UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: ECON4160 – Econometrics – Modeling and Systems Estimation

Date of exam: Friday, December 13, 2019 **Grades are given:** January 2, 2020

Time for exam: 09.00 a.m. – 13.00 noon (4 hours)

The problem set covers 5 pages (incl. cover sheet and tables)

Resources allowed:

- Open book examination where all printed and written resources, in addition to two alternative calculators are allowed.

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

Guidelines:
In the grading, question A gets 33 %, B 33 % and C 33 %.

Question A (1/3)

Consider the following three-equation model of the endogenous variables nominal wage level (w_t) price level (p_t) and unemployment rate (u_t).

$$\Delta w_t = \alpha_0 + \alpha_1 \Delta p_t + \alpha_2 (w-p)_{t-1} + \alpha_3 u_t + \alpha_4 x1_t + \alpha_5 x4_t + \epsilon_{wt} \quad (1)$$
$$\Delta p_t = \gamma_0 + \gamma_1 \Delta w_t + \gamma_2 (w-p)_{t-1} + \gamma_3 x2_t + \epsilon_{pt} \quad (2)$$
$$u_t = \beta_0 + \beta_1 (w-p)_{t-1} + \beta_2 u_{t-1} + \beta_3 x3_t + \epsilon_{ut} \quad (3)$$

The three endogenous variables are measured in natural logarithms. Hence $(w-p)_t$ is the log of the real wage in period t . Δ is the difference operator, hence $\Delta w_t = w_t - w_{t-1}$ and $\Delta p_t = p_t - p_{t-1}$.

The disturbance terms ϵ_{jt} $(j = w, p, u)$ are jointly Gaussian white-noise variables. They may be contemporaneously correlated (the covariances between the disturbances are **not** restricted to be zero).

xi_t $(i = 1, 2, 3, 4)$ are observable time series variables that are independent of the disturbances ϵ_{jt} $(j = w, p, u)$.

All variables in the model are stationary time series.

1. Explain why all equations in model (1)-(3) are identified. You can base your answer on the assumption that all the coefficients in the model are different from zero.

2. Assume that the main parameters of interest are the coefficients of equation (1), which we will refer to as the wage equation of the model. Table 1 contains three estimations of the wage equation. In part A of the table, OLS results are reported. Part B and C show the results of two different 2SLS estimations.

   (a) First compare the results in Part A and Part B. You may focus on the estimated coefficient of the change in the price level (the Dp variable in the estimation results). What can explain the difference between the OLS and the 2SLS estimates of α_1?

   (b) Table 1, part C, shows the results of a second 2SLS estimation. Compared to part B, this 2SLS estimation gives different results for the coefficients, and in particular for α_3, the coefficient of the unemployment rate. Can you suggest an explanation for this difference?

3. If the research purpose was to estimate only the coefficients in equation (3). Will OLS give consistent estimators of the coefficients β_j, $j = 0, 1, 2, 3$?

Question B (1/3)

Consider the model:

$$WS_t = c_0 + c_{wu}U_t + \epsilon_{wt},\ c_{wu} \leq 0 \tag{4}$$

$$U_t = d_0 + d_{uw}WS_t + d_{uu}U_{t-1} + \epsilon_{ut},\ d_{uw} \geq 0,\ d_{uu} > 0 \tag{5}$$

where WS_t is the wage share of the economy and U_t is the unemployment rate. The economic interpretation of the model is that an increase in unemployment can reduce labour's share of GDP (hence $c_{wu} \leq 0$), while an increase in the labour share can lead to higher unemployment, (hence $d_{uw} \geq 0$). ϵ_{wt} and ϵ_{ut} are white-noise time series.

The reduced form of the model is:

$$WS_t = \frac{c_0 + c_{wu}d_0}{(1 - c_{wu}d_{uw})} + \frac{c_{wu}d_{uu}}{(1 - c_{wu}d_{uw})}U_{t-1} + \frac{c_{wu}\epsilon_{ut} + \epsilon_{wt}}{(1 - c_{wu}d_{uw})} \tag{6}$$

$$U_t = \frac{d_0 + d_{uw}c_0}{(1 - c_{wu}d_{uw})} + \frac{d_{uu}}{(1 - c_{wu}d_{uw})}U_{t-1} + \frac{\epsilon_{ut} + d_{uw}\epsilon_{wt}}{(1 - c_{wu}d_{uw})} \tag{7}$$

In the following you shall take (6) and (7) as given, you shall not derive them.

1. Explain why the condition of stationarity of U_t is:

$$-1 < \frac{d_{uu}}{(1 - c_{wu}d_{uw})} < 1 \tag{8}$$

2. What is the condition for stationarity of WS_t?

3. Assuming stationarity, what is the expression for the expectation of U_t?

4. Assume that you know the values of the coefficients (c_0, c_{wu}, d_0,d_{uw},d_{uu}) and the value of unemployment in period T, U_T. Assume that the conditional expectation function $E(U_{T+h} \mid U_T)$ is used to generate dynamic forecasts of U_{T+h}, $h = 1, 2, .., H$.

   (a) Sketch a graph of the forecast in the case of:

   $$0 < \frac{d_{uu}}{(1 - c_{wu}d_{uw})} < 1 \text{ and } U_T > U^*$$

   where U^* denotes the expectation of U_t.

   (b) Sketch a graph of the forecast in the case of:

   $$\frac{d_{uu}}{(1 - c_{wu}d_{uw})} = 1,$$

   depending on what you assume about the sign of the constant term in (7).

5. What are the typical features of the variance of the forecast errors in the two cases in B. 4) ?

Question C (1/3)

1. Explain how you can use the information in Table 2 at the back of the question set to support the assumption that the two time series $LCO2$ (log of CO_2 level in the atmosphere) and $LGDP$ (log of world GDP) are integrated of order one, $I(1)$.

2. Assume that a friend shows you the results in Table 3. He says that he thinks it represents strong evidence for a relationship between CO_2 and world GDP, because the t-value of $LGDP$ is very high and because robust standard errors have been used. How would you explain to him that this in fact represents no formal evidence for a relationship, and that he may become criticized for reporting a spurious regression?

3. Explain a correct testing procedure of the null hypothesis of no cointegrating relationship between $LCO2_t$ and $LGDP_t$.

Tables

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**********************************************************
  A  Estimating the wage equation by OLS
**********************************************************
       The estimation sample is: 2 - 101
Equation for: Dw
                 Coefficient  Std.Error  t-value  t-prob
Dp                  0.987501    0.02643     37.4  0.0000
(w-p)_1            -0.320808    0.02982    -10.8  0.0000
u                  0.0387146    0.05192    0.746  0.4577
x1                  0.959641    0.06900     13.9  0.0000
x4                 -0.371072    0.04717    -7.87  0.0000
Constant          -0.0526795    0.08706   -0.605  0.5465

**********************************************************
 B   Estimating the wage equation by 2SLS
**********************************************************
     Endogenous right-hand side variable: Dp
     Exogenous and Instruments: (w-p)_1, u, x1, x2, x4, Constant

     The estimation sample is: 2 - 101
Equation for: Dw
                 Coefficient  Std.Error  t-value  t-prob
Dp                  0.812189    0.05926     13.7  0.0000
(w-p)_1            -0.286912    0.03722    -7.71  0.0000
u                  0.0470528    0.06262    0.751  0.4543
x1                   1.23475     0.1143     10.8  0.0000
x4                 -0.468555    0.06328    -7.40  0.0000
Constant           -0.125322     0.1070    -1.17  0.2442

**********************************************************
  C      Estimating the wage equation by 2SLS
**********************************************************
     Endogenous right-hand side variable: Dp, u.
     Exogenous and Instruments: (w-p)_1, x1, x2, x3, x4, Constant

     The estimation sample is: 2 - 101
Equation for: Dw
                 Coefficient  Std.Error  t-value  t-prob
Dp                  0.798373    0.06716     11.9  0.0000
(w-p)_1            -0.188763    0.05132    -3.68  0.0004
u                  -0.223944    0.09773    -2.29  0.0241
x1                   1.25084     0.1282     9.76  0.0000
x4                 -0.530557    0.07553    -7.02  0.0000
Constant           -0.130630     0.1191    -1.10  0.2754
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Table 1: OLS and 2SLS results for the wage equation (1).

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*********************************************************
Unit-root tests: LCO and LGDP
*********************************************************

The sample is: 1970 - 2018 (51 observations)

LCO2: ADF tests (T=49, Constant; 5%=-2.92 1%=-3.57)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob
 1       3.244       1.0108 0.001338   -0.06406   0.9492
 0       3.732       1.0107 0.001323

LGDP: ADF tests (T=49, Constant; 5%=-2.92 1%=-3.57)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob
 1      -1.233      0.99424  0.01326      1.443   0.1557
 0      -1.668      0.99242  0.01341

*********************************************************
Unit-root tests: DLCO and DLGDP
*********************************************************

DLCO2: ADF tests (T=49, Constant; 5%=-2.92 1%=-3.57)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob
 1      -4.290**    0.21674 0.001483     0.1032   0.9183
 0      -5.434**    0.22838 0.001467

DLGDP: ADF tests (T=49, Constant; 5%=-2.92 1%=-3.57)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob
 1      -5.451**    0.12061  0.01320      1.404   0.1671
 0      -5.577**    0.24764  0.01334
```

Table 2: Dickey-Fuller tests of unit-root in $LCO2$ and $LGDP$ (levels) and in $DLCO2$ and $DLGDP$ (differenced variables). Annual data.

```
EQ(1) Modelling LCO2 by OLS
      The estimation sample is: 1970 - 2018

                 Coefficient  Std.Error      HACSE  t-HACSE  t-prob
Constant             5.30109   0.008089    0.01801     294.  0.0000
LGDP                0.158148   0.002159   0.004887     32.4  0.0000

sigma             0.00632872  RSS             0.00188247485
R^2                 0.991314  F(1,47) =      5364 [0.000]**
Adj.R^2             0.991129  log-likelihood        179.563
no. of observations       49  no. of parameters           2
mean(LCO2)           5.88983  se(LCO2)            0.0671933

AR 1-2 test:      F(2,45)    =   53.037 [0.0000]**
ARCH 1-1 test:    F(1,47)    =   83.953 [0.0000]**
Normality test:   Chi^2(2)   =   20.628 [0.0000]**
Hetero test:      F(2,46)    =   17.436 [0.0000]**
Hetero-X test:    F(2,46)    =   17.436 [0.0000]**
```

Table 3: Results for the regression between $LCO2$ and $LGDP$. Annual data. (HACSE is an acronym for heteroscedasticity and autocorrelation consistent standard errors, i.e., robust standard errors).