Exam in: ECON 4160: Econometrics: Modelling and Systems Estimation

Day of exam: 30 November 2016

Time of day: 09:00—12:00

This is a 3 hour school exam.

Guidelines:
In the grading, question A gets 40 %, B 30 % and C 30 %.

Question A (40 %)

We have annual observations of the two variables *pci* and *pmi* for the period 1950 to 2015. *pci* is Norwegian inflation, in percent, and *pmi* is the change (also in percent) in an import price index, so called imported inflation.

1. Explain why the evidence in Table 1 gives reason to conclude that neither *pci* nor *pmi* contain a unit-root (they are not $I(1)$ series).

```
Unit-root tests

The sample is: 1950 - 2015 (68 observations and 2 variables)

pci: ADF tests (T=66, Constant; 5%=-2.91 1%=-3.53)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob
  1     -3.491*      0.65840    2.505     0.5587  0.5783
  0     -3.563**     0.67909    2.492

pmi: ADF tests (T=66, Constant; 5%=-2.91 1%=-3.53)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob
  1     -4.432**     0.42707    4.821     0.5411  0.5904
  0     -4.894**     0.46361    4.795
```

Table 1: Augmented Dickey-Fuller (ADF) tests to determine the order of integration of pci_t and pmi_t.

The Diskey-Fuller test rejects at 1 percent lever in the case with no augmentation. Which is admissible from the t-DYlag column. But also augmented version rejects I(1) at 5 % level.

2. Table 2 shows the result of estimation of the following ADL model for Norwegian inflation:

$$(1)\quad pci_t = \phi_0 + \phi_1 pci_{t-1} + \beta_1 pmi_t + \beta_2 pmi_{t-1} + \epsilon_t, \qquad t = 1950, ..., 2015$$

```
     Modelling pci by OLS
     The estimation sample is: 1950 - 2015

                 Coefficient  Std.Error  t-value  t-prob
Constant             1.39333     0.4039     3.45  0.0010
pci_1               0.367467    0.09465     3.88  0.0003
pmi                 0.286378    0.04869     5.88  0.0000
pmi_1               0.133322    0.06304     2.11  0.0385

sigma                1.86419  RSS                215.461777
R^2                  0.71277  F(3,62) =     51.28 [0.000]**
Adj.R^2             0.698871  log-likelihood       -132.693
no. of observations       66  no. of parameters           4
mean(pci)            4.62354  se(pci)               3.39714

AR 1-2 test:      F(2,60)    =   1.3208 [0.2746]
ARCH 1-1 test:    F(1,64)    =  0.37217 [0.5440]
Normality test:   Chi^2(2)   =   1.6559 [0.4369]
```

Table 2: Results for estimation of equation (1).

(a) Based on the information in the table, does the column labelled "t-probability" provide reliable statistical evidence about the significance of the individual variables? Explain briefly.

Three of the standard misspecification tests are reported. None of them indicate significant departures from the assumtions that underlie the statistical inference theory that for example the t-value of 5.88 can be used to test the hypthesis that $\beta_1 = 0$ *for example. by comparison with the critcal value with 62 df, or with the standard normal since the number of observations is relatively large.*

(b) Assume that *pmi* is increased permanently by one unit (one percentage point). Based on Table 1, answer the following questions:

i. What is the impact effect on domestic inflation?
Increase in period of the unit increse (for example in period T*)*: 0.29

ii. What is the second year effect? (To save time, you can do the algebra with only two decimals.)
Effect:

$$0.13 + 0.29 + 0.29 * 0.37 = 0.5273$$

iii. What is the long-run effect?
It is: $\frac{0.29+0.13}{1-0.37} = 0.67$

(c) Can the estimated effect in b.ii) be biased if pmi_t is not strongly exogenous? Explain briefly.

Yes. If pmi is Granger-caused by pci, the calculated second year effect is biased because the number 0.29 *should be replaced by a number that takes into account that* X_{T+1} *(if the period of the shock is* T*) is increased by* 1 *pluss/minus the effect that the first period increase* 0.55 *has on* X_{T+1}*, i.e.:*

$$0.13 + 0.29(1 + x) + 0.29 * 0.37 = ?$$

where x *is the effect that is due to joint Granger causality.*

(d) Re-write (1) in ECM-form.
This is straight forward.

(e) Using the coefficient estimates in Table 2, what are the coefficients estimates of the ECM equation?

$$\begin{aligned}\widehat{\Delta pci}_t = & \ \hat{\beta}_0 + (\hat{\phi}_1 - 1)pci_{t-1} + \hat{\beta}_1 \Delta pmi_t + (\hat{\beta}_1 + \hat{\beta}_2)pmi_{t-1} \\ & = 1.93 + (0.37 - 1)pci_t + 0.29\Delta pmi_t + (0.29 + 0.13)pmi_{t-1} \\ & = 1.93 - (0.63)pci_t + 0.29\Delta pmi_t + (0.42)pmi_{t-1}\end{aligned}$$

(f) Show that you can use the empirical ECM equation to confirm you answer to the question about long-run effect of a permanent increase in pmi.

$$\frac{0.42}{0.63} = 0.67$$

(g) Explain briefly how you could test the null hypothesis that the long-run effect of a permanent increase in pmi is one? In particular, what extra regression output would you need?

Use delta-method to calculate variance

$$\hat{\theta} = \frac{\overbrace{(\hat{\beta}_1 + \hat{\beta}_2)}^{\hat{\gamma}}}{\underbrace{\phi_1 - 1}_{\hat{\alpha}}}$$

After estimation of the ECM-form we get the estimated $Var(\hat{\gamma})$ and $Var(\hat{\alpha})$ directly from the output. If we in addition get hold of the estimated $Cov(\hat{\alpha}, \hat{\gamma})$ we have what we need to calculate the variance of $\hat{\theta}$, and use that to test $H_0 : \theta - 1 = 0$.

3. Assume that, for a different data set with two variables, the unit-root tests lead to the conclusion that both variables were $I(1)$.

   (a) Describe how you could test the hypothesis of no cointegration in that case.

   Use the ECM and test $H_0 : \alpha = 0$ using the critical values for the ECM-test, for example from Ericsson and MacKinnon. These critical values are larger in absolute values ofthe $N(0,1)$ or t-distribution that we would use in the stationary case.

   (b) If the outcome of your test was rejection of the hypothesis of no cointegration, how would you estimate the cointegrating parameters?

   Write the cointegration relationship as

$$pci = \mu + \theta pmi$$

   Estimate θ is the same way as before, as $\hat{\theta} = \hat{\gamma}/\hat{\alpha}$ and μ as $\hat{\mu} = \hat{\beta}_0/\hat{\alpha}$.

Question B (30 %)

Consider the VAR:

$$\begin{pmatrix} pci_t \\ pmi_t \end{pmatrix} = \begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix} \begin{pmatrix} pci_{t-1} \\ pmi_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \tag{2}$$

where the two disturbances are jointly normally distributed, with zero expectations and with covariance matrix:

$$\mathbf{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma \\ \sigma & \sigma_2^2 \end{pmatrix}. \tag{3}$$

Assume that this VAR is the statistical system that has generated the data series pci_t and pmi_t that we used in Question A.

1. When (2) is estimated on the 1950-2015 sample, we get the estimated residual covariance matrix:

   $$\hat{\mathbf{\Sigma}} = \begin{pmatrix} (2.3084)^2 & 6.6644 \\ 6.6644 & (4.8240)^2 \end{pmatrix}. \tag{4}$$

   Show that the estimate of β_1 in Table 2 can be confirmed by using the information in (4).

   Since we assume that the VAR has generated the data, equation (1) is the conditional model of pci_t, given pmi_t (and pci_{t-1} and pmi_{t-1} which are already condtioned on in the VAR). Then from the properties of the conditional expectation $E(pci_t \mid pm_i,\text{and lags})$, the partial regression coefficent is

   $$\beta_1 = \frac{Cov(\varepsilon_{1,t}, \varepsilon_{2,t})}{Var(\varepsilon_{2,t})}$$

   Estimate β_1 by using the elements in $\hat{\Sigma}$ matrix:

   $$\hat{\beta}_1 = \frac{6.6644}{(4.8240)^2} = 0.286\,38$$

2. Table 3 contains more estimation results for the VAR:

Estimating the system by OLS
The estimation sample is: 1950 - 2015

URF equation for: pci

		Coefficient	Std.Error	t-value	t-prob
pci_1		0.400283	0.1170	3.42	0.0011
pmi_1		0.251545	0.07399	3.40	0.0012
Constant	U	1.87936	0.4895	3.84	0.0003

sigma = 2.30837 RSS = 335.6989835

URF equation for: pmi

		Coefficient	Std.Error	t-value	t-prob
pci_1		0.114588	0.2445	0.469	0.6409
pmi_1		0.412824	0.1546	2.67	0.0096
Constant	U	1.69716	1.023	1.66	0.1021

sigma = 4.82403 RSS = 1466.088488

log-likelihood -328.666187
no. of observations 66 no. of parameters 6

Table 3: Results for estimation of the VAR in equation (2).

When we estimate an econometric model of the VAR, with (1) as the first equation, and with

$$pmi_t = \gamma_0 + \gamma_1 pmi_{t-1} + v_t \tag{5}$$

as the second equation, the estimated log-likelihood is -328.78. (Estimation is by OLS on each equation). How can you use this result to test the validity of the restriction(s) on the system that the model consisting of (1) and (5) implies?

(Hint: The 5 % critical value for a $\chi^2(1)$ distribution is 3.8).

There are 4 parameters in (1) and 2 in (5), 6 in all. There is no covariance between the diturbances. In the VAR there are 6 coefficient but also a covaraince between the disturbances, so we actually count 7 parameters for the VAR. To test the statistical significance of the restriction:

$$-2((-328.78-(-328.666187)) = 0.227\,63$$

which is insignificant.

3. Does the evidence support the hypothesis that pmi_t is strongly exogenous?

   Yes. Strong exogeneity requires that pmi_t is not Granger-caused by pci_t. And that is not rejected by the test.

4. Are the impulse responses of the model consisting of (1) and (5) identified?

   Since estimation is by "1SLS" the structural disturbances are uncorrelated by construction. This is enough to identify the umpulse responses. But in addition, pmi_t is strongly exogenous, which makes for even stronger identification.

Question C (30 %)

A researcher wants to estimate a more complete simultaneous equation model (SEM) of Norwegian inflation. She wants to bring in two other domestic variables: Domestic wage inflation, wi, and the unemployment rate, UR. Both variables are measured in percent. She specifies the following theoretical model:

$$pci_t + \beta_{12}wi_t + \beta_{14}pmi_t = \beta_{10} + \phi_{11}pci_{t-1} + \epsilon_{1t} \tag{6}$$

$$\beta_{21}pci_t + wi_t + \beta_{23}UR_t = \beta_{20} + \phi_{22}wi_{t-1} + \epsilon_{2t} \tag{7}$$

$$\beta_{31}pci_t + UR_t - \beta_{31}pmi_t = \beta_{30} + \phi_{33}UR_{t-1} + \epsilon_{3t} \tag{8}$$

$$pmi_t = \beta_{40} + \phi_{44}pmi_{t-1} + \epsilon_{4t} \tag{9}$$

All the coefficients are assumed to be non-zero. There are no theoretical restrictions on the covariance matrix of the disturbances

1. In the researcher's theory, equation (6) is a price equation, and (7) is a wage equation. Discuss the identification of each of these two equations.

	pci_t	wi_t	UR_t	pmi_t	*1*	pci_{t-1}	wi_{t-1}	$UR_{t.-1}$	pmi_{t-1}
P	*1*	β_{12}	*0*	β_{14}	β_{10}	ϕ_{11}	*0*	*0*	*0*
W	β_{21}	*1*	β_{23}	*0*	β_{20}	*0*	ϕ_{22}	*0*	*0*
U	β_{31}	*0*	*1*	$-\beta_{31}$	β_{30}	*0*	*0*	ϕ_{33}	*0*
PM	*1*	*0*	*0*	*0*	β_{40}	*0*	*0*	*0*	ϕ_{44}

Since we know that all coefficients are non-zero, we concentrate on the order condition. With 4 equations, a single equaion is identified if the number of excluded variables is $4 - 1 = 3$*, or higher.*

Price eq: Number of excluded is 4. So overidentified.

W-eq: Also here the number of excluded is 4. Degree of overidentification is 1 here as well.

2. Based on your conclusions about identification, explain in words how you would estimate the identified equation(s) using single equation estimation (i.e., without estimation the complete structural model).

   Take P-eq as example. Formally there are two endogenous variables in that equation: wi_t *and* pmi_t*. So in that interpretation the rlevant instrumets to use are:* wi_{t-1}, UR_{t-1} *and* pmi_{t-1}*. 2SLS uses uses the OLS predicted values of* wi_t *and* pmi_t *(conditional on these instruments) in the "second step" LS estimation of the P-eq.*

   However, miaght also say that pmi_t *in P-eq is a predetermined variable, so we do not need to use and instrument for* pmi_t*. Hence some may say that we estimate with 2SLS because* wi_t *is endogenous, and use* wi_{t-1}, UR_{t-1} *as instrumental variabls. But thsi interpretation assumes that* ϵ_{1t}*and* ϵ_{4t} *are uncorrelated, and the text says nothing about that. So small "minus" for not nothing that* pmi_t *can be correlated with* ϵ_{1t} *via the covariance matrix of distubances.*

   But main point here is to explain how 2SLS "works"

   If we choose W-eq as our example it is more definite that there are two endogenous: pci_t *and* UR_t*. Relevant instryments are:* pci_{t-1}, UR_{t-1} *and* pmi_{t-1}*.*