

***UNIVERSITY OF OSLO***  
***DEPARTMENT OF ECONOMICS***

Exam: **ECON4160 – Econometrics: Modelling and Systems Estimation**

Date of exam: Monday, November 26, 2018

**Grades are given:** December 17, 2018

Time for exam: 09.00 a.m. – 12.00 noon

The problem set covers 7 pages

Resources allowed:

- Open book exam. All written and printed resources, in addition to one out of two different calculators is allowed.

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

**Exam in:** ECON 4160: Econometrics: Modelling and Systems Estimation

**Day of exam:** 26 November 2018

**Time of day:** 09:00—12:00

This is a 3 hour school exam.

**Guidelines:**

In the grading, question A gets 33 %, B 33 % and C 33 %.

## Question A (1/3)

1. Use the information in Table 1 to decide the order of integration of the two equity price indices  $LPA$  (Norway) and  $LPAW$  (world), both measured in natural logarithms.
2. In this question you can take for granted that  $LPAW_t$  is a strongly exogenous variable in the conditional model for  $DLPA_t$  shown in Table 2.
  - (a) Based on the mis-specification tests reported, is there any indication that statistical inference based on the  $t$ -values will be unreliable?
  - (b) Assume that the test situation is:

$H_0$  : No relationship between  $DLPA_t$  and  $DLPAW_t$   
against

$H_1$  : There is a relationship between  $DLPA_t$  and  $DLPAW_t$ .

Based on the information in the table, what is your conclusion?

- (c) A business school student says that the column labelled  $t$ -prob (which contains  $p$ -values) supports that there is a long-run relationship between  $LPA_t$  and  $LPAW_t$ , because the  $p$ -values of both  $LPA_{t-1}$  and  $LPAW_{t-1}$  show that the coefficients are significantly different from zero when a 10 % significance level is used.

Explain why this test method can lead to a spurious relationship.

- (d) Explain why the ECM-test is a valid test of the  $H_0$  of absence of cointegration between  $LPA_t$  and  $LPAW_t$ , and why that test will not reject  $H_0$  at the 10 % level of significance (see Table 2 for critical values).
- (e) Assume that the outcome of the test of the  $H_0$  of absence of cointegration between  $LPA_t$  and  $LPAW_t$  was different: that the  $H_0$  had been rejected by the ECM-test. In this case, what would the estimated long-run elasticity of  $PA_t$  with respect to  $PAW_t$  be?

## Question B (1/3)

Assume that the time series variable  $Y_t$  is generated by the linear difference equation:

$$(1) \quad Y_t = \phi_0 + \phi_1 Y_{t-1} + \varepsilon_t,$$

where  $\varepsilon_t$  is a Gaussian white noise variable with variance  $\sigma^2$ , hence  $\varepsilon_t \sim N(0, \sigma^2)$  for all  $t$ .

1. Under which condition on  $\phi_1$  can the stable solution for  $Y_t$  be written in terms of a past value  $Y_{t-1-j}$  (initial condition), and the white noise terms:  $\varepsilon_t, \varepsilon_{t-1}, \dots, \varepsilon_{t-j}$ ?
2. Under the stability condition in QB1, derive the solution for  $Y_t$  when  $j = 2$ .
3. Under the stability condition in QB1, what is the expression for the solution for  $Y_t$  when  $j \rightarrow \infty$ ?
4. Assume that we are interested in estimating the parameters  $\phi_0$  and  $\phi_1$ .
  - Explain why  $Y_{t-1}$  is a pre-determined variable in (1).
  - Denote the OLS estimator by  $\hat{\phi}_1$ . Explain why  $\hat{\phi}_1$  is a biased estimator in any finite sample, but that  $plim(\hat{\phi}_1 - \phi_1) = 0$  under the stability condition.

5. Assume that we are interested in forecasting  $Y_{T+h}$ , using (1). For simplicity, we assume that  $\phi_0$ ,  $\phi_1$ ,  $\sigma^2$  and  $Y_T$  are known numbers. Since we do not know the future white noise variables, the forecast is made by replacing  $\varepsilon_{T+1}, \varepsilon_{T+2}, \dots, \varepsilon_{T+h}$  by zeros (expected values). Denote the sequence of (point) forecasts by  $Y_{T+h}^f$ ,  $h = 1, 2, \dots, H$ .

Show that:

$$(2) \quad Y_{T+h|T}^f \rightarrow \frac{\phi_0}{1 - \phi_1} \text{ as } h \rightarrow \infty$$

under the stability condition.

6. Assume that right after you published your forecast, there is a structural break so that (1) changes to:

$$Y_{t+h} = (\phi_0 + d) + \phi_1 Y_{t+h-1} + \varepsilon_{t+h}, \text{ for } t = T \text{ and } h = 1, 2, \dots, H$$

- (a) Assume that  $d > 0$ . How will the forecast-error ( $Y_{T+1} - Y_{T+1|T}^f$ ) be affected by this structural break?
- (b) Will (2) still hold in this case?

### Question C (1/3)

Consider the macro model:

$$(3) \quad C_t = c_0 + c_1 GDP_t + c_2 C_{t-1} + \varepsilon_{Ct},$$

$$(4) \quad J_t = d_0 + d_1 GDP_t + d_2 GDP_{t-1} + d_3 J_{t-1} + \varepsilon_{Jt},$$

$$(5) \quad GDP_t = C_t + J_t + G_t.$$

The endogenous variables are:  $C_t$  (private consumption),  $GDP_t$  (gross domestic product),  $J_t$  (private investment).  $G_t$  (public expenditure) is determined outside the system, it is an exogenous variable.

Assume that the coefficients of the model are different from zero. Assume that the two error-terms are Gaussian white noise variables. We assume that the covariance matrix of the error terms (ie  $\mathbf{\Omega}$ ) is invertible, but it is not necessarily a diagonal matrix.

1. What are the conditions for stationarity of the time series variable  $GDP_t$  in this model? (No derivations are required in the answer)

2. Explain why the OLS estimator of  $c_1$  is an inconsistent estimator.
3. Are (3) and (4) identified on the order condition?
4. Explain why 2SLS is a more efficient method of estimation than IV for an over-identified structural equation.
5. Imagine that you have been able to estimate the SEM (3)-(5) by FIML. Will the estimates be identical to the 2SLS estimates? If not why?
6. Assume that you are interested in testing the hypothesis  $H_0: d_2 = -d_1$  (the restriction implying that  $J_t$  depends on  $\Delta GDP_t$ ). Explain in words how you can test this hypothesis by the use of FIML or 2SLS estimation.

# Tables

## Unit-root tests

The dataset is: C:\SW20\ECON4160\H2018\Exam\MODobligexam.in7

The sample is: 1973(2) - 2018(2) (185 observations and 4 variables)

LPA: ADF tests (T=181, Constant; 5%=-2.88 1%=-3.47)

D-lag	t-ADF	beta	Y <sub>-1</sub>	sigma	t-DY <sub>-lag</sub>	t-prob
3	-1.093	0.99148	0.1065		2.481	0.0140
2	-0.9773	0.99228	0.1080		-1.636	0.1037
1	-1.083	0.99142	0.1085		4.241	0.0000
0	-0.8944	0.99259	0.1135			

LPAW: ADF tests (T=181, Constant; 5%=-2.88 1%=-3.47)

D-lag	t-ADF	beta	Y <sub>-1</sub>	sigma	t-DY <sub>-lag</sub>	t-prob
3	-0.6086	0.99715	0.05602		1.564	0.1197
2	-0.5399	0.99746	0.05625		-2.180	0.0306
1	-0.6487	0.99692	0.05684		5.691	0.0000
0	-0.3780	0.99806	0.06162			

DLPA: ADF tests (T=181, Constant; 5%=-2.88 1%=-3.47)

D-lag	t-ADF	beta	Y <sub>-1</sub>	sigma	t-DY <sub>-lag</sub>	t-prob
3	-6.544**	0.25367	0.1056		2.025	0.0444
2	-6.222**	0.35127	0.1065		-2.436	0.0158
1	-8.997**	0.20990	0.1080		1.705	0.0900
0	-9.868**	0.29873	0.1086			

DLPAW: ADF tests (T=181, Constant; 5%=-2.88 1%=-3.47)

D-lag	t-ADF	beta	Y <sub>-1</sub>	sigma	t-DY <sub>-lag</sub>	t-prob
3	-6.093**	0.34188	0.05601		0.6527	0.5148
2	-6.466**	0.37264	0.05592		-1.542	0.1249
1	-8.687**	0.29101	0.05613		2.216	0.0280
0	-8.853**	0.39075	0.05674			

Table 1: Test results for  $LPA_t$ , the natural logarithm of the Oslo Stock Exchange Index, and LPAW, the log of a world equity price index.

EQ(1) Modelling DLPA by OLS

The dataset is: C:\SW20\ECON4160\H2018\Exam\MODobligexam.in7

The estimation sample is: 1985(1) - 2018(2)

	Coefficient	Std.Error	t-value	t-prob
DLPA_1	0.118291	0.04598	2.57	0.0113
LPA_1	-0.0355075	0.01952	-1.82	0.0713
LPAW_1	0.0432321	0.02383	1.81	0.0721
DLPAW	0.899811	0.09305	9.67	0.0000
DLSPoilUSD	0.153493	0.03232	4.75	0.0000
DVOLUSA	-0.00389039	0.001015	-3.83	0.0002
DPADUM	0.975029	0.1063	9.17	0.0000
Constant	0.00718436	0.007280	0.987	0.3256
sigma	0.0491035	RSS		0.303805505
R <sup>2</sup>	0.799467	F(7,126) =	71.76	[0.000]**
Adj.R <sup>2</sup>	0.788327	log-likelihood		217.839
no. of observations	134	no. of parameters		8
AR 1-5 test:	F(5,121) =	1.3873	[0.2338]	
ARCH 1-4 test:	F(4,126) =	0.68241	[0.6054]	
Normality test:	Chi <sup>2</sup> (2) =	1.5245	[0.4666]	
Hetero-X test:	F(34,99) =	1.3095	[0.1536]	

Critical values of ECM-test: -3.21 (5 %), -2.91 (10 %)

Table 2: Estimation results for a model of DLPA which is the first difference of  $LPA_t$ . DLPAW is the first difference of LPAW. The other variables are the change in the log of the oil price ( $DLSPoilUSD$ ), a measure of volatility ( $DVOLUSA$ ) and a dummy ( $DPADUM$ ).