UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Postponed exam: **ECON4160 – Econometrics – Modeling and systems estimation**

Date of exam: Wednesday, January 8, 2014

Time for exam: 09:00 a.m. – 12:00 noon

The problem set covers 7 pages (incl. cover sheet)

Resources allowed:

- All written and printed resources, as well as calculator, is allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

Postponed exam in: ECON 4160: Econometrics: Modelling and Systems Estimation

Day of exam: 8 January 2014,

Time of day: 9:00—12:00

This is a 3 hour school exam.

Guidelines:
In the grading, question A will count 20 %, and B and C will count 40 % each .

Question A (20 %)

Let $\mathbf{y}$ be a vector $(n \times 1)$ with n observations of some variable y, and let $\mathbf{X}$ be a $n \times k$ matrix with observations of k explanatory variables. Consider the linear relationship

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{1}$$

where $\boldsymbol{\varepsilon}$ is the $n \times 1$ vector with disturbances, and $\boldsymbol{\beta}$ is the $k \times 1$ vector with parameters.

1. Assume that $(\mathbf{X}'\mathbf{X})$ is non-singular. Explain why the OLS estimator of $\boldsymbol{\beta}$ is given by:

   $$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}. \tag{2}$$

2. Show that

   $$\boldsymbol{X}'\hat{\boldsymbol{\varepsilon}} = \mathbf{0}$$

   where $\hat{\boldsymbol{\varepsilon}}$ is the vector of OLS residuals. Use this result to explain why the OLS estimator is identical to the method of moments estimator of $\boldsymbol{\beta}$.

3. Show that

   $$\mathbf{y} = \hat{\mathbf{y}} + \hat{\boldsymbol{\varepsilon}}$$

   where $\hat{\mathbf{y}}$ contains the OLS predictions for the y variable**.** Give a brief interpretation.

Question B (40 %)

Assume that we know that the time series $\{Y_t; t = 0, \pm 1, \pm 2, ...\}$ has been generated by one of the following three processes:

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \varepsilon_t, \ -1 < \phi_1 < 1 \tag{3}$$

$$Y_t = \phi_0 + Y_{t-1} + \delta t + \varepsilon_t \tag{4}$$

$$Y_t = \phi_0 + \delta t + \varepsilon_t, \ \ \delta \neq 0 \tag{5}$$

In all the three possible DGPs, it is assumed that ε_t is Gaussian white-noise, which we write as $\varepsilon_t \sim IIN(0; \sigma^2)$ for all t.

1. In each of the cases (3)-(5), characterize Y_t as weakly stationary (covariance stationary) or non-stationary. Explain what motivates your answer.
2. What is meant by a trend-stationary variable? Is Y_t trend-stationary in any of the DGPs (3)-(5)?
3. Assume that you have a data sample $\{Y_t; t = 1, 2, ..., T\}$. Describe how you could investigate empirically which of (3)-(5) is the true DGP.

Question C (40 %)

At the end of the question set, in *Display 1*, you find estimation results of a VAR in the two variables *Lmenns* (the natural logarithm of the male suicide rate in Norway) and *LU* (the natural logarithm of the Norwegian unemployment rate). We assume that both series are covariance stationary when we condition on the year dummies *I:1916*, *I:1921*,*I:1941* and *I:1945*.

1. The VAR estimation results in *Display 1* are obtained by OLS. Under which assumptions are the reported *t-prob* reliable for conducting inference on the individual significance of the parameters?
2. Under which assumptions do least squares estimation give approximate ML estimators of the VAR parameters?
3. *Display 2* shows an econometric model of the VAR that has been estimated by FIML. Which assumptions regarding identification have been imposed on the model in *Display 2*?

4. Explain the calculation of the "LR test of over identifying restrictions" in *Display 2*. How do you interpret the result from this test?

5. Explain how you would proceed to test the hypothesis of one-way Granger causality, from the rate of unemployment to the male suicide rate.

6. Consider next the results in *Display 3,* where each of the equations of this model of the VAR have been estimated by OLS (referred to as 1SLS in the display). What is the interpretation of the different results of the "LR test of over identifying restrictions" in *Display 3* and in *Display 2*?

7. In the light of the results in the displays, how would you prefer to calculate the effects of a shock to the suicide rate (*Lmenns*)?

   (No complete calculations are expected (because of the complicated dynamics), just an explanation of the method).

Empirical results for question C

Display 1: Estimation results for the unrestricted VAR

```
SYS(1) Estimating the system by OLS
       The dataset is: D:\sw20\ECON5101\Suicide\selvm2rny.in7
       The estimation sample is: 1907 - 2004

URF equation for: Lmenns
                 Coefficient  Std.Error  t-value  t-prob
Lmenns_1            0.569744    0.07992     7.13  0.0000
Lmenns_2            0.239402    0.08740     2.74  0.0075
Lmenns_3            0.162957    0.08094     2.01  0.0472
LU_1               0.0164866    0.03565    0.462  0.6449
LU_2              -0.0218148    0.05093   -0.428  0.6694
LU_3              -0.0227240    0.03605   -0.630  0.5302
I:1916             -0.352406    0.09660    -3.65  0.0004
I:1921              0.289300    0.09907     2.92  0.0045
I:1941             -0.470867    0.09453    -4.98  0.0000
I:1945              0.562765    0.09405     5.98  0.0000
Constant            0.109393    0.08188     1.34  0.1850

sigma = 0.0921814   RSS = 0.7392741107

URF equation for: LU
                 Coefficient  Std.Error  t-value  t-prob
Lmenns_1            0.290383     0.1854     1.57  0.1208
Lmenns_2           -0.186992     0.2027   -0.923  0.3588
Lmenns_3          -0.0247408     0.1877   -0.132  0.8954
LU_1                 1.03238    0.08267     12.5  0.0000
LU_2               -0.326933     0.1181    -2.77  0.0069
LU_3                0.232311    0.08361     2.78  0.0067
I:1916             -0.894987     0.2240    -3.99  0.0001
I:1921               1.39092     0.2298     6.05  0.0000
I:1941             -0.717391     0.2192    -3.27  0.0015
I:1945          -0.000432653     0.2181 -0.00198  0.9984
Constant           -0.127183     0.1899   -0.670  0.5048

sigma = 0.213779   RSS = 3.976036374

log-likelihood     118.430493  -T/2log|Omega|     396.542446
```

Display 2: FIML estimation results for a model of the VAR in Display 1

```
MOD( 1) Estimating the model by FIML
       The dataset is: D:\sw20\ECON5101\Suicide\selvm2rny.in7
       The estimation sample is: 1907 - 2004

 Equation for: Lmenns
                  Coefficient  Std.Error  t-value  t-prob
LU                   0.656369     0.2351     2.79  0.0065
Lmenns_1             0.379138     0.1556     2.44  0.0169
Lmenns_2             0.362154     0.1625     2.23  0.0284
Lmenns_3             0.179174     0.1437     1.25  0.2158
LU_1                -0.661135     0.2444    -2.71  0.0082
LU_2                 0.192772     0.1128     1.71  0.0909
LU_3                -0.175203    0.07942    -2.21  0.0300
I:1916               0.235034     0.2750    0.855  0.3951
I:1921              -0.623674     0.3736    -1.67  0.0986
I:1945               0.562771    0.09347     6.02  0.0000
Constant             0.192910     0.1487     1.30  0.1980

sigma = 0.164533

Equation for: LU
                  Coefficient  Std.Error  t-value  t-prob
Lmenns_1             0.290394     0.1842     1.58  0.1185
Lmenns_2            -0.187017     0.2012   -0.930  0.3551
Lmenns_3           -0.0247057     0.1858   -0.133  0.8945
LU_1                  1.03238    0.08220     12.6  0.0000
LU_2                -0.326930     0.1174    -2.78  0.0066
LU_3                 0.232307    0.08312     2.79  0.0064
I:1916              -0.894986     0.2228    -4.02  0.0001
I:1921                1.39095     0.2281     6.10  0.0000
I:1941              -0.717381     0.2179    -3.29  0.0014
Constant            -0.127241     0.1866   -0.682  0.4970

sigma = 0.212561

log-likelihood     118.430491  -T/2log|Omega|      396.542443
no. of observations        98  no. of parameters           21

LR test of over-identifying restrictions: Chi^2(1)  =4.4323e-006 [0.9983]

correlation of structural residuals (standard deviations on diagonal)
                   Lmenns          LU
Lmenns            0.16453    -0.83065
LU               -0.83065     0.21256
```

Display 3: 1SLS estimation results for a model of the VAR in Display 1

```
MOD(2) Estimating the model by 1SLS
       The dataset is: D:\sw20\ECON5101\Suicide\selvm2rny.in7
       The estimation sample is: 1907 - 2004

Equation for: Lmenns
                  Coefficient  Std.Error  t-value  t-prob
LU                  0.0838731    0.04863     1.72  0.0881
Lmenns_1             0.531749    0.08997     5.91  0.0000
Lmenns_2             0.248884    0.09789     2.54  0.0128
Lmenns_3             0.191907    0.08997     2.13  0.0357
LU_1               -0.0864602    0.06280    -1.38  0.1722
LU_2                0.0287205    0.05815    0.494  0.6226
LU_3               -0.0604446    0.04109    -1.47  0.1449
I:1916              -0.286667     0.1164    -2.46  0.0158
I:1921               0.178165     0.1297     1.37  0.1732
I:1945               0.572034     0.1048     5.46  0.0000
Constant             0.110700    0.09151     1.21  0.2297

sigma = 0.102175

Equation for: LU
                  Coefficient  Std.Error  t-value  t-prob
Lmenns_1             0.290394     0.1842     1.58  0.1185
Lmenns_2            -0.187017     0.2012   -0.930  0.3551
Lmenns_3           -0.0247057     0.1858   -0.133  0.8945
LU_1                  1.03238    0.08220     12.6  0.0000
LU_2                -0.326930     0.1174    -2.78  0.0066
LU_3                 0.232307    0.08312     2.79  0.0064
I:1916              -0.894986     0.2228    -4.02  0.0001
I:1921                1.39095     0.2281     6.10  0.0000
I:1941              -0.717381     0.2179    -3.29  0.0014
Constant            -0.127241     0.1866   -0.682  0.4970

sigma = 0.212561

log-likelihood     107.735613  -T/2log|Omega|     385.847565
no. of observations        98  no. of parameters          21

LR test of over-identifying restrictions: Chi^2(1)  =    21.390 [0.0000]**

correlation of structural residuals (standard deviations on diagonal)
                   Lmenns          LU
Lmenns            0.10218     0.00000
LU                0.00000     0.21256
```