t}, \\
(1-\beta_2)C_{2t} &= \alpha_2 + \beta_2(C_{1t}+Q_t) + \gamma_2 P_{2t} + u_{2t},
\end{aligned}
$$

and hence

$$
\begin{aligned}
&(2.4) \qquad & C_{1t} &= \frac{\alpha_1}{1-\beta_1} + \frac{\beta_1}{1-\beta_1}(C_{2t}+Q_t) + \frac{\gamma_1}{1-\beta_1}P_{1t} + \frac{1}{1-\beta_1}u_{1t}, \\
&(2.5) \qquad & C_{2t} &= \frac{\alpha_2}{1-\beta_2} + \frac{\beta_2}{1-\beta_2}(C_{1t}+Q_t) + \frac{\gamma_2}{1-\beta_2}P_{2t} + \frac{1}{1-\beta_2}u_{2t}.
\end{aligned}
$$

2A. Discuss the identification of the equations in model (2.1)–(2.3). It has been suggested to estimate $\beta_1, \gamma_1, \beta_2, \gamma_2$ by:

- Estimate $\beta_1/(1-\beta_1)$ and $\gamma_1/(1-\beta_1)$ from (2.4), using Q_t as instrument for $C_{2t}+Q_t$.
- Estimate $\beta_2/(1-\beta_2)$ and $\gamma_2/(1-\beta_2)$ from (2.5), using Q_t as instrument for $C_{1t}+Q_t$.

The following objection has been raised against this procedure: "Using Q_t as an instrument for $C_{2t}+Q_t$ and $C_{1t}+Q_t$ is invalid. Because both the latter variables contain Q_t, observations on Q_t are exploited both in constructing two variables and as an instrument. This will not work." Do you agree? State briefly the reason for your answer.

2B. Another procedure is to estimate equation (2.1) and (2.2) by the Two-Stage Least Squares (2SLS). Explain briefly – estimation formulae not required. Would you prefer this procedure to the one proposed in question **2A**? Explain.

2C. Make now the same changed assumption about a capacity constrained economy as the one made in question **1C** and consider Y_t as exogenously determined from the supply side, along with P_{1t} and P_{2t}, and Q_t as endogenous (it may well contain exogenous parts). Explain how you would estimate the two-equation system (2.1)–(2.2) under this new set of assumptions.

PROBLEM 3 *(weight: 20 %)*

We want to model a simple equation explaining a measure of money velocity ("pengenes omløpshastighet") by means of a measure of the interest rate. Quarterly, seasonally adjusted time series for 75 quarters (1980.4–1999.2) are available for:

```
M3E11:  Euro-zone M3 in 1000 000 000 Euro
YnE11:  Euro-zone nominal GDP in 1000 000 000 Euro
YrE11:  Euro-zone real GDP in 1000 000 000 Euro
RLDE:   Government bond yield, Germany
```

from which we have constructed observations on the variables

```
logm  = log(M3E11)       = log(nominal monetary stock)
logy  = log(YrE11)       = log(real GDP)
logp  = log(YnE11/YrE11) = log(price index GDP)
logvc = logm-logp-logy   = log(money velocity)
r     = RLDE
```

The interest rate is measured as a decimal number (0.05 represents a 5 % pro anno rate etc.) The sample means and standard deviation of `r` are 0.0682986 and 0.0148256, respectively. (The actual data series are longer than 75 quarters. Up to four additional observations are used to construct lags.)

For simplicity, `r` is considered as exogenous.

Slightly edited printouts from PcGive for four specifications estimated by the OLS, are given at the end of the problem set. x_i indicates that the variable x is lagged i periods. Printout EQ(4) is from a regression where a smoothed interest rate, `rs`, is used as regressor. It is calculated as:

```
rs = [5*r + 4*r(-1) + 3*r(-2) + 2*r(-3) + r(-4)]/15.
```

3A. Compute from printouts EQ(2) – EQ(4) estimates of the short-run and the long-run effect of the German interest rate on the log of the money velocity in the Euro zone. Does the negative sign of most of the estimates agree with your expectations?

3B. The estimated short-run effects in printout EQ(2) – EQ(4) differ considerably across the specifications. The long-run effects are more similar and are also more in line with the effect estimated from the static equation, EQ(1). Can you explain this?

```
EQ(1) Modelling logvc by OLS

                  Coefficient  Std.Error  t-value  t-prob
Constant              1.12091    0.02137     52.4  0.0000
r                   -0.911422     0.2975    -3.06  0.0031

sigma               0.0416628  RSS             0.126712736
R^2                  0.113931  F(1,73) =     9.386 [0.003]**
no. of observations        75  no. of parameters         2
mean(logvc)           1.05711  se(logvc)         0.0439603

AR 1-5 test:      F(5,68)   =   267.03 [0.0000]**
Normality test:   Chi^2(2)  =   5.8369 [0.0540]

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EQ(2) Modelling logvc by OLS

                  Coefficient  Std.Error  t-value  t-prob
logvc_1              0.982860    0.02353     41.8  0.0000
Constant            0.0226242    0.02664    0.849  0.3986
r                  -0.0490073    0.06311   -0.777  0.4400

sigma              0.00835232  RSS           0.00502281168
R^2                  0.964877  F(2,72) =       989 [0.000]**
no. of observations        75  no. of parameters         3
mean(logvc)           1.05711  se(logvc)         0.0439603

AR 1-5 test:      F(5,67)   =   1.0162 [0.4152]
Normality test:   Chi^2(2)  =   2.7275 [0.2557]

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EQ(3) Modelling logvc by OLS

                  Coefficient  Std.Error  t-value  t-prob
Constant              1.12349    0.02541     44.2  0.0000
r                   -0.645174      1.007   -0.641  0.5238
r_1                 -0.202624      1.464   -0.138  0.8903
r_2                 -0.195377      1.443   -0.135  0.8927
r_3                  0.241373      1.438    0.168  0.8672
r_4                 -0.143070     0.9969   -0.144  0.8863

sigma               0.0428162  RSS             0.126492401
R^2                  0.115472  F(5,69) =       1.802 [0.124]
no. of observations        75  no. of parameters         6
mean(logvc)           1.05711  se(logvc)         0.0439603

AR 1-5 test:      F(5,64)   =   328.10 [0.0000]**
Normality test:   Chi^2(2)  =   5.9522 [0.0510]

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EQ(4) Modelling logvc by OLS

                  Coefficient  Std.Error  t-value  t-prob
Constant              1.12572    0.02320     48.5  0.0000
rs                  -0.969060     0.3206    -3.02  0.0035

sigma               0.0417263  RSS             0.127099203
R^2                  0.111228  F(1,73) =     9.136 [0.003]**
no. of observations        75  no. of parameters         2
mean(logvc)           1.05711  se(logvc)         0.0439603

AR 1-5 test:      F(5,68)   =   341.09 [0.0000]**
Normality test:   Chi^2(2)  =   5.8387 [0.0540]
```