UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON4160 – Econometrics – Modeling and Systems Estimation**

Date of exam: Tuesday, November 28, 2017 **Grades are given:** December 19, 2017

Time for exam: 09.00 a.m. – 12.00 noon

The problem set covers 5 pages (incl. cover sheet)

Resources allowed:
- Open book exam, where all written and printed resources – including calculator - is allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

Exam in: ECON 4160: Econometrics: Modelling and Systems Estimation

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This is a 3 hour school exam.

Guidelines:
In the grading, question A gets 20 %, B 40 % and C 40 %.

Question A (20 %)

Consider the linear in parameters system consisting of the four random variables X_i, Y_i, W_i, Z_i. Assume that you have a sample of n mutually independent quadruplets (X_i,Y_i, W_i,Z_i) that are identically distributed. Analyze the difference between the "plim" of the OLS estimate of the total derivative of Y with respect to X and the true partial derivative of Y with respect to X.

Question B (40 %)

A white-noise time series $\{\varepsilon_t; t = 1, 2, ...T\}$ is characterized by:

$$E(\varepsilon_t) = 0 \tag{1}$$

$$Var(\varepsilon_t) = \sigma_\varepsilon^2 > 0 \tag{2}$$

$$Cov(\varepsilon_t, \varepsilon_{t-j}) = 0, \text{ for } j = \pm 1, \pm 2, ... \tag{3}$$

Assume that the DGP for the time series Y_t is:

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \varepsilon_t. \tag{4}$$

Based on (1)-(4), answer the following questions:

1. What are the expressions for $E(Y_t \mid Y_{t-1})$ and $Var(Y_t \mid Y_{t-1})$?
2. What are the expressions for $E(Y_{t+1} \mid Y_{t-1})$ and $Var(Y_{t+1} \mid Y_{t-1})$?

3. Give the expression for $E(Y_t)$ in the case where Y_t is a covariance stationary time series, denoted $Y_t \sim I(0)$.

4. Give the expression for $E(Y_t \mid Y_0)$ when Y_t is a unit-root non stationary time series, $I(1)$. Regard Y_0 as a fixed number.

5. Show that $Var(Y_t)$ is finite if $Y_t \sim I(0)$, but that $Var(Y_t)$ is infinite if $Y_t \sim I(1)$.

6. Assume that ε_t is Gaussian white-noise in (4).

   (a) Explain in words why the OLS estimator $\hat{\phi}_1$ of ϕ_1 can be interpreted as a Maximum Likelihood estimator.

   (b) Explain in words why $\hat{\phi}_1$ is a biased estimator in finite samples, but that it is also a consistent estimator.

7. Replace assumptions (1)-(3), by

   (5) $$\varepsilon_t = \tau\varepsilon_{t-1} + \nu_t\,, \quad -1 < \tau < 1,$$

   where ν_t is white-noise. Will a regression of Y_t on Y_{t-1} and a constant give a consistent estimator of ϕ_1 in (4) in this case? Explain your answer.

8. Under the same assumption as in Q7, can you suggest a method that can be used for consistent estimation of the characteristic roots of the DGP for Y_t in this case?

Question C (40 %)

Consider the simultaneous equations model (SEM) with two endogenous variables, Y_t and X_t, and two strongly exogenous variables W_t and Z_t.

(6) $$Y_t - \beta_{10}X_t = \beta_{11}X_{t-1} + \phi Y_{t-1} + \beta_{20}Z_t + \epsilon_{1t},$$

(7) $$-\gamma_{10}Y_t + X_t = \alpha X_{t-1} + \gamma_{11}Y_{t-1} + \gamma_{20}W_t + \epsilon_{2t}.$$

ϵ_{1t} and ϵ_{2t} are white-noise disturbances which may be correlated, $Cov(\epsilon_{1t}, \epsilon_{2t}) \neq 0$. In Q1 and Q2 below we assume that both W_t and Z_t are $I(0)$ variables, and that the reduced form of (6) and (7) is a stationary VAR.

1. Discuss the identification of the two structural equations.

2. Assuming that equation (6) in the SEM is identified:

   (a) Explain why OLS gives inconsistent estimation of β_{10} (and the other coefficients in (6)).

   (b) Explain why IV estimation gives consistent estimators.

   We now change the assumption about the stationarity of W_t and Z_t, and assume instead that both are $I(1)$ variables.

3. Explain why Y_t and X_t are in general $I(1)$ variables when W_t and Z_t are $I(1)$.

4. Consider the model (6) and (7) under the assumptions: $\beta_{10} = \gamma_{10} = \gamma_{11} = \gamma_{20} = 0$ and $\alpha = 1$. Explain how you use the results in Table 1 (at the end of the question set) to do a valid statistical test of the null hypothesis of no cointegration. Give your conclusion, and the estimated cointegration relationship if you reject the null hypothesis.

5. Finally, consider the model (6) and (7) under the assumptions: $\beta_{20} = \gamma_{20} = 0$. Explain briefly what properties the reduced form must have for the two endogenous variables to be $I(1)$ in this case. How many cointegration relationships can we have, at most, in this case?

Modelling DY by OLS

The estimation sample is: 3 - 101

	Coefficient	Std.Error	t-value	t-prob
Constant	-0.123867	0.08941	-1.39	0.1692
DX	0.252379	0.08154	3.10	0.0026
Y_1	-0.558797	0.08878	-6.29	0.0000
X_1	0.388778	0.06430	6.05	0.0000
Z	0.799442	0.07869	10.2	0.0000

sigma	0.801969	RSS	60.4565135
R^2	0.683199	F(4,94) =	50.68 [0.000]**
Adj.R^2	0.669718	log-likelihood	-116.062
no. of observations	99	no. of parameters	5
mean(DY)	-0.319047	se(DY)	1.39545

AR 1-2 test:	F(2,92)	=	1.7649 [0.1770]
ARCH 1-1 test:	F(1,97)	=	0.39606 [0.5306]
Normality test:	Chi^2(2)	=	0.43364 [0.8051]
Hetero test:	F(8,90)	=	0.73809 [0.6576]
Hetero-X test:	F(14,84)	=	0.69692 [0.7713]

Table 1: ECM type model for DY_t. Difference is symbolized by D, i.e, $DY = \Delta Y_t$ and $DX = \Delta X_t$