Exam in: ECON 4160: Econometrics: Modelling and Systems Estimation–ANSWER NOTES

Day of exam: 2 December 2013

Time of day: 14:00—17:00

This is a 3 hour school exam.

Guidelines:
In the grading, question A will count 1/3, and question B will count 2/3 .

Question A (1/3)

1. (a) Since the parameters of interest (POIs) are in the conditional expectation ("regression model"), and the classical disturbance properties hold, the OLS estimators of the POIs are efficient: There is no extra information about the POIs in the marginal model for X_t. Hence X_t is WE for the POIs ϕ_0 and β.

   (b) X_t is SU for, for example β, if X_t is WE for β, and β is invariant to structural breaks elsewhere in the joint statistical model of Y_t and $X_{t.}$

   As a regression parameter β can be invariant, but it needs not be. For example writing it as

$$\beta = Corr(Y, X)\frac{\sigma_Y}{\sigma_X}$$

   shows that β is invariant if $Corr(Y, X)$ changes proportionally to a break in σ_X. It is possible to frame the answer more in the direction of testing for invariance, by the use of the stability, or not, of $\hat{\beta}$ in periods with (empirically) identified breaks in the marginal model, which is also relevant, of course.

2. If X_t is not Granger-caused by Y_t, the characteristic root that determines whether we have stationarity or not is simply ϕ_1. But if there is joint Granger causality, the relevant characteristic roots are more complicated, derived parameters of the VAR (i.e. the system). Hence

X_t is not WE in general, only if there is one-way Granger causality (X_t is then also strongly exogenous).

3. Optimal MM is OLS

$$\hat{\boldsymbol{\beta}}_{OLS} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$$

   (a) Optimal MM is the IV estimator with the instrumental variable matrix $\mathbf{W}$

$$\hat{\boldsymbol{\beta}}_{IV} = (\mathbf{W}'\mathbf{X})^{-1}\mathbf{W}'\mathbf{y}$$

   (b) Optimal MM is GLS:

$$\hat{\boldsymbol{\beta}}_{GLS} = (\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\Omega}^{-1}\mathbf{y}$$

   Of course, there is a lot more that can be said here, including exact identification and extension to overidentification in b), and potentially also to GMM, and feasibility of GLS in c). Positive if student mention this, if all the other questions are covered.

Question B (2/3)

1. (a) Exact identification by the order cond., and by the rank cond. if $\eta_{21} \neq 0$ and $\eta_{12} \neq 0$.

   (b) Overidentification (of the second equation), assuming that rank is also fulfilled

2. IV in a). 2SLS (also called GIVE) in b). One overid., restriction that can also be tested by LR test (URF versus restricted). If ϵ_{1t} and ϵ_{2t} are contemporaneously correlated, FIML is more efficient than 2SLS and IV, and should then definitively be used.

3. The null-hypothesis that Y is not Granger-causing X: "t-value" associated with t-ratio 0.163 does not reject at 5 % or 10 % level of significance.

   X is not Granger-causing Y: t-value associated with t-ratio 6.71 clearly rejects at very low (<0.001) significance levels.

   The information given about the residuals diagnostics is relevant to mention here, since autocorrelation and or non-normality for example, would damage the reliability of these t-tests.

4. The model

$$Y_t = \beta_0 + \rho Y_{t-1} + \beta_1 X_t + \beta_2 X_{t-1} + \varepsilon_t$$

is a conditional regression model derived from the bivariate VAR(1) that we have estimation results for here. Since Granger causation is found to be one-way from X to Y in Q3, X is strongly-exogenous in the ARDL, and ΔX_t will also be in the ECM version of this equation.

   (a) Based on the VAR we can formulate the condition-marginal equation system:

   $$\begin{aligned} Y_t &= \beta_0 + \rho Y_{t-1} + \beta_1 X_t + \beta_2 X_{t-1} + \varepsilon_t \\ X_t &= a_{20} + a_{21} Y_{t-1} + a_{22} X_{t-1} + \varepsilon_{2,t} \end{aligned}$$

   which is a re-parameterization of the VAR. For example $Cov(\varepsilon_t, \varepsilon_{2,t}) = 0$. The maximised "log-likelihood" of this model is the same as for the unrestricted VAR. The same is true for the model we have estimation result for, namely

   $$\begin{aligned} \Delta Y_t &= \beta_0 + (\rho - 1) Y_{t-1} + \beta_1 \Delta X_t + (\beta_1 + \beta_2) X_{t-1} + \varepsilon_t \\ \Delta X_t &= a_{20} + a_{21} Y_{t-1} + a_{22} X_{t-1} + \varepsilon_{2,t} \end{aligned}$$

   since the two disturbances are unaffected by this (second) re-paramerization that changes the left-hand side variables form levels to differences. The increase in parameters from 6 to 7 is because the correlation of the VAR disturbances has been "moved to" the regression parameter β_1. The endogenous variables are different in the two models.

   (b) If not already noted: No: The disturbances of the (valid) conditional model and the marginal model are uncorrelated by construction. This is not affected by writing the model in ECM form.

5. Since we have one-way Granger-causality we can base inference on the conditional model for ΔY_t. As a matter of fact ΔX_t is WE for the cointegration parameter (if it exists).

   Based on that: Under null hypothesis of no-cointegration must have $(\rho - 1) = 0$ since Y_{t-1} must then be uncorrelated asymptotically with ΔY_t. But cannot use t-prob. directly from the table to test $(\rho - 1) = 0$, since they are based on stationary variables, Need the critical values of the relevant Dickey-Fuller type distributions, which are given. -8.41 rejects at both levels of significance.

6. Under the assumption of cointegration (and one way causality) the estimated long-run coefficient is 0.80. To apply the delta-method to construct an asymptotic confidence interval, need only the estimated covariance of the two level coefficients in the equation for ΔY_t.