Postponed exam in: ECON 4160: Econometrics: Modelling and Systems Estimation

Day of exam: 17 January 2020

Time of day: 09:00—13:00

This is a 4 hour school exam.

Guidelines:
In the grading, question A gets 20 %, B 30 % and C 50 %.

Question A (20 %)

Consider the following stochastic difference equation:

$$Y_t = 1.3Y_{t-1} - 0.4Y_{t-2} + 0.2 + \epsilon_t, \quad t = 1, 2, ..., T \tag{1}$$

where ϵ_t is a white-noise time series.

1. It can be shown that the associated characteristic roots are 0.8 and 0.5. What do they tell us about the stationarity (or non-stationarity) of Y_t?
   A: With reference to theorems in the book (curriculum): Since both roots are less than one in magnitude, the homogenous difference equation is globally asymptotically stable. It then follows that the time series Y_t generated by (1) is stationary.

2. The partial derivative of Y_t with respect to ϵ_t is 1. Calculate the first and second dynamic multipliers (impulse responses).
   A:

$$\begin{aligned}
\delta_0 &= \frac{\partial Y_t}{\partial \epsilon_t} = 1 \\
\delta_1 &= \frac{\partial Y_{t+1}}{\partial \epsilon_t} = 1.3 * \frac{\partial Y_t}{\partial \epsilon_t} = 1.3 \\
\delta_2 &= \frac{\partial Y_{t+2}}{\partial \epsilon_t} = 1.3\frac{\partial Y_{t+1}}{\partial \epsilon_t} - 0.4 = 1.3 * 1.3 - 0.4 = 1.29
\end{aligned}$$

3. Assume that we use equation (1) to forecast $Y_{T+h}, h = 1, 2, ..., H$. Calculate the optimal (minimum MSFE) forecasts of Y_{T+1}, Y_{T+2} and Y_{T+3} conditional on $Y_T = 1$ and $Y_{T-1} = 0.5$.
   A: Since ϵ is white noise the optimal (minimum) MSFE) forecast is obtained as the conditional expectations:

$$\begin{aligned}
E(Y_{T+1} \mid Y_T, Y_{T-1}) &= 1.3 * 1 - 0.4 * 0.5 + 0.2 = 1.3 \\
E(Y_{T+2} \mid Y_T, Y_{T-1}) &= 1.3 * E(Y_{T+1} \mid Y_T, Y_{T-1}) - 0.4 * 1 + 0.2 = \\
&= 1.3 * 1.3 - 0.4 * 1 + 0.2 = 1.49 \\
E(Y_{T+2} \mid Y_T, Y_{T-1}) &= 1.3 * 1.49 - 0.4 * 1.3 + 0.2 = 1.617
\end{aligned}$$

   Possible additional remarks. The conditional forecast is optimal under assumption of a symmetric loss function.

4. What is the long-run forecast of Y_{T+H}, i.e., when $H \approx \infty$?
   A: Due to stationarity, the conditional expectation is asymptotically equivalent to the expectation, hence:

$$E(Y_{T+\infty}) = E(Y_t) = \frac{0.2}{1 - 1.2 + 0.4} = 1.0$$

Question B (30 %)

Table 1 shows unit-root tests for three different interest rates:

- RBO: The yield on 5-year Norwegian treasury bills.
- $RLBOLIGH$: The interest rate on house loans (mortgage rate) in Norway.
- RSH: The Norwegian money market interest rate.

All three variables are measured in percent.

1. Make use of the information in Table 1 to decide the order of integration of each of the three time series variables.
   A: With reference to the curriculum we choose the ADF for D-lag 1 as the most reliable for testing the null hypothesis of unit-root for all three variables. The conclusion is that H_0 is rejected at the 1 % or 5 % level for all three interest rates-

2. Table 2 shows estimation results for a conditional ADL model of $RLBOLIGH$ given RBO and RSH. There are also two dummy variables in the model: $BASELIII$ is a dummy that captures the introduction of new banking regulations: It is zero until 2011(3) and 1 after. CRISIS09Q1 is an impulse dummy which is one in 2009(1) and zero elsewhere.

   (a) Does Table 2 represent reliable evidence for the view that BASELIII banking regulation has increased the interest rate on housing loans?
   A: The validity of the underlying statistical assumptions of the regression model are supported by the battery of mis specification tests. (The only departure being the Hetero-X-test). We can therefore regard the t-value of BASSELIII, and its t-prob (same as p-value) as reliable. Economically it seem reasonable to assume that the relationship in all important respects goes from BASEL-regulation regimes and to Norwegian interest rate setting. This does not rule our there "is a connection" between the introduction of BASELIII and the history of interest rates, but to capture that formally would require a much more complex set up. Hence it is reasonable to concluded that the model captures an effect from regulation to the interest rate.

   (b) Show that the estimated long-run equation becomes:

$$RLBOLIGH = 0.25RBO + 0.74RSH + 1.06BASELIII + 1.05 \qquad (2)$$

   (Rounding errors are not important here, as long as you show the right method.)
   A:

$$(1 - 0.63233)RLBOLIGH = 0.38093 + (-0.0393621 + 0.128756)RBO + (0.559226 - 0.290578)R$$

$$0.367\,67RLBOLIGH = (0.08939\,4)RBO + 0.268\,65RSH + 0.385553BASELIII + 0.38093$$

$$RLBOLIGH = \frac{0.08939\,4}{0.36767}RBO + \frac{0.26865}{0.36767}RSH + \frac{0.385553}{0.36767}BASELIII + \frac{0.38093}{0.36767}$$

   (c) Assume that both RBO and RSH are increased permanently by 1 percentage point (e.g., from 2 percent to 3 percent).

   i. What is the estimated impact response of $RLBOLIGH$ to the change?
   A: Impact effect:

$$-0.0393621 + 0.559116 = 0.5197539$$

   ii. What is the estimated long-run response to the change?
   A: Long-run effect:

$$\frac{0.089394}{0.36767} + \frac{0.26865}{0.36767} = 0.97382$$

Question C (50 %)

Assume that the three time series variables: w_t: nominal wage level, p_t: price level, u_t: unemployment rate are measured in natural logarithms and that they are generated by the VAR:

$$\begin{pmatrix} w_t \\ p_t \\ u_t \end{pmatrix} = \begin{pmatrix} (1-\phi_{wp}) & \phi_{wp} & 0 \\ \phi_{pw} & (1-\phi_{pw}) & 0 \\ \phi_{uw} & -\phi_{uw} & (1-\phi_{uu}) \end{pmatrix} \begin{pmatrix} w_{t-1} \\ p_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{wt} \\ \varepsilon_{pt} \\ \varepsilon_{ut} \end{pmatrix} \tag{3}$$

where the vector with VAR error terms $(\varepsilon_{wt} \quad \varepsilon_{pt} \quad \varepsilon_{ut})'$ is Gaussian white-noise with expectation zero and covariance matrix $\mathbf{\Sigma}$. We do *not* assume that $\mathbf{\Sigma}$ is a diagonal matrix.

1. It can be shown that the eigenvalues of the autoregressive matrix can be expressed as: 1, $(1-\phi_{wp}-\phi_{pw})$ and $(1-\phi_{uu})$. Assume that $0 < \phi_{wp}+\phi_{pw} < 1$ and $0 < \phi_{uu} < 1$.

   (a) What does this imply for the order of integration of the three time series variables?
   A: Under the assumptions, the autoregressive matrix has one eigenvalue equal to one, and two the eigenvalues that are less than one in magnitude. This means the vector time series is non-stationary, i.e., at least one of the variables are non-stationary.

   (b) What does this imply for the number of long-run relationships between the variables?
   A: Since there in only one unit root, not three, it follows that there are two cointegrating relationships. Note that one variable (u_t?) can be cointegrated with itself, and that w_t and p_t are a cointegrated pair.

2. Assume that we are interested in the relationship between the real wage, $(w-p)_t = w_t - p_t$, and the rate of unemployment, u_t.

   Show that (3) implies the following VAR for $(w-p)_t$ and u_t:

$$\begin{pmatrix} (w-p)_t \\ u_t \end{pmatrix} = \begin{pmatrix} (1-\phi_{wp}-\phi_{pw}) & \phi_{wu} \\ \phi_{uw} & (1-\phi_{uu}) \end{pmatrix} \begin{pmatrix} (w-p)_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{wpt} \\ \varepsilon_{ut} \end{pmatrix} \tag{4}$$

   where the coefficient of u_{t-1} in the first row (ϕ_{wu}) is implied to be: $\phi_{wu} = 0$ and the error-term ε_{wpt} is implied to be: $\varepsilon_{wpt} = \varepsilon_{wt} - \varepsilon_{pt}$.
   A: Subtract the second row from the first row. It is then seen that logically (and using the notation in the question): $\phi_{wu} = 0$ and $\varepsilon_{wpt} = \varepsilon_{wt} - \varepsilon_{pt}$.

3. What are the orders of integration of $(w-p)_t$ and u_t?
   A: Using the information above (in the question set), it follows that $(w-p)_t$ is defined by a stable AR(1) process. Hence $(w-p)_t \sim I(0)$. And, as a consequence, $u_t \sim I(0)$ from the second row of the bivariate VAR.

4. A data set for $(w-p)_t$ and u_t has been generated in accordance with the specification above. Table 3 shows estimation results for an **unrestricted** VAR of $(w-p)_t$ and u_t.
   Assume that the true data generating process was unknown to you and that you used Table 3 to test:

$$H_0 : \phi_{wu} = 0 \;\text{ against }\; H_1 : \phi_{wu} \neq 0$$

   Explain how you would conclude. (Hint: You can take for granted that none of the standard mis-specification tests are significant).
   A: We can use the t-value 0.235 and its t-prob to conclude that H_0 cannot be rejected.

5. Consider another test situation:

$$H_0 : \phi_{uw} = 0 \text{ against } H_1 : \phi_{uw} \neq 0$$

How would you conclude?
A: Again: Directly fron the unrestricted VAR results: we use t-value 1.77 and its t-prob 0.08 to conclude that H_0 can be rejected at the 10 % level, but not at levels lower then 8 %.

6. In the data generation, $\mathbf{\Sigma}$ was specified as:

$$\mathbf{\Sigma} = \begin{pmatrix} 1 & 0.5 & -0.3 \\ 0.5 & 1 & 0 \\ -0.3 & 0 & 1 \end{pmatrix}.$$

Show that

$$Cov(\varepsilon_{wpt}, \varepsilon_{ut}) = -0.3.$$

A:

$$\begin{aligned} Cov(\varepsilon_{wpt}, \varepsilon_{ut}) &= E(\varepsilon_{wpt}\varepsilon_{ut}) = E((\varepsilon_{wt} - \varepsilon_{pt})\varepsilon_{ut}) = \\ &= E((\varepsilon_{wt}\varepsilon_{ut} - \varepsilon_{pt}\varepsilon_{ut}) = -0.3 - 0 = -0.3. \end{aligned}$$

7. Table 4 shows estimation results for a model of the VAR.

   (a) Explain why the reported log-likelihood is the same for this empirical model as for the VAR in Table 3
   A: The model is 1-1 reparameterization of the VAR system,. The second equation is the second row of the VAR, it the marginal model equation, and the first model equation the correct conditional model (again given the bivariate VAR as the system). Therefore the log likelihoods of the VAR and the conditional model of the VAR must be identical.

   (b) Show how the estimated coefficient of u_t in the first equation in Table 4 can be obtained by use of the information in Table 3. A: From OLS algebra:

$$-0.30038 * \frac{0.91454}{1.0585} = -0.25953$$

Tables

Unit-root tests

The sample is: 1994(1) - 2019(3) (106 observations and 3 variables)

RBO: ADF tests (T=103, Constant+Trend; 5%=-3.45 1%=-4.05)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob
2	-3.301	0.83113	0.3681	-1.549	0.1245
1	-4.140**	0.80183	0.3707	3.768	0.0003
0	-3.106	0.84678	0.3944		

RLBOLIGH: ADF tests (T=103, Constant+Trend; 5%=-3.45 1%=-4.05)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob
2	-3.288	0.85894	0.4380	-0.2293	0.8191
1	-3.555*	0.85586	0.4359	4.906	0.0000
0	-2.296	0.89920	0.4836		

RSH: ADF tests (T=103, Constant+Trend; 5%=-3.45 1%=-4.05)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob
2	-3.234	0.88293	0.4640	-0.5394	0.5908
1	-3.662*	0.87615	0.4623	6.400	0.0000
0	-2.043	0.91994	0.5470		

Table 1: Dickey Fuller tests of unit-root in *RBO*, *RLBOLIGH* and *RSH* (levels). Quarterly data.

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EQ(1) Modelling RLBOLIGH by OLS

      The estimation sample is: 1994(1) - 2019(2)

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
RLBOLIGH_1           0.636233    0.02956     21.5  0.0000   0.8328
RBO                -0.0393621    0.02885    -1.36  0.1758   0.0196
RBO_1                0.128756    0.02912     4.42  0.0000   0.1737
RSH                  0.559116    0.02648     21.1  0.0000   0.8274
RSH_1               -0.290578    0.03815    -7.62  0.0000   0.3841
BASELIII             0.385553    0.04873     7.91  0.0000   0.4023
CRISIS09Q1          -0.752125     0.1179    -6.38  0.0000   0.3045
Constant             0.380293    0.05127     7.42  0.0000   0.3717

sigma                 0.10117  RSS              0.951892533
R^2                  0.997316  F(7,93) =      4320 [0.000]**
Adj.R^2              0.997085  log-likelihood        93.6563
no. of observations       102  no. of parameters           8
mean(RLBOLIGH)        5.03518  se(RLBOLIGH)          1.87393

AR 1-5 test:      F(5,88)    =   1.0916 [0.3708]
ARCH 1-4 test:    F(4,94)    =  0.24759 [0.9105]
Normality test:   Chi^2(2)   =  0.48073 [0.7863]
Hetero test:      F(14,86)   =  0.86052 [0.6030]
Hetero-X test:    F(31,69)   =   1.8576 [0.0170]*
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Table 2: Estimation results for an ADL model of $RLBOLIGH$.

```
      Estimating the VAR by OLS
      he estimation sample is: 2 - 101

VAR equation for: (w-p)
                Coefficient  Std.Error  t-value  t-prob
(w-p)_1            0.389369    0.09327     4.17  0.0001
u_1               0.0115088    0.04903    0.235  0.8149

sigma = 0.914543   RSS = 81.96606412

VAR equation for: u
                Coefficient  Std.Error  t-value  t-prob
(w-p)_1            0.190940     0.1079     1.77  0.0800
u_1                0.827843    0.05674     14.6  0.0000

sigma = 1.05846

log-likelihood    -273.787634

correlation of VAR residuals (standard deviations on diagonal)

             (w-p)           u
(w-p)      0.91454    -0.30038
u         -0.30038      1.0585
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Table 3: Estimation results for unrestricted VAR of $(w-p)_t$ and u_t.

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     Estimating the model by 1SLS
     The estimation sample is: 2 - 101

Equation for: (w-p)
                 Coefficient  Std.Error  t-value  t-prob
(w-p)_1             0.438925    0.09083     4.83  0.0000
u                  -0.259536    0.08368    -3.10  0.0025
u_1                 0.226364    0.08371     2.70  0.0081

sigma = 0.872309

Equation for: u
                 Coefficient  Std.Error  t-value  t-prob
(w-p)_1             0.190940     0.1079     1.77  0.0800
u_1                 0.827843    0.05674     14.6  0.0000

sigma = 1.05846

log-likelihood    -273.787634
no. of observations       100  no. of parameters           5

correlation of model residuals (standard deviations on diagonal)
             (w-p)           u
(w-p)       0.87231     0.00000
u           0.00000      1.0585
```

Table 4: Estimation results for a model of $(w-p)_t$ and u_t.