UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON4160 – Econometrics – Modeling and systems estimation**

Date of exam: Friday, December 19, 2014 **Grades are given: January 6, 2015**

Time for exam: 09.00 a.m. – 12.00 noon

The problem set covers 11 pages (incl. cover sheet)

Resources allowed:
- All written and printed resources, as well as calculator, is permitted.

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

Exam in: ECON 4160: Econometrics: Modelling and Systems Estimation

Day of exam: 19 December 2014

Time of day: 09:00—12:00

This is a 3 hour school exam.

Guidelines:
In the grading, question A will count 1/4, and question B will count 3/4 .

Question A (1/4)

Let $\mathbf{y}$ be a vector $(n \times 1)$ with n observations of a variable Y, and let $\mathbf{X}$ be a $n \times k$ matrix with observations of k explanatory variables. Consider the linear relationship

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{1}$$

where $\boldsymbol{\varepsilon}$ is a $n \times 1$ vector with disturbances, and $\boldsymbol{\beta}$ is the $k \times 1$ vector with parameters.

1. Assume that the sample moments $(\boldsymbol{X}'\boldsymbol{X})^{-1}$ and $\boldsymbol{X}'\boldsymbol{y}$ are given by:

$$(\boldsymbol{X}'\boldsymbol{X})^{-1} = \begin{bmatrix} 1 & -10 \\ 10 & 1 \end{bmatrix}$$

   and

$$\boldsymbol{X}'\boldsymbol{y} = \begin{bmatrix} 1 \\ 10 \end{bmatrix}$$

   (a) What are the OLS estimates of the parameters in this example?

   (b) Explain why these estimates are identical to the estimates you would have obtained if you had used the method of moments as the estimation principle.

2. Assume that you get data for a new variable, Z, where the n observations are collected in the $n \times 1$ vector $\mathbf{z}$. Assume that all k elements in the vector $(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\mathbf{z}$ are close to zero.

(a) How will the OLS estimate of $\boldsymbol{\beta}$ be affected if you decide to include Z as a new regressor in equation (1)?

(b) An economic theorist suggests that it would be a good idea to use to Z as an instrumental variable "in the regression (1)" in order to avoid suspected simultaneity bias. How would you (as an econometrician) respond to his suggestion?

Question B (3/4)

In this question, we study the relationship between Norwegian exports of traditional goods and an export market indicator. The data set is quarterly, and the variables are seasonally unadjusted.

The export variable is denoted $ATRAD_t$, and the market indicator is denoted MII_t. Both are measured in real terms, i.e. they are volumes. We will use the log-transformed time series, which we denote $LATRAD_t$ and $LMII_t$ and the differenced series $DLATRAD_t = LATRAD_t - LATRAD_{t-1}$ and $DLII_t = LII_t - LII_{t-1}$. For reference, Figure 1 shows the time plots of the two level variables and their differences.

1. Table 1 shows Augmented Dickey Fuller test statistics. Explain how you can use the results to test the unit-root hypothesis for $LATRAD_t$ and LII_t, and give your conclusions.

2. We define the vector $\boldsymbol{y}_t$ as $\boldsymbol{y}_t = (\ ATRAD_t \quad LMII_t\ )'$, and the vector $\Delta\boldsymbol{y}_t$ with the differenced variables as $\Delta\boldsymbol{y}_t = (\ DATRAD_t \quad DLMII_t\ )'$. We first analyse the differenced data, and formulate a bivariate first order VAR as

   (2) $$\Delta\boldsymbol{y}_t = \boldsymbol{A}\Delta\boldsymbol{y}_{t-1} + \boldsymbol{C}\boldsymbol{G}_t + \boldsymbol{\varepsilon}_t$$

   where $\boldsymbol{A}$ and $\boldsymbol{C}$ are matrices with parameters, and $\boldsymbol{G}_t$ is a matrix with deterministic variables: An intercept, three (centered) seasonals and one or more indicator variables (impulse dummies) for structural breaks. The vector $\boldsymbol{\varepsilon}_t$ with VAR disturbances is assumed to follow a bivariate normal distribution.

   Table 2 shows estimation results for (2) when $\boldsymbol{G}_t$ includes three indicator variables, for 1980q2, 2008q4 and 2009q1.

(a) The roots of the estimated companion matrix of this system are -0.55 and 0.41 (two real roots). What does this indicate about the dynamic stability of the system, and does it confirm or contradict your conclusion in Question B.1?

(b) Since the disturbances in $\boldsymbol{\varepsilon}_t$ are generally correlated, they are not structural disturbances. A fellow student claims that the structural disturbances can be achieved by putting zero restrictions on one of the off-diagonal elements in $\boldsymbol{A}$ (so that the matrix becomes either upper triangular or lower triangular). Explain why this is not correct.

(c) Consider the estimated model in Table 3, and explain why this is an example of identification of the structural impulse response functions by the use of the Cholesky-decomposition.

(d) Another identification scheme assumes, first, that there should be no seasonal indicator variables in the structural equation for $DLMII$, since $LMII$ is a variable which is a broad average of seasonally adjusted GDP data series (of Norway's trading partner). Second, since the three dummies for 1980q2, 2008q4 and 2009q1 represent structural changes in export marked growth, they should not be included in the structural equation for $DLATRAD_t$. Explain why this identification scheme implies that the degree of over-identification is 4, as shown in Table 4.

(e) Show how the test of over-identifying restrictions in Table 4 can be calculated, using the information provided in the tables. Explain why the test of over-identifying restrictions can be interpreted as an encompassing test.

(f) What is the estimated impact effect on $DLATRAD$ of a positive shock to $DLMII$, when you use the identified model in Table 3, and what is the estimate if you instead consider the over-identified model in Table 4.

3. Table 5 shows a single equation model of $DLATRAD_t$ estimated by OLS, where we for simplicity have omitted $DLMII_{t-1}$. Figure 2 contains some relevant recursive graphs for this model. Consider the coefficient of $DLMII_t$ as the parameter of interest. Would you say that there is evidence of super-exogeneity of $DLMII_t$ with respect to the

parameter of interest? Give a brief motivation for your answer (without formal tests)

4. Finally, we consider the possibility that $LATRAD_t$ is cointegrated with $LMII_t$ and a third $I(1)$ variable $LREX_t$, which is the natural logarithm of the Norwegian real exchange rate. Use the results in Table 6 to test formally the null of no cointegration, against the alternative of a single cointegration relationship. Relevant critical values are -3.62 (1 %), -3.00 (5 %) and -2.26 (10 %). If you conclude with rejection of the null hypothesis, what is the estimated cointegration relationship?

5. An important theoretical result in time series econometrics says that the method used in Question B.4 is statistically efficient only if $LMII_t$ and $LREX_t$ are weakly exogenous variables with respect to the cointegration parameters. Try to summarize, in a few sentences, a method that would allow the hypothesis of no cointegration between $LATRAD$, $LMII$ and $LREX$ to be tested without making the assumption of weak exogeneity.

Graphs and estimation results for question B.

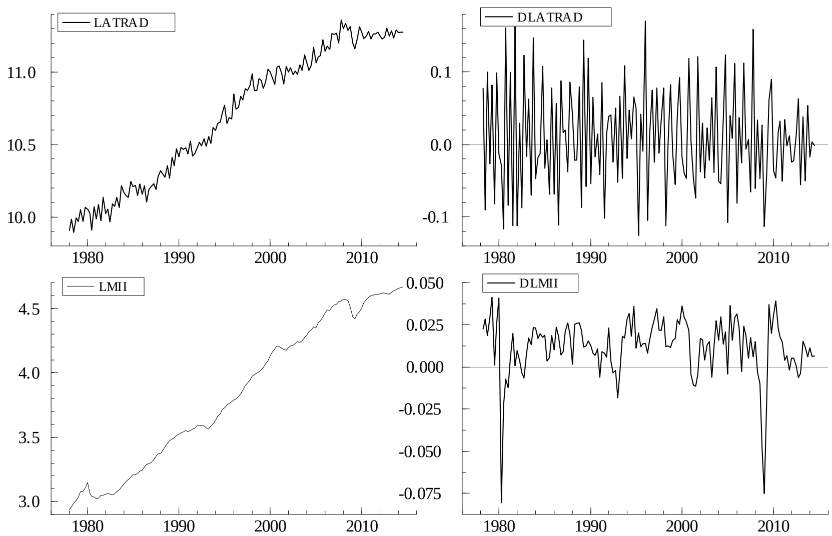

Figure 1: Time plots of the export and marked indicator data.

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Unit-root tests
The dataset is: C:\SW20\ECON4160\H2014\exam\ATRAD.in7
The sample is: 1979(1) - 2014(3) (147 observations and 2 variables)

LATRAD: ADF tests (T=143, Constant+Trend; 5%=-3.44 1%=-4.02)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob      AIC   F-prob
 3    -0.2352       0.98687  0.04888    -6.696  0.0000    -5.996
 2     -1.766       0.89061  0.05610   -0.8195  0.4139    -5.727  0.0000
 1     -2.071       0.87674  0.05604    -7.020  0.0000    -5.736  0.0000
 0     -4.799**     0.69990  0.06499                      -5.446  0.0000

LMII: ADF tests (T=143, Constant+Trend; 5%=-3.44 1%=-4.02)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob      AIC   F-prob
 3     -2.656       0.94712  0.01547     1.470  0.1439    -8.296
 2     -2.360       0.95421  0.01554    0.7365  0.4627    -8.295  0.1439
 1     -2.255       0.95748  0.01551     6.340  0.0000    -8.305  0.2617
 0     -1.142       0.97592  0.01755                      -8.065  0.0000
```

Table 1: Tests of the null hypothesis that $LATRAD$ and $LMII$ are integrated of order 1 ($I(1)$)

```
SYS( 1) Estimating the system by OLS
       The dataset is: C:\SW20\ECON4160\H2014\exam\ATRAD.in7
       The estimation sample is: 1978(3) - 2014(3)

URF equation for: DLATRAD
                 Coefficient  Std.Error  t-value  t-prob
DLATRAD_1           -0.553622    0.07135    -7.76  0.0000
DLMII_1              0.638051     0.2259     2.82  0.0055
I:1980q2           -0.0535959    0.04478    -1.20  0.2334
I:2008q4           -0.0521030    0.04456    -1.17  0.2444
I:2009q1           -0.0815688    0.04639    -1.76  0.0810
Constant      U    0.00811030   0.004566     1.78  0.0779
CSeasonal     U    -0.0453698    0.01344    -3.38  0.0010
CSeasonal_1   U    -0.0675976    0.01046    -6.46  0.0000
CSeasonal_2   U    -0.0917899    0.01087    -8.45  0.0000

sigma = 0.0436634   RSS = 0.2592829231

URF equation for: DLMII
                 Coefficient  Std.Error  t-value  t-prob
DLATRAD_1         -0.00543558    0.01833   -0.297  0.7673
DLMII_1              0.413869    0.05804     7.13  0.0000
I:1980q2            -0.106701    0.01150    -9.28  0.0000
I:2008q4           -0.0528273    0.01145    -4.61  0.0000
I:2009q1           -0.0640580    0.01192    -5.37  0.0000
Constant      U    0.00846113   0.001173     7.21  0.0000
CSeasonal     U -3.07642e-005   0.003453 -0.00891  0.9929
CSeasonal_1   U   0.000290695   0.002686    0.108  0.9140
CSeasonal_2   U   -0.00178677   0.002792   -0.640  0.5233

sigma = 0.011218   RSS = 0.01711469135

log-likelihood      703.614779  -T/2log|Omega|     1115.10695
|Omega|         2.0903052e-007  log|Y'Y/T|        -14.0665117
no. of observations        145  no. of parameters          18

correlation of URF residuals (standard deviations on diagonal)
                  DLATRAD        DLMII
DLATRAD           0.043663     0.098072
DLMII             0.098072     0.011218

Vector AR 1-5 test:      F(20,250) =   1.8691 [0.0751]
Vector Normality test:   Chi^2(4)  =   1.5407 [0.8194]
Vector Hetero test:      F(21,379) =  0.95430 [0.5204]
```

Table 2: Estimation results for a VAR of $DLATRAD$ and $DLMII$

```
MOD(2) Estimating the model by 1SLS
       The dataset is: C:\SW20\ECON4160\H2014\exam\ATRAD.in7
       The estimation sample is: 1978(3) - 2014(3)

Equation for: DLATRAD
                 Coefficient  Std.Error  t-value  t-prob
DLMII               0.381721     0.3334     1.15  0.2542
DLATRAD_1          -0.551548    0.07129    -7.74  0.0000
DLMII_1             0.480069     0.2645     1.81  0.0717
I:1980q2          -0.0128658    0.05715   -0.225  0.8222
I:2008q4          -0.0319377    0.04787   -0.667  0.5058
I:2009q1          -0.0571165    0.05102    -1.12  0.2649
Constant          0.00488051   0.005363    0.910  0.3644
CSeasonal         -0.0453580    0.01342    -3.38  0.0009
CSeasonal_1       -0.0677085    0.01044    -6.48  0.0000
CSeasonal_2       -0.0911079    0.01087    -8.38  0.0000

sigma = 0.0434529

Equation for: DLMII
                 Coefficient  Std.Error  t-value  t-prob
DLATRAD_1        -0.00543558    0.01833   -0.297  0.7673
DLMII_1             0.413869    0.05804     7.13  0.0000
I:1980q2           -0.106701    0.01150    -9.28  0.0000
I:2008q4          -0.0528273    0.01145    -4.61  0.0000
I:2009q1          -0.0640580    0.01192    -5.37  0.0000
Constant          0.00846113   0.001173     7.21  0.0000
CSeasonal       -3.07642e-005  0.003453 -0.00891  0.9929
CSeasonal_1      0.000290695   0.002686    0.108  0.9140
CSeasonal_2      -0.00178677   0.002792   -0.640  0.5233

sigma = 0.011218

log-likelihood     703.614779  -T/2log|Omega|     1115.10695
no. of observations       145  no. of parameters          19
No restrictions imposed

correlation of structural residuals (standard deviations on diagonal)
                  DLATRAD        DLMII
DLATRAD          0.043453      0.00000
DLMII             0.00000     0.011218
```

Table 3: Estimation results for an identified version of the VAR in Table 2

```
MOD(3) Estimating the model by FIML
       The dataset is: C:\SW20\ECON4160\H2014\exam\ATRAD.in7
       The estimation sample is: 1978(3) - 2014(3)

Equation for: DLATRAD
                  Coefficient  Std.Error  t-value  t-prob
DLMII                0.748048     0.3287     2.28  0.0244
DLATRAD_1           -0.548349    0.07063    -7.76  0.0000
DLMII_1              0.397331     0.2597     1.53  0.1284
Constant          0.000814805   0.004849    0.168  0.8668
CSeasonal          -0.0457105    0.01311    -3.49  0.0007
CSeasonal_1        -0.0659676    0.01023    -6.45  0.0000
CSeasonal_2        -0.0903711    0.01073    -8.42  0.0000

sigma = 0.0434738

Equation for: DLMII
                  Coefficient  Std.Error  t-value  t-prob
DLATRAD             0.0155810    0.02807    0.555  0.5798
DLATRAD_1          0.00331812    0.02222    0.149  0.8815
DLMII_1              0.400981    0.06172     6.50  0.0000
I:1980q2            -0.103934    0.01156    -8.99  0.0000
I:2008q4           -0.0528994    0.01112    -4.76  0.0000
I:2009q1           -0.0638377    0.01161    -5.50  0.0000
Constant           0.00836328   0.001170     7.15  0.0000

sigma = 0.0111112

log-likelihood     702.885884  -T/2log|Omega|     1114.37806
no. of observations       145  no. of parameters          14

LR test of over-identifying restrictions: Chi^2(4)  =   1.4578 [0.8341]

correlation of structural residuals (standard deviations on diagonal)
                   DLATRAD        DLMII
DLATRAD           0.043474     -0.15532
DLMII             -0.15532     0.011111
```

Table 4: FIML estimation results for an identified version of the VAR in Table 2.

EQ(1) Modelling DLATRAD by OLS
The dataset is: C:\SW20\ECON4160\H2014\exam\ATRAD.in7
The estimation sample is: 1978(3) - 2014(3)

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
DLMII	0.904256	0.2104	3.69	0.0003	0.0891
DLATRAD_1	-0.520404	0.07026	-7.41	0.0000	0.2830
Constant	0.00495056	0.004420	1.12	0.2646	0.0089
CSeasonal	-0.0491638	0.01328	-3.70	0.0003	0.0898
CSeasonal_1	-0.0669509	0.01035	-6.47	0.0000	0.2312
CSeasonal_2	-0.0916259	0.01082	-8.47	0.0000	0.3402

sigma	0.0438589	RSS	0.267381208
R^2	0.621887	F(5,139) =	45.72 [0.000]**
Adj.R^2	0.608286	log-likelihood	250.7
no. of observations	145	no. of parameters	6
mean(DLATRAD)	0.0088978	se(DLATRAD)	0.0700767

Table 5: Estimation results for a conditional model of $DLATRAD_t$

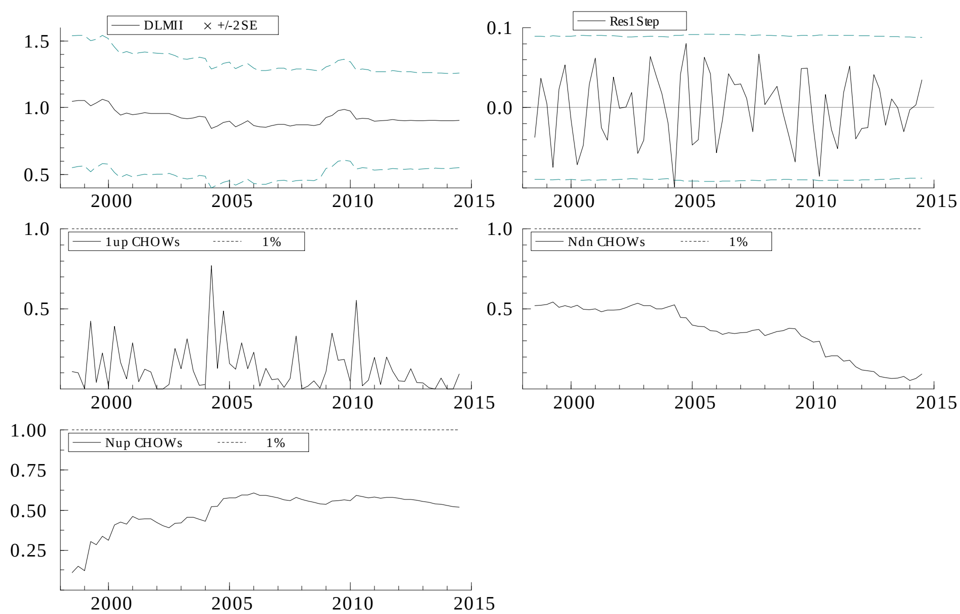

Figure 2: Recursive plots of for the model in Table 5.

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EQ(2) Modelling DLATRAD by OLS
      The dataset is: C:\SW20\ECON4160\H2014\exam\ATRAD.in7
      The estimation sample is: 1978(3) - 2014(3)
```

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
DLATRAD_1	-0.418093	0.07860	-5.32	0.0000	0.1754
Constant	1.79707	0.5304	3.39	0.0009	0.0795
DLMII	0.665952	0.2301	2.89	0.0044	0.0593
DLMII_1	0.389232	0.2321	1.68	0.0958	0.0207
DLREX	0.221008	0.1884	1.17	0.2429	0.0102
DLREX_1	-0.245433	0.1878	-1.31	0.1934	0.0127
LATRAD_1	-0.248506	0.07283	-3.41	0.0009	0.0805
LMII_1	0.217816	0.06436	3.38	0.0009	0.0793
LREX_1	0.213545	0.09305	2.30	0.0233	0.0381
CSeasonal	-0.0435038	0.01295	-3.36	0.0010	0.0782
CSeasonal_1	-0.0594354	0.01037	-5.73	0.0000	0.1980
CSeasonal_2	-0.0875664	0.01063	-8.24	0.0000	0.3379

sigma	0.0421299	RSS	0.236065277
R^2	0.666172	F(11,133) =	24.13 [0.000]**
Adj.R^2	0.638562	log-likelihood	259.732
no. of observations	145	no. of parameters	12
mean(DLATRAD)	0.0088978	se(DLATRAD)	0.0700767

Table 6: OLS estimates for an unrestricted equilibirum correction equation for $LATRAD_t$