Exam in: ECON 4160: Econometrics: Modelling and Systems Estimation

Day of exam: 13 December 2019—ANSWER NOTES

Time of day: 09:00—13:00

This is a 4 hour school exam.

Guidelines:
In the grading, question A gets 33 %, B 33 % and C 33 %.

1 Question A (1/3)

1. Since all coefficients of the model are assumed to be non-zero, the order condition can be applied equation by equation. (3) is clearly identified since it only has exogenous or predetermined variables on the RHS For the purpose of discussing identification (1) and (2) can regarded as a sub-system where u is predetermined (but it is not necessary, the conclusions will be the same). Both (1) and (2) exclude more variables than necessary to be exactly identified. They are over-identified equations.

2. It is important to note precisely how the model has been specified in the question to get this right:

   (a) Simultaneity bias, since (1) and (2) constitute a SEM, there will be a bias due to correlation between Dp and the disturbance $\epsilon_p t$ which induces a bias in the OLS estimate of α_1 that does not go away even when the sample size increases towards infinity.

   (b) The 2SLS results in part B of Table 1 are based on treating u as exogenous. But u is only weakly exogenous for α_3 if the disturbance ϵ_{ut} is uncorrelated with ϵ_{wt} (and/or ϵ_{pt}). This independence is not secured by the model specification given in the exercise (it is not assumed that the disturbances are independent). The 2SLS estimation results in Part C in Table 3 treats u as an endogenous RHS variable in (6), and also correctly extends the IV list by $x3_t$. But the main point is to see that (based on the model

3. Since all RHS variables are predetermined, OLS gives consistent estimations. In fact under the assumption given in the question (Gaussian white noise disturbances), OLS is FIML for (3). It is positive if the inevitable Hurwicz bias is mentioned.

2 Question B (1/3)

1. The RF equation for U_t is also the final equation for U_t. This equation is a first order stochastic difference equation. It can be written as

$$U_t = \phi_0 + \phi_1 U_{t-1} + \varepsilon_t$$

   Since ε_t is stationary we know that the stationarity condition is $-1 < \phi_1 < 1$, which becomes

$$-1 < \frac{d_{uu}}{(1 - c_{wu} d_{uw})} < 1$$

   in this case. We see that instability is a possible also if $d_{uu} < 1$. Stability is a system property. Equivalent to the stated condition is:

$$d_{uu} < (1 - c_{wu} d_{uw})$$

2. It is the same generally, since (3) with $c_w u > 0$ shows that WS must be stationary of U_t is stationary, and vise versa. Small point: if $c_{wu} = 0$, WS_t can be stationarity at the same time as:

$$-1 < d_{uu} = 1$$

which implies that U_t is I(1).

3. We can take the (un)conditional expectation on both sides the RF equation for U_t. With reference to stationarity we can set $E(U_{t-1} = E(U_t)$ on the RHS. We then get:

$$E(U_t)(1 - \frac{d_{uu}}{(1 - c_{wu}d_{uw})}) = \frac{d_0 + d_{uw}c_0}{(1 - c_{wu}d_{uw})}$$
$$E(U_t)(\frac{1 - c_{wu}d_{uw} - d_{uu}}{1 - c_{wu}d_{uw}}) = \frac{d_0 + d_{uw}c_0}{(1 - c_{wu}d_{uw})}$$
$$E(U_t) = \frac{d_0 + d_{uw}c_0}{1 - c_{wu}d_{uw} - d_{uu}}$$

4. the function and function value is the (point) forecast.

   The sketched plot of this function can start in $(h = 0, U_T)$. The graph will fall monotonously towards Y^*. Since the autoregressive coefficient is positive there will be no oscillations in this case.

   (b) Since the conditional expectation is well defined also in this case, where U_t is I(1), we can use it to forecast for a finite forecast horizon. The drift term is positive under the assumptions given, so the forecasted U_{T+h} will be increasing with h.

5. In the stationarity case, the variance of the forecast errors increases with h When H is large, the forecast error variance is approximately equal to $Var(U_t)$. In the non-stationary case, the forecast error variance grows without limit.

3 Question C (1/3)

1. The Table 1 tests for LCO2 and LGDP can be used to test the H_0 of each one of them being $I(1)$. For LCO2 the conclusion about non-rejection of a unit-root does not depend on which one the three unit root tests we choose: All of them even are even positively signed ! For LGDP the t-adf are negative, but with values that are insignificant compared to the critical values given in the table. The critical values are from the Dickey-Fuller distribution. Using critical values of N(0,1) would lead to over-rejection of H_0. The tests in the second part of the table reject the H_0 of the differences being $I(1)$.

2. The evidence is not strong because under the H_0 of no cointegration, the distribution of the t-statistic from the static regression is neither normally distributed not t-distributed. What we know from Monte Carlo simulations is that the Type-I error probability will grow towards 1 if inference about existence of a relationship is based on that t-statistic. Use of robust standard errors does not help. It would have helped if the two variables where I(0) (although modelling the dynamics with ADL or ECM models would be much better).

   The friend's procedure may be a classical example of spurious regression. In general, taking linear combinations of two $I(1)$ variables delivers another I(1) variable, which will the disturbance of his static regression. To avoid that a valied testing procedure must be used. The standard critical values of the t-ratio (with or without robust SE) are not valid.

3. We have learned about two test approaches. The method uses the frends static regression as a first step, but the correct procedure is to base inference on a test of the H_0 that the residuals from the static regression is I(1). The critical values of this test need to take account of the estimation (that we test $\hat{e}_t$ not e_t), hence the critical values are larger in magnitude than the ADF critical values for a observable time series. The second testing procedure is known as the ECM-test of no cointegration. We then simple formulate a unrestricted ECM for $\Delta LCO2$ and test the significance of the coefficient of $LCO2_{t-1}$, using the critical values in the cited paper by Ericsson and MacKinnon for example. In general the ECM test is preferable, as it usually has larger power. The Type-I error probabilities more or less the same for the two tests.