Exam in: ECON 4160: Econometrics: Modelling and Systems Estimation—POSTPONED

Day of exam: 20 January 2021

Time of day: 09:00—14:00

This is a 5 hour home exam.

Guidelines:
In the grading, question A, B and C get equal weights (1/3).

Question A (1/3)

In this question we use time series data for real consumption (C), real disposable income (RDI) and a real interest rate (R) for the USA economy. The data set is quarterly and is the same that Campbell and Mankiw used in their journal article from 1990.

We will use the first differences of the natural logarithms of C and RDI:

$$DLC_t = LC_t - LC_{t-1} = log(C_t) - log(C_{t-1})$$
$$DLRDI_t = LRDI_t - LRDI_{t-1} = log(RDI_t) - log(RDI_{t-1}).$$

1. Table 1 at the back of the exam set shows unit-root tests for DLC and $DLRDI$. Explain how you test the hypothesis of DLC being I(1) against DLC being I(0) and give your conclusion. Test also $DLRDI$ being I(1) against $DLRDI$ being $I(0)$.

   To save space we do not test R_t for a unit-root. In the following you can take as given that $R_t \sim I(0)$.

2. Permanent income rational expectations theory implies that log consumption is a random-walk process with a drift term that may depend on the interest rate. With the variables introduced above, this consumption Euler equation can be expressed as:

   $$DLC_t = \alpha + \beta_1 R_t + \epsilon_t \tag{1}$$

   It is custom to estimate Euler equations by the instrumental variables method (IV). When (1) is estimated on the sample 1957(1) - 1985(4) the results are (standard errors of the estimated coefficients are in parentheses):

   $$DLC_t = \underset{(0.000164)}{-\ 0.0005295}\ R_t + \underset{(0.00109)}{0.007893} \tag{2}$$

   Estimation by IV $\quad$ 1957(1) − 1985(4)
   Instruments: $\quad DLC_{t-1}, DLRDI_{t-1}, R_{t-1}$
   Sargan specification test : $\quad \chi^2(2) = 4.3949[0.1111]$

   You can take as given that the model in (2) is not misspecified.

   (a) OLS estimation of the consumption Euler equation (1) could be subject to the simultaneity bias critique. Explain what is meant by simultaneity bias and how IV estimation may be used to solve this problem.

   (b) Is R_t significant in (2)? Explain your answer.

   (c) R_t is measured in percent. Assume that R_t is increased by one unit (e.g., from 1 to 2) for one period. What is the estimated responses of DLC and LC in the period that the increase occurs and in the period after the increase?

   (d) How do you interpret the "Sargan specification test" reported with (2)?

3. Table 2 contains estimation results for a VAR of DLC, $DLRDI$ and R. Table 3 contains estimation results for a model of the VAR where (1) is the first equation and the two other equations are the second and third equations of the VAR.

   (a) In Table 3 estimation is by 2SLS. Why are the 2SLS estimation results for the consumption Euler equation the same as the IV results in (2)?

   (b) What is the interpretation of the "LR test of over-identifying restrictions" shown at the bottom of Table 3?

4. A modification of the Euler equation is to allow for so called rule-of-thumb consumers. In our context this is achieved by including $DLRDI_t$ in the structural equation for DLC_t. The results are:

$$DLC_t \;=\; -\underset{(0.000164)}{0.0003589}\; R_t \;+\; \underset{(0.135)}{0.2544}\; DLRDI_t \;+\; \underset{(0.00152)}{0.005605} \tag{3}$$

   and the log-likelihood of the model with (2) replaced by (3) becomes 701.375567. What is the "LR test of over-identifying restrictions" for this model of the VAR?

Question B (1/3)

Assume that the three time series variables: w_t: nominal wage level, z_t: labour productivity level , u_t: unemployment rate are measured in natural logarithms and that they are generated by the VAR:

$$\begin{pmatrix} w_t \\ u_t \\ z_t \end{pmatrix} = \begin{pmatrix} \varphi_{w0} \\ \varphi_{u0} \\ \varphi_{z0} \end{pmatrix} + \begin{pmatrix} \varphi_{ww} & \varphi_{wu} & \varphi_{wz} \\ \varphi_{uw} & \varphi_{uu} & \varphi_{uz} \\ \varphi_{zw} & \varphi_{zu} & \varphi_{zz} \end{pmatrix} \begin{pmatrix} w_{t-1} \\ u_{t-1} \\ z_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{wt} \\ \varepsilon_{ut} \\ \varepsilon_{zt} \end{pmatrix} \tag{4}$$

where the vector with VAR error-terms $(\varepsilon_{wt} \;\; \varepsilon_{ut} \;\; \varepsilon_{zt})'$ is Gaussian white-noise with expectation zero and covariance matrix $\mathbf{\Sigma}$. We do *not* assume that $\mathbf{\Sigma}$ is a diagonal matrix.

1. Assume that the VAR is a stationary system, and that the stationary solution can be obtained from given initial conditions $(w_0, u_0, z_0)'$ and the history of the error-terms. What does this imply for the eigenvalues of the matrix with autoregressive coefficients?

2. Consider the following model equation for w_t (in ADL form):

$$w_t = \beta_0 + \beta_{10} u_t + \beta_{11} u_{t-1} + \beta_{20} z_t + \beta_{21} z_{t-1} + \phi_1 w_{t-1} + \epsilon_t \tag{5}$$

   (a) Explain in your own words why $Cov(u_t, \epsilon_t) = Cov(z_t, \epsilon_t) = 0$ in (5) even though u_t and z_t are endogenous variables in the VAR system specified above.

   (b) Assume that you have 101 observations of the triplet $(w_t, u_t, z_t)'$ and that you estimate the coefficients in (5) by OLS. Explain why the OLS estimators are biased and consistent (for example $E(\hat{\phi}_1) \neq \phi_1$, $\text{plim}(\hat{\phi}_1) = \phi_1$).

   (c) Imagine that we are interested in estimating the dynamic multipliers of w_t with respect to a change in z_t.

      i. Explain why strong exogeneity of z_t is required for the dynamic multipliers to be estimated consistently from (5).

      ii. Expain how you would test the strong exogeneity of z_t.

Question C (1/3)

Consider a different VAR for $(w_t, u_t, z_t)'$:

$$\begin{pmatrix} w_t \\ u_t \\ z_t \end{pmatrix} = \begin{pmatrix} \varphi_{w0} \\ \varphi_{u0} \\ \varphi_{z0} \end{pmatrix} + \begin{pmatrix} \varphi_{ww} & \varphi_{wu} & \varphi_{wz} \\ 0 & \varphi_{uu} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} w_{t-1} \\ u_{t-1} \\ z_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{wt} \\ \varepsilon_{ut} \\ \varepsilon_{zt} \end{pmatrix} \tag{6}$$

where the assumptions about the error-terms and the matrix $\mathbf{\Sigma}$ are the same as in Question B. The eigenvalues of the autoregressive matrix in (6) can be shown to be 1 , φ_{uu} and φ_{ww}.

1. Explain why $z_t \sim I(1)$ in this VAR.
2. Show that the conditional expectation of $E(z_t \mid z_0)$ is a deterministic trend with slope coefficient φ_{z0}, and that the conditional variance of z_t is increasing in time.
3. Assume that $0 < \varphi_{uu} < 1$. What does this imply for the time series properties of u_t?
4. Assume also that $\varphi_{wz} > 0$. What is implied about the degree of integration of w_t?
5. Use the estimation results in Table 4 to test the null hypothesis of no cointegration between w_t and z_t. If you conclude that the H_0 of no long-run relationship can be rejected, what is the estimated cointegration relationship?

Tables with results

Table 1: Dickey-Fuller tests of unit-root in *DLC* and *DLRDI*.

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The sample is: 1957(1) - 1985(4) (120 observations)

DLC: ADF tests (T=116, Constant; 5%=-2.89 1%=-3.49)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob
 3      -3.756**    0.47626 0.005080    -0.2281  0.8200
 2      -4.129**    0.46485 0.005059     -2.660  0.0090
 1      -6.085**    0.29523 0.005193    -0.8071  0.4213
 0      -8.370**    0.23764 0.005185

DLRDI: ADF tests (T=116, Constant; 5%=-2.89 1%=-3.49)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob
 3      -4.655**    0.20690 0.009579     0.6615  0.5097
 2      -4.829**    0.25387 0.009555     -1.970  0.0513
 1      -6.984**   0.086738 0.009676    -0.4264  0.6706
 0      -10.18**   0.047917 0.009641
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Table 2: VAR results. Estimation by OLS. Sample 1957(1) - 1985(4).

URF equation for: DLC

	Coefficient	Std.Error	t-value	t-prob
DLC_1	0.149563	0.1112	1.35	0.1812
DLRDI_1	0.0208739	0.06014	0.347	0.7292
R_1	-0.000450032	0.0001575	-2.86	0.0051
Constant	0.00659221	0.001215	5.42	0.0000

URF equation for: DLRDI

	Coefficient	Std.Error	t-value	t-prob
DLC_1	0.679951	0.1991	3.41	0.0009
DLRDI_1	-0.194377	0.1077	-1.80	0.0739
R_1	-0.000482288	0.0002820	-1.71	0.0900
Constant	0.00562295	0.002177	2.58	0.0111

URF equation for: R

	Coefficient	Std.Error	t-value	t-prob
DLC_1	29.8290	20.34	1.47	0.1453
DLRDI_1	7.25586	11.00	0.659	0.5110
R_1	0.966469	0.02881	33.5	0.0000
Constant	0.0561826	0.2224	0.253	0.8010

log-likelihood 702.802606

Table 3: Results for a model of the VAR in Table 2. Estimation by 2SLS. Sample 1957(1) - 1985(4).

Equation for: DLC

	Coefficient	Std.Error	t-value	t-prob
Constant	0.00789320	0.001093	7.22	0.0000
R	-0.000529542	0.0001648	-3.21	0.0017

sigma = 0.00520023

Equation for: DLRDI

	Coefficient	Std.Error	t-value	t-prob
Constant	0.00562295	0.002167	2.59	0.0107
DLC_1	0.679951	0.1982	3.43	0.0008
DLRDI_1	-0.194377	0.1073	-1.81	0.0726
R_1	-0.000482288	0.0002808	-1.72	0.0886

sigma = 0.00900076

Equation for: R

	Coefficient	Std.Error	t-value	t-prob
Constant	0.0561826	0.2214	0.254	0.8002
DLC_1	29.8290	20.25	1.47	0.1435
DLRDI_1	7.25586	10.96	0.662	0.5091
R_1	0.966469	0.02868	33.7	0.0000

sigma = 0.919445

log-likelihood 699.540718
LR test of over-identifying restrictions: Chi^2(2) = 6.5238 [0.0383]*

Table 4: Estimation results for a model of Dw_t conditional on Du_t and Dz_t.

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     Modelling Dw by OLS
                    Coefficient  Std.Error  t-value
Constant              0.163065    0.07467     2.18
Du                   -0.154668    0.07940    -1.95
Dz                   0.0707870    0.04984     1.42
w_1                  -0.277020    0.03310    -8.37
u_1                  -0.143024    0.04428    -3.23
z_1                   0.269691    0.03011     8.96

sigma                 0.493687  RSS        22.910312
R^2                   0.480392  F(5,94) =  17.38 [0.000]**
no. of observations        100  no. of parameters: 6

AR 1-2 test:      F(2,92)    =   2.6987 [0.0726]
ARCH 1-1 test:    F(1,98)    = 0.032887 [0.8565]
Normality test:   Chi^2(2)   =   3.0120 [0.2218]
Hetero test:      F(10,89)   =  0.56565 [0.8377]
Hetero-X test:    F(20,79)   =  0.81442 [0.6893]
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