This is a 5 hour home exam.

Guidelines:
The problem set covers 4 pages (incl. tables and figures).
In the grading, question A gets 40 %, B 40 % and C 20 %.

Question A (40 %)

1. Consider the stochastic difference equation:
$$Y_t = 1 + 0.5Y_{t-1} + \epsilon_{yt}, \;\; t = 1, 2, ..., T \tag{1}$$
where ϵ_{yt} is white noise. What is the characteristic root associated with this equation?
2. Explain why the time series generated by (1) is stationary.
3. What is the expectation of Y_t?
4. Calculate the variance of Y_t under the assumption that $Var(\epsilon_{yt}) = 1$.
5. Let ζ_j denote the autocorrelation function (ACF) of Y_t. What are the values of ζ_1, ζ_2 and ζ_{10}?
6. Consider another equation:
$$X_t = 1 - 0.5X_{t-1} + \epsilon_{xt}, \;\; t = 1, 2, ..., T \tag{2}$$
where ϵ_{xt} is white noise. Sketch a graph of the ACF of X_t given by (2) together with a graph of the ACF of Y_t given by (1). How will you characterize Y_t and X_t as negatively or positively autocorrelated?
7. Imagine that the model in question A1 is used to forecast Y_{T+1}, Y_{T+2},...,Y_{T+H}. For simplicity we abstract from parameter estimation uncertainty (i.e., we assume that the model is correctly specified within-sample). Under the assumption of a quadratic cost function and assuming $Y_T = 4$, what are the optimal point forecasts of Y_{T+1}, Y_{T+2} and Y_{T+10}?
8. Calculate 95% forecast confidence intervals for the forecasts of Y_{T+1} and Y_{T+2}, when it is assumed that $\varepsilon_{T+h} \sim IIN(0, 1)$, $h = 1, 2$.
9. Imagine that when we get to evaluate the two first forecasts, we observe that Y_{T+1} and Y_{T+2} (the actuals) are well outside the forecast confidence intervals. How would you suggest that the forecasts of Y_{T+3},...Y_{T+10} can be revised in the light of the outcomes (actuals) for period $T + 1$ and $T + 2$?

Question B (40 %)

1. Assume that a time series Y_t is generated by:
$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \epsilon_{yt}, \;\; t = 1, 2, ..., T \tag{3}$$
where $\epsilon_{yt} \sim IIN(0, \sigma_y^2)$ for all t. (3) can be re-written as:
$$\Delta Y_t = \phi_0 - \pi Y_{t-1} - \phi_2 \Delta Y_{t-1} + \epsilon_{yt}, \;\; t = 1, 2, ..., T \tag{4}$$

where

$$\pi \quad = \quad 1 - \phi_1 - \phi_2. \tag{5}$$

Imagine that you estimate (4) by OLS and that the t-value of $-\hat{\pi}$ is -2.1. Can you reject the null hypothesis of $Y_t \sim I(1)$ at the 5 % level?

You can take as given that the number of observations is large enough to validate the use of asymptotic critical values.

2. Table 1 at the back of the exam set shows unit-root tests for $TEMP$, a quarterly time series of western-hemisphere temperatures measured as deviations from 1950-1980 means. The table also contains results for $DTEMP$ (the difference of $TEMP$). Explain how you reach a conclusion about $TEMP$ being I(0) or I(1) based on the tests.

3. We want to investigate empirically whether there is a relationship between $TEMP$ and another time series variable, $CO2$, which measures CO_2 in the atmosphere. You can take as given that $CO2$ is an I(1) variable.

   Imagine that a friend, who studies at a business school, suggests that you can regress $TEMP$ on $CO2$ and a constant term, and compare the t-value of the regression coefficient to a 5 % critical value from the t-distribution.

   How would you explain to him that his method would put you in acute danger of falling into the spurious regression trap?

4. Explain why a correct test of the null hypothesis of no relationship between $TEMP$ and $CO2$ can be based on the model estimated in Table 2. What is the result of the test?

5. Table 3 shows estimation results after the lags of $TEMP$ and $CO2$ have been replaced by the variable TEMP-1.4*CO2_1. Show how the coefficient 1.4 has been calculated, and explain how it should be interpreted.

6. Figure 1 shows a selection of graphs that can be used to evaluate the degree of parameter constancy of the model of $DTEMP$ in Table 3.

   It can be shown that there are significant structural breaks in the marginal equation of $DCO2$, namely in 1991(3), 2010(1), 2015(4), 2016(2), 2016(3), 2016(4) and 2018(4). When the set of indicator variables (i.e., dummies) for these seven quarters are added to the conditional equation of $DTEMP$, the residual sum of squares becomes: $RSS = 30697.8882$ (the log-likelihood becomes $log - likelihood = -717.918$).

   What can be concluded about the relative invariance of the model estimated in Table 3?
   HINT: In the $F(7, 161)$ distribution, the 5 % critical value is 2.06. In the $\chi^2(7)$ it is 14.

Question C (20 %)

1. Explain what is meant by simultaneity bias of OLS when used to estimate the coefficients of an identified structural equation in a SEM.

2. Explain why estimation by IV or 2SLS gives consistent estimation of the coefficients of the equation.

Table 1: Dickey-Fuller tests of unit-root in $TEMP$ and $DTEMP$.

The sample is: 1959(4) - 2019(4) (247 observations and 1 variables)

TEMP: ADF tests (T=241, Constant; 5%=-2.87 1%=-3.46)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob
4	-1.012	0.97326	0.1602	0.8884	0.3753
3	-0.8975	0.97653	0.1601	-2.788	0.0057
2	-1.340	0.96492	0.1624	-5.428	0.0000
1	-2.311	0.93721	0.1718	-2.830	0.0051
0	-2.981*	0.91992	0.1743		

Unit-root tests

The sample is: 1959(4) - 2019(4) (246 observations and 1 variables)

DTEMP: ADF tests (T=241, Constant; 5%=-2.87 1%=-3.46)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob
3	-9.806**	-0.85801	16.02	-0.7548	0.4511
2	-13.94**	-0.95408	16.00	2.970	0.0033
1	-17.35**	-0.64283	16.26	5.791	0.0000
0	-19.20**	-0.21570	17.33		

Table 2: Estimation results for a model of $DTEMP$ conditional on $DCO2$ (the first difference of $CO2$).

Modelling DTEMP by OLS

The estimation sample is: 1975(1) - 2019(4)

	Coefficient	Std.Error	t-value	t-prob
DTEMP_1	0.0474593	0.09568	0.496	0.6205
DTEMP_2	0.0250048	0.08436	0.296	0.7673
DTEMP_3	0.0734019	0.07503	0.978	0.3293
Constant	-254.928	53.03	-4.81	0.0000
DCO2	9.14585	3.421	2.67	0.0083
DCO2_1	5.42229	3.536	1.53	0.1271
DCO2_2	-2.47831	3.571	-0.694	0.4886
DCO2_3	8.04115	3.448	2.33	0.0209
TEMP_1	-0.561265	0.09522	-5.89	0.0000
CO2_1	0.784512	0.1438	5.45	0.0000
Seasonal	-46.6643	26.17	-1.78	0.0764
Seasonal_1	-21.3409	36.24	-0.589	0.5567
Seasonal_2	18.1724	25.68	0.708	0.4802

sigma	13.8428	RSS	32000.9952
R^2	0.500221	F(12,167) =	13.93 [0.000]**
Adj.R^2	0.464308	log-likelihood	-721.66
no. of observations	180	no. of parameters	13

AR 1-5 test:	F(5,162)	=	0.25670 [0.9359]
ARCH 1-4 test:	F(4,172)	=	2.4967 [0.0446]*
Normality test:	Chi^2(2)	=	4.2469 [0.1196]
Hetero test:	F(21,158)	=	1.8113 [0.0215]*
Hetero-X test:	F(57,122)	=	1.2197 [0.1813]

Table 3: Estimation results for a model of $DTEMP$ conditional on $DCO2$, with $TEMP\_1$ and $CO2\_1$ replaced by $TEMP - 1.4 * CO2\_1$.

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 Modelling DTEMP by OLS
      The estimation sample is: 1975(1) - 2019(4)

                  Coefficient  Std.Error  t-value  t-prob
DTEMP_1              0.0475378    0.09534    0.499  0.6187
DTEMP_2              0.0251967    0.08369    0.301  0.7637
DTEMP_3              0.0736135    0.07424    0.992  0.3228
Constant              -255.339      49.73    -5.13  0.0000
DCO2                   9.12105      3.235     2.82  0.0054
DCO2_1                 5.39361      3.297     1.64  0.1037
DCO2_2                -2.50755      3.325   -0.754  0.4518
DCO2_3                 8.01630      3.263     2.46  0.0150
TEMP-1.4*CO2_1       -0.561248    0.09494    -5.91  0.0000
Seasonal              -46.6725      26.09    -1.79  0.0754
Seasonal_1            -21.3284      36.12   -0.590  0.5557
Seasonal_2             18.1921      25.59    0.711  0.4782

sigma                  13.8015  RSS               32001.0957
R^2                   0.500219  F(11,168) =   15.29 [0.000]**
Adj.R^2               0.467495  log-likelihood       -721.66
no. of observations        180  no. of parameters         12

AR 1-5 test:      F(5,163)   =  0.25726 [0.9356]
ARCH 1-4 test:    F(4,172)   =   2.4898 [0.0451]*
Normality test:   Chi^2(2)   =   4.2626 [0.1187]
Hetero test:      F(19,160)  =   2.0083 [0.0106]*
Hetero-X test:    F(47,132)  =   1.3249 [0.1093]
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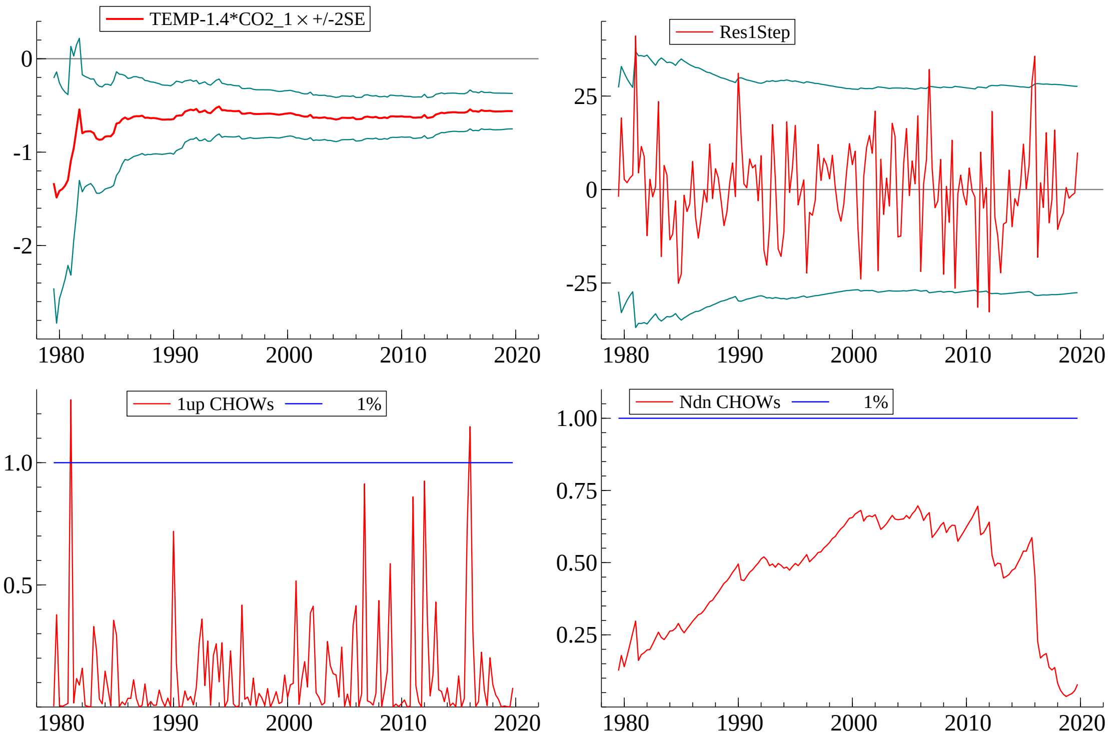

Figure 1: Some recursive graphs of the model estimated in Table 3.