Exam in: ECON 4160: Econometrics: Modelling and Systems Estimation —Postponed exam

Day of exam: 9 December 2015

Time of day: 09:00—12:00

This is a 3 hour school exam.

Guidelines:
In the grading, question A will count 40%, B will count 20% and C will count 40% .

Question A (40%)

1. Write down the log likelihood function for the two following cases:

   (a) The n random variables $(Y_1, Y_2, \dots, , Y_n)$ are independent and have identical normal distributions.
   A: Ch 2.4.4 with $\beta_1 = 0$

   (b) The time series $(Y_1, Y_2, \dots, , Y_T)$ can be represented by the model equation

   $$Y_t = \phi_0 + \phi_1 Y_{t-1} + \varepsilon_t, \quad -1 < \phi_1 < 1 \tag{1}$$

   where ε_1,ε_2,...,ε_T are independent and have identical normal distributions. Consider Y_0 as a fixed parameter.
   A: Ch 4.6.1

2. In case 1(a), write down the expressions for the ML estimator of the location parameter and the dispersion parameter of the distribution.

   For the rest of Question A, we consider only the model in case 1(b).

   A: $\hat{\beta}_0 = \bar{Y}$ and $\hat{\sigma}^2 = n^{-1} \sum (Y_i - \bar{Y})^2$

3. Assume that $\phi_0 = 2$ and $\phi_1 = 0.5$, and $Var(\varepsilon_t) = 1$ and calculate $E(Y_t)$, $Var(Y_t)$ and $E(Y_{t+2} \mid Y_t)$.

   A: $E(Y_t) = 2/(1-0.5) =$,$Var(Y_t) = 1/(1-0.5^2) =$, $E(Y_{t+2} \mid Y_t) = E(2(1+0.5) + 0.5^2 Y_{t+2} + \varepsilon_{t+2} + 0.5\varepsilon_{t+1}) \mid Y_t = \ 2.5 + 0.5^2 Y_t$

4. Explain why the ML estimator of ϕ_1 is biased and consistent.

   A: Y_{t-1} is predetermined, it is correlated with past disturbances (implies bias), but is uncorrelated with current and future distubances, implying consistency in the stationary case.

5. Which (if any) of the moments $E(Y_t)$, $Var(Y_t)$ and $E(Y_{t+2} \mid Y_t)$ exist if we instead of $-1 < \phi_1 < 1$ assume that Y_t is integrated of order one?

   A: $E(Y_{t+2} \mid Y_t)$ exists for any value of ϕ_1.

Question B (20 %)

Consider the time series $(Y_1, X_1), \ldots, (Y_T, X_T)$ where Y_t and X_t separately follow simple random-walk processes (without drift terms). Assume that the system can be written as:

$$Y_t + \gamma_1 X_t = u_{1t}, \quad u_{1t} = u_{1t-1} + \varepsilon_{1t} \tag{2}$$

$$Y_t + \gamma_2 X_t = u_{2t}, \quad u_{2t} = \phi_2 u_{2t-1} + \varepsilon_{2t}, \ \ |\phi_2| < 1 \tag{3}$$

where ε_{1t} and ε_{2t} are two independent white-noise processes.

1. Why is $\gamma_2 \neq 0$ the cointegrating parameter in the system of equations given by (2) and (3)?

   A. See Box 10.1 about this exercise.

2. Explain briefly why the OLS estimator of γ_2 is a consistent estimator under the given assumptions.

   A. See Box 10.1 about this exercise.

3. Assume that the assumption about the parameter ϕ_2 in (3) is changed to $\phi_2 = 1$ and that instead of asserting that $\gamma_2 \neq 0$, we are interested in testing the null hypothesis of $\gamma_2 = 0$. Assume that it is suggested to test the null hypothesis of $\gamma_2 = 0$ by the use of conventional critical values from the standard normal (or a t-distribution). Will this lead to reliable inference? Motivate your answer briefly.

   A: Unreliable inference. The null hypothesis is rejected incorrectly (Type I error) with a higher probability than indicated by the conventional critical values (spurious relationship). For reliable inference, use the ECM test or another valid test of absence of cointegration.

Question C (40 %)

In this question we consider three time series variables Y_t, Xa_t and Xb_t.

1. Use the estimation results in Table 1 to sketch graphs that show the dynamic multipliers of Y with respect to unit changes in Xa and Xb.

   *A: Base answer on the assumption that both roots are less than one in magnitude, and that changes are for one period. Impact multiplier of Xa is zero. First dynamic multiplier is 0.93. Second is 0.99. Third is -0.41+1.06 =0.60. Fourth is 0.6*1.07-0.41*0.99 etc. Converges to zero.*

   *Impact multiplier of Xb is -0.55. First dynamic multiplier is -0.55*1.07+0.55=-0.03. Higher order are all small numerically*

2. Explain why the model in Table 2 is a reparameterization of the model in Table 1.

   A. ECM transformation.

```
EQ(1) Modelling Y by OLS
      The dataset is: C:postexam.xls
      The estimation sample is: 1960 - 2015

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Y_1                   1.06824    0.07261     14.7  0.0000   0.8124
Y_2                 -0.412416    0.07053    -5.78  0.0000   0.4002
Xa_1                 0.929774     0.1197     7.77  0.0000   0.5468
Xb                  -0.547075     0.1223    -4.47  0.0000   0.2858
Xb_2                 0.548419     0.1292     4.24  0.0001   0.2643
Constant           0.00890672     0.1313   0.0685  0.9456   0.0001

sigma                0.954286  RSS                 45.5331315
R^2                  0.934739  F(5,50) =     143.2 [0.000]**
Adj.R^2              0.928213  log-likelihood          -73.667
no. of observations        56  no. of parameters             6
mean(Y)              0.394624  se(Y)                   3.36169

AR 1-2 test:      F(2,48)   =0.0247843 [0.9553]
ARCH 1-1 test:    F(1,54)   =  0.48901 [0.4878]
Normality test:   Chi^2(2)  =  0.92925 [0.6284]
Hetero test:      F(10,45)  =  0.60514 [0.7580]
Hetero X test:    F(20,35)  =  0.78508 [0.7117]
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Table 1: Results for an ARDL model of Y.

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EQ(2) Modelling DY by OLS
      The dataset is: C:postexam.xls
      The estimation sample is: 1960 - 2015

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
DY_1                 0.407416    0.07053     5.78  0.0000   0.4002
Y_1                 -0.339182    0.04327    -7.84  0.0000   0.5514
Xa_1                 0.929774     0.1197     7.77  0.0000   0.5468
D2Xb                -0.547075     0.1223    -4.47  0.0000   0.2858
Xb_2               0.00134336     0.1247   0.0108  0.9915   0.2000
Constant          -0.00899672     0.1313  -0.0685  0.9456   0.2001

sigma                0.954286  RSS              45.5331315
R^2                   0.79073  F(5,50) =      37.79 [0.000]**
Adj.R^2              0.769803  log-likelihood        -73.667
no. of observations        56  no. of parameters           6
mean(DY)             0.115824  se(DY)                1.98897

AR 1-2 test:      F(2,48)   = 0.0047043 [0.9953]
ARCH 1-1 test:    F(1,54)   =   0.48801 [0.4878]
Normality test:   Chi^2(2)  =   0.92925 [0.6284]
Hetero test:      F(10,45)  =   0.64288 [0.7093]
Hetero-X test:    F(20,35)  =   0.78598 [0.7117]
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Table 2: Results for an ECM of Y.

3. Assume that Xa_t and Xb_t are weakly exogenous for the cointegration parameter. Explain the meaning of this assumption.

   A: Neither of them equilibrium correct with respect to deviations from the long-run relationship.

4. Use Table 2 to show that the null hypothesis of no cointegration can be rejected at a conventionally low significance level.

   (The critical values of the relevant test are: 5%: -3.00 and 1%: 3.62).

   A: -7.84 rejects the null of no relationship at the 1 % level.

5. What are the estimated cointegration parameters of Xa_t and Xb_t?

   A: Xa: 2.74 Xb: practically zero.

6. Why does the estimated cointegration parameter of Xa_t have interpretation as a so called long-run parameter?

   A: If Xa is permanently increased by 1 unit, the long-run response of Y is the same as the cointegration parameter.

7. Explain how you would formally test the hypothesis that the long-run parameter of Xb_t is zero. In particular, what additional computer output would you need?

   A: Use the Delta method. Need the estimated covariance between the estimators of the long-run cofficients of Xa_t *and* Xb_t.