UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Postponed exam: ECON4160 – Econometrics – Modeling and Systems Estimation

Date of exam: Friday, January 17, 2020

Time for exam: 09:00 a.m. – 13:00 noon (4 hours)

The problem set covers 7 pages (incl. cover sheet)

Resources allowed:

- All written and printed resources – as well as two alternative calculators - are allowed (except if you have been granted use of a dictionary from the Faculty of Social Sciences)

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

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This is a 4 hour school exam.

Guidelines:
In the grading, question A gets 20 %, B 30 % and C 50 %.

Question A (20 %)

Consider the following stochastic difference equation:

$$Y_t = 1.3Y_{t-1} - 0.4Y_{t-2} + 0.2 + \epsilon_t, \quad t = 1, 2, ..., T \tag{1}$$

where ϵ_t is a white-noise time series.

1. It can be shown that the associated characteristic roots are 0.8 and 0.5. What do they tell us about the stationarity (or non-stationarity) of Y_t?
2. The partial derivative of Y_t with respect to ϵ_t is 1. Calculate the first and second dynamic multipliers (impulse responses).
3. Assume that we use equation (1) to forecast $Y_{T+h}, h = 1, 2, ..., H$. Calculate the optimal (minimum MSFE) forecasts of Y_{T+1}, Y_{T+2} and Y_{T+3} conditional on $Y_T = 1$ and $Y_{T-1} = 0.5$.
4. What is the long-run forecast of Y_{T+H}, i.e., when $H \approx \infty$?

Question B (30 %)

Table 1 shows unit-root tests for three different interest rates:

- RBO: The yield on 5-year Norwegian treasury bills.
- $RLBOLIGH$: The interest rate on house loans (mortgage rate) in Norway.
- RSH: The Norwegian money market interest rate.

All three variables are measured in percent.

1. Make use of the information in Table 1 to decide the order of integration of each of the three time series variables.
2. Table 2 shows estimation results for a conditional ADL model of $RLBOLIGH$ given RBO and RSH. There are also two dummy variables in the model: $BASELIII$ is a dummy that captures the introduction of new banking regulations: It is zero until 2011(3) and 1 after. CRISIS09Q1 is an impulse dummy which is one in 2009(1) and zero elsewhere.
   (a) Does Table 2 represent reliable evidence for the view that BASELIII banking regulation has increased the interest rate on housing loans?
   (b) Show that the estimated long-run equation becomes:

$$RLBOLIGH = 0.25RBO + 0.74RSH + 1.06BASELIII + 1.05 \tag{2}$$

   (Rounding errors are not important here, as long as you show the right method.)

(c) Assume that both RBO and RSH are increased permanently by 1 percentage point (e.g., from 2 percent to 3 percent).

i. What is the estimated impact response of $RLBOLIGH$ to the change?

ii. What is the estimated long-run response to the change?

Question C (50 %)

Assume that the three time series variables: w_t: nominal wage level, p_t: price level, u_t: unemployment rate are measured in natural logarithms and that they are generated by the VAR:

$$\begin{pmatrix} w_t \\ p_t \\ u_t \end{pmatrix} = \begin{pmatrix} (1-\phi_{wp}) & \phi_{wp} & 0 \\ \phi_{pw} & (1-\phi_{pw}) & 0 \\ \phi_{uw} & -\phi_{uw} & (1-\phi_{uu}) \end{pmatrix} \begin{pmatrix} w_{t-1} \\ p_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{wt} \\ \varepsilon_{pt} \\ \varepsilon_{ut} \end{pmatrix} \tag{3}$$

where the vector with VAR error terms $\begin{pmatrix} \varepsilon_{wt} & \varepsilon_{pt} & \varepsilon_{ut} \end{pmatrix}'$ is Gaussian white-noise with expectation zero and covariance matrix $\mathbf{\Sigma}$. We do *not* assume that $\mathbf{\Sigma}$ is a diagonal matrix.

1. It can be shown that the eigenvalues of the autoregressive matrix can be expressed as: 1, $(1-\phi_{wp}-\phi_{pw})$ and $(1-\phi_{uu})$. Assume that $0 < \phi_{wp}+\phi_{pw} < 1$ and $0 < \phi_{uu} < 1$.

   (a) What does this imply for the order of integration of the three time series variables?

   (b) What does this imply for number of long-run relationships between the variables?

2. Assume that we are interested in the relationship between the real wage, $(w-p)_t = w_t - p_t$, and the rate of unemployment, u_t.

   Show that (3) implies the following VAR for $(w-p)_t$ and u_t:

$$\begin{pmatrix} (w-p)_t \\ u_t \end{pmatrix} = \begin{pmatrix} (1-\phi_{wp}-\phi_{pw}) & \phi_{wu} \\ \phi_{uw} & (1-\phi_{uu}) \end{pmatrix} \begin{pmatrix} (w-p)_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{wpt} \\ \varepsilon_{ut} \end{pmatrix} \tag{4}$$

   where the coefficient of u_{t-1} in the first row (ϕ_{wu}) is implied to be: $\phi_{wu} = 0$ and the error-term ε_{wpt} is implied to be: $\varepsilon_{wpt} = \varepsilon_{wt} - \varepsilon_{pt}$.

3. What are the orders of integration of $(w-p)_t$ and u_t?

4. A data set for $(w-p)_t$ and u_t has been generated in accordance with the specification above. Table 3 shows estimation results for an **unrestricted** VAR of $(w-p)_t$ and u_t.

   Assume that the true data generating process was unknown to you and that you used Table 3 to test:

$$H_0 : \phi_{wu} = 0 \text{ against } H_1 : \phi_{wu} \neq 0$$

   Explain how you would conclude. (Hint: You can take for granted that none of the standard mis-specification tests are significant).

5. Consider another test situation:

$$H_0 : \phi_{uw} = 0 \text{ against } H_1 : \phi_{uw} \neq 0$$

   How would you conclude?

6. In the data generation, $\mathbf{\Sigma}$ was specified as:

$$\mathbf{\Sigma} = \begin{pmatrix} 1 & 0.5 & -0.3 \\ 0.5 & 1 & 0 \\ -0.3 & 0 & 1 \end{pmatrix}.$$

   Show that

$$Cov(\varepsilon_{wpt}, \varepsilon_{ut}) = -0.3.$$

7. Table 4 shows estimation results for a model of the VAR.

   (a) Explain why the reported log-likelihood is the same for this empirical model as for the VAR in Table 3

   (b) Show how the estimated coefficient of u_t in the first equation in Table 4 can be obtained by use of the information in Table 3.

Tables

Unit-root tests

The sample is: 1994(1) - 2019(3) (106 observations and 3 variables)

RBO: ADF tests (T=103, Constant+Trend; 5%=-3.45 1%=-4.05)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob
2	-3.301	0.83113	0.3681	-1.549	0.1245
1	-4.140**	0.80183	0.3707	3.768	0.0003
0	-3.106	0.84678	0.3944		

RLBOLIGH: ADF tests (T=103, Constant+Trend; 5%=-3.45 1%=-4.05)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob
2	-3.288	0.85894	0.4380	-0.2293	0.8191
1	-3.555*	0.85586	0.4359	4.906	0.0000
0	-2.296	0.89920	0.4836		

RSH: ADF tests (T=103, Constant+Trend; 5%=-3.45 1%=-4.05)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob
2	-3.234	0.88293	0.4640	-0.5394	0.5908
1	-3.662*	0.87615	0.4623	6.400	0.0000
0	-2.043	0.91994	0.5470		

Table 1: Dickey Fuller tests of unit-root in RBO, $RLBOLIGH$ and RSH (levels). Quarterly data.

EQ(1) Modelling RLBOLIGH by OLS

The estimation sample is: 1994(1) - 2019(2)

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
RLBOLIGH_1	0.636233	0.02956	21.5	0.0000	0.8328
RBO	-0.0393621	0.02885	-1.36	0.1758	0.0196
RBO_1	0.128756	0.02912	4.42	0.0000	0.1737
RSH	0.559116	0.02648	21.1	0.0000	0.8274
RSH_1	-0.290578	0.03815	-7.62	0.0000	0.3841
BASELIII	0.385553	0.04873	7.91	0.0000	0.4023
CRISIS09Q1	-0.752125	0.1179	-6.38	0.0000	0.3045
Constant	0.380293	0.05127	7.42	0.0000	0.3717

sigma	0.10117	RSS	0.951892533
R^2	0.997316	F(7,93) =	4320 [0.000]**
Adj.R^2	0.997085	log-likelihood	93.6563
no. of observations	102	no. of parameters	8
mean(RLBOLIGH)	5.03518	se(RLBOLIGH)	1.87393

AR 1-5 test:	F(5,88)	=	1.0916 [0.3708]
ARCH 1-4 test:	F(4,94)	=	0.24759 [0.9105]
Normality test:	Chi^2(2)	=	0.48073 [0.7863]
Hetero test:	F(14,86)	=	0.86052 [0.6030]
Hetero-X test:	F(31,69)	=	1.8576 [0.0170]*

Table 2: Estimation results for an ADL model of $RLBOLIGH$.

Estimating the VAR by OLS
he estimation sample is: 2 - 101

VAR equation for: (w-p)

	Coefficient	Std.Error	t-value	t-prob
(w-p)_1	0.389369	0.09327	4.17	0.0001
u_1	0.0115088	0.04903	0.235	0.8149

sigma = 0.914543 RSS = 81.96606412

VAR equation for: u

	Coefficient	Std.Error	t-value	t-prob
(w-p)_1	0.190940	0.1079	1.77	0.0800
u_1	0.827843	0.05674	14.6	0.0000

sigma = 1.05846

log-likelihood -273.787634

correlation of VAR residuals (standard deviations on diagonal)

	(w-p)	u
(w-p)	0.91454	-0.30038
u	-0.30038	1.0585

Table 3: Estimation results for unrestricted VAR of $(w-p)_t$ and u_t.

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     Estimating the model by 1SLS
     The estimation sample is: 2 - 101

Equation for: (w-p)
                 Coefficient  Std.Error  t-value  t-prob
(w-p)_1             0.438925    0.09083     4.83  0.0000
u                  -0.259536    0.08368    -3.10  0.0025
u_1                 0.226364    0.08371     2.70  0.0081

sigma = 0.872309

Equation for: u
                 Coefficient  Std.Error  t-value  t-prob
(w-p)_1             0.190940     0.1079     1.77  0.0800
u_1                 0.827843    0.05674     14.6  0.0000

sigma = 1.05846

log-likelihood    -273.787634
no. of observations       100  no. of parameters           5

correlation of model residuals (standard deviations on diagonal)
             (w-p)            u
(w-p)       0.87231      0.00000
u           0.00000       1.0585
```

Table 4: Estimation results for a model of $(w-p)_t$ and u_t.