Exam in: ECON 4160: Econometrics: Modelling and Systems Estimation—POSTPONED—notes to evaluators.

Day of exam: 20 January 2021

Time of day: 09:00—14:00

This is a 5 hour home exam.

Guidelines:
In the grading, question A, B and C get equal weights (1/3).

Question A (1/3)

In this question we use time series data for real consumption (C), real disposable income (RDI) and a real interest rate (R) for the USA economy. The data set is quarterly and is the same that Campbell and Mankiw used in their journal article from 1990.

We will use the first differences of the natural logarithms of C and RDI:

$$DLC_t = LC_t - LC_{t-1} = log(C_t) - log(C_{t-1})$$
$$DLRDI_t = LRDI_t - LRDI_{t-1} = log(RDI_t) - log(RDI_{t-1}).$$

1. Table 1 at the back of the exam set shows unit-root tests for DLC and $DLRDI$. Explain how you test the hypothesis of DLC being I(1) against DLC being I(0) and give your conclusion. Test also $DLRDI$ being I(1) against $DLRDI$ being $I(0)$.

   To save space we do not test R_t for a unit-root. In the following you can take as given that $R_t \sim I(0)$.
   A: In the column labelled ADF-test, all values are significant at the 1 % level of significance. This implies that the rejection of the H_0 a unit root is robust to the degree of augmentation of the Dickey-Fuller regression, see equation (9.22) in DEEMM.

2. Permanent income rational expectations theory implies that log consumption is a random-walk process with a drift term that may depend on the interest rate. With the variables introduced above, this consumption Euler equation can be expressed as:

   $$DLC_t = \alpha + \beta_1 R_t + \epsilon_t \tag{1}$$

   It is custom to estimate Euler equations by the instrumental variables method (IV). When (1) is estimated on the sample 1957(1) - 1985(4) the results are (standard errors of the estimated coefficients are in parentheses):

   $$DLC_t = \underset{(0.000164)}{-0.0005295}\, R_t + \underset{(0.00109)}{0.007893} \tag{2}$$

Estimation by IV	$1957(1) - 1985(4)$
Instruments:	$DLC_{t-1}, DLRDI_{t-1}, R_{t-1}$
Sargan specification test :	$\chi^2(2) = 4.3949[0.1111]$

   You can take as given that the model in (2) is not misspecified.

   (a) OLS estimation of the consumption Euler equation (1) could be subject to the simultaneity bias critique. Explain what is meant by simultaneity bias and how IV estimation may be used to solve this problem.
   A: If R_t is correlated with ϵ_t in (1), the OLS estimator of β_1 (in particular) is inconsistent, see Ch. 7.9.1 in DEEMM. The IV estimator is a consistent

estimator if a valid instrumental variable exists, see Ch. 2.5 in DEEMM. If Z_t denotes the instrumental variable, the IV estimator of β_1 in (1) is by definition:

$$\hat{\beta}_1^{IV} = \frac{\sum_{t=1}^{T} DLC_t(Z_t - \bar{Z})}{\sum_{t=1}^{T}(R_t - \bar{R})(Z_t - \bar{Z})}$$

Insertion from (1) gives:

$$\hat{\beta}_1^{IV} = \beta_1 + \frac{T^{-1}\sum_{t=1}^{T} \epsilon_t(Z_t - \bar{Z})}{T^{-1}\sum_{t=1}^{T}(R_t - \bar{R})(Z_t - \bar{Z})}$$

If Z_t is a valid and relevant instrument, the bias term is zero asymptotically (probalibty limit (plim) is zero). In this case there are three instruments, and the IV estimator used is the generalized IV estimator, see Ch. 2.6.8 and 7.9.3 in DEEMM, i.e., we can think of Z_t as the optimal linear combination of the three instruments.

(b) Is R_t significant in (2)? Explain your answer.
A: We can compare the t-value -3.22 with the critical values of the $N(0,1)$ and reject the H_0 of $\beta_1 = 0$ at very low significance level. Since the model equation in not-misspecified, it is allowed to base the test on the result that the t-value has an asymptotic distribution $N(0,1)$.

(c) R_t is measured in percent. Assume that R_t is increased by one unit (e.g., from 1 to 2) for one period. What is the estimated responses of DLC and LC in the period that the increase occurs and in the period after the increase?
A: DLC: Response is given by $\hat{\beta}_1$ in the period of the increase ("shock"). After that: zero response. LC is changed by $\hat{\beta}_1$ in the period of the shock. In the period after the shock the response is also $\hat{\beta}_1$. This is because the model equation can be written as

$$LC_t = \alpha + \beta_1 R_t + LC_{t-1} + \epsilon_t$$

in other words: a random walk where the drift parameter is affected by R_t.

(d) How do you interpret the "Sargan specification test" reported with (2)?
A: See Ch 7.9.5 in DEEMM.

3. Table 2 contains estimation results for a VAR of DLC, $DLRDI$ and R. Table 3 contains estimation results for a model of the VAR where (1) is the first equation and the two other equations are the second and third equations of the VAR.

(a) In Table 3 estimation is by 2SLS. Why are the 2SLS estimation results for the consumption Euler equation the same as the IV results in (2)?
A: See Ch 7.9.2 in DEEMM, implying that 2SLS is equivalent to the GIV estimator that was used in (2).

(b) What is the interpretation of the "LR test of over-identifying restrictions" shown at the bottom of Table 3?
A: It is the likelihood-ratio test of over-identifying restrictions that the model in Table 3 implies on the unrestricted VAR.

$$-2 \cdot (699.540718 - 702.802606) = 6.5238$$

Since the degree of over-identification is 2, the value of the test statistic is compared to critical values of the Chi-square distribution with two degrees of freedom. The test is significant at the 5 % significance level, which indicates that the model does not encompass the VAR. There is statistical information in the VAR that the model does not use in an efficient way.

4. A modification of the Euler equation is to allow for so called rule-of-thumb consumers. In our context this is achieved by including $DLRDI_t$ in the structural equation for DLC_t. The results are:

$$DLC_t = \underset{(0.000164)}{-\,0.0003589}\ R_t + \underset{(0.135)}{0.2544}\ DLRDI_t + \underset{(0.00152)}{0.005605} \qquad (3)$$

and the log-likelihood of the model with (2) replaced by (3) becomes 701.375567. What is the "LR test of over-identifying restrictions" for this model of the VAR?
A:

$$-2 \cdot (701.375567 - 702.802606) = 2.8541$$

which is insignificant.

Question B (1/3)

Assume that the three time series variables: w_t: nominal wage level , z_t: labour productivity level , u_t: unemployment rate are measured in natural logarithms and that they are generated by the VAR:

$$\begin{pmatrix} w_t \\ u_t \\ z_t \end{pmatrix} = \begin{pmatrix} \varphi_{w0} \\ \varphi_{u0} \\ \varphi_{z0} \end{pmatrix} + \begin{pmatrix} \varphi_{ww} & \varphi_{wu} & \varphi_{wz} \\ \varphi_{uw} & \varphi_{uu} & \varphi_{uz} \\ \varphi_{zw} & \varphi_{zu} & \varphi_{zz}) \end{pmatrix} \begin{pmatrix} w_{t-1} \\ u_{t-1} \\ z_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{wt} \\ \varepsilon_{ut} \\ \varepsilon_{zt} \end{pmatrix} \qquad (4)$$

where the vector with VAR error-terms $(\varepsilon_{wt} \ \ \varepsilon_{ut} \ \ \varepsilon_{zt})'$ is Gaussian white-noise with expectation zero and covariance matrix $\mathbf{\Sigma}$. We do *not* assume that $\mathbf{\Sigma}$ is a diagonal matrix.

1. Assume that the VAR is a stationary system, and that the stationary solution can be obtained from given initial conditions $(w_0, u_0, z_0)'$ and the history of the error-terms. What does this imply for the eigenvalues of the matrix with autoregressive coefficients?
   A: All three eigenvalues are less than one in magnitude.

2. Consider the following model equation for w_t (in ADL form):

$$w_t = \beta_0 + \beta_{10}u_t + \beta_{11}u_{t-1} + \beta_{20}z_t + \beta_{21}z_{t-1} + \phi_1 w_{t-1} + \epsilon_t \qquad (5)$$

   (a) Explain in your own words why $Cov(u_t, \epsilon_t) = Cov(z_t, \epsilon_t) = 0$ in (5) even though u_t and z_t are endogenous variables in the VAR system specified above.
   A: They are logical implications of valid conditioning.

   (b) Assume that you have 101 observations of the triplet $(w_t, u_t, z_t)'$ and that you estimate the coefficients in (5) by OLS. Explain why the OLS estimators are biased and consistent (for example $E(\hat{\phi}_1) \neq \phi_1$, $\text{plim}(\hat{\phi}_1) = \phi_1$).
   A: The keywords here are pre-determined variables as opposed to strictly exogenous variables, and therefore consistency of OLS despite finite sample bias (Hurwicz-bias) of the OLS estimator.

   (c) Imagine that we are interested in estimating the dynamic multipliers of w_t with respect to a change in z_t.

      i. Explain why strong exogeneity of z_t is required for the dynamic multipliers to be estimated consistently from (5).
      A: If there is joint Granger causality, the impulse responses of the conditional model become incorrect, exactly because they ignore mutual dynamic dependencies.

      ii. Expain how you would test the strong exogeneity of z_t.
      A: Test the relevant zero-restrictions in the thord (and potentially second) row of the VAR (by t and/or F tests). Specifically, $\varphi_{zw} = 0$

Question C (1/3)

Consider a different VAR for $(w_t, u_t, z_t)'$:

$$\begin{pmatrix} w_t \\ u_t \\ z_t \end{pmatrix} = \begin{pmatrix} \varphi_{w0} \\ \varphi_{u0} \\ \varphi_{z0} \end{pmatrix} + \begin{pmatrix} \varphi_{ww} & \varphi_{wu} & \varphi_{wz} \\ 0 & \varphi_{uu} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} w_{t-1} \\ u_{t-1} \\ z_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{wt} \\ \varepsilon_{ut} \\ \varepsilon_{zt} \end{pmatrix} \quad (6)$$

where the assumptions about the error-terms and the matrix $\boldsymbol{\Sigma}$ are the same as in Question B. The eigenvalues of the autoregressive matrix in (6) can be shown to be 1 , φ_{uu} and φ_{ww}.

1. Explain why $z_t \sim I(1)$ in this VAR.
   A: z_t is a first order difference equation with autoregressive coefficient $= 1$

   $$z_t = \varphi_{z0} + z_{t-1} + \varepsilon_{zt}$$

   also known as random walk with drift. The first difference of z_t is:

   $$\Delta z_t = \varphi_{z0} + \varepsilon_{zt}$$

   which is I(0) by assumption, hence $z_t \sim I(1)$.

2. Show that the conditional expectation of $E(z_t \mid z_0)$ is a deterministic trend with slope coefficient φ_{z0}, and that the conditional variance of z_t is increasing in time.
   A: Solution after substitution $j - 1$ periods backwards in time:

   $$z_t = j\varphi_{z0} + z_{t-j} + \sum_{i=0}^{j-1} \varepsilon_{zt-i}$$

   Define $t - j = 0$ and hence $t = j$

   $$z_t = t\varphi_{z0} + z_0 + \sum_{i=0}^{t-1} \varepsilon_{zt-i}$$

   $$\begin{aligned} E(z_t) &= t\varphi_{z0} + z_0 \\ Var(z_t) &= t\sigma_z^2 \end{aligned}$$

3. Assume that $0 < \varphi_{uu} < 1$. What does this imply for the time series properties of u_t?
   A: With reference to Ch. 4, u_t is stationary, and I(0), with expectation, variance and autocovariances that do not depend on t. u_t is positively autocorrelated, the ACF decays monotonously toward zero as a function of the lag lenngth.

4. Assume also that $\varphi_{wz} > 0$. What is implied about the degree of integration of w_t?
   A: The I(1)-ness of z_t will be "passed on", hence $w_t \sim I(1)$

5. Use the estimation results in Table 4 to test the null hypothesis of no cointegration between w_t and z_t. If you conclude that the H_0 of no long-run relationship can be rejected, what is the estimated cointegration relationship?
   A: The ECM test of cointegrated is $ECM = -8.4$ and gives clear rejection of the H_0 of no cointegration when the relevant critical value from the Ericsson and MacKinnon article is used. Estimate long-run relationship:

   $$w = 0.59 - 0.52y + 0.97z$$

Tables with results

Table 1: Dickey-Fuller tests of unit-root in DLC and $DLRDI$.

The sample is: 1957(1) - 1985(4) (120 observations)

DLC: ADF tests (T=116, Constant; 5%=-2.89 1%=-3.49)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob
3	-3.756**	0.47626	0.005080	-0.2281	0.8200
2	-4.129**	0.46485	0.005059	-2.660	0.0090
1	-6.085**	0.29523	0.005193	-0.8071	0.4213
0	-8.370**	0.23764	0.005185		

DLRDI: ADF tests (T=116, Constant; 5%=-2.89 1%=-3.49)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob
3	-4.655**	0.20690	0.009579	0.6615	0.5097
2	-4.829**	0.25387	0.009555	-1.970	0.0513
1	-6.984**	0.086738	0.009676	-0.4264	0.6706
0	-10.18**	0.047917	0.009641		

Table 2: VAR results. Estimation by OLS. Sample 1957(1) - 1985(4).

URF equation for: DLC

	Coefficient	Std.Error	t-value	t-prob
DLC_1	0.149563	0.1112	1.35	0.1812
DLRDI_1	0.0208739	0.06014	0.347	0.7292
R_1	-0.000450032	0.0001575	-2.86	0.0051
Constant	0.00659221	0.001215	5.42	0.0000

URF equation for: DLRDI

	Coefficient	Std.Error	t-value	t-prob
DLC_1	0.679951	0.1991	3.41	0.0009
DLRDI_1	-0.194377	0.1077	-1.80	0.0739
R_1	-0.000482288	0.0002820	-1.71	0.0900
Constant	0.00562295	0.002177	2.58	0.0111

URF equation for: R

	Coefficient	Std.Error	t-value	t-prob
DLC_1	29.8290	20.34	1.47	0.1453
DLRDI_1	7.25586	11.00	0.659	0.5110
R_1	0.966469	0.02881	33.5	0.0000
Constant	0.0561826	0.2224	0.253	0.8010

log-likelihood 702.802606

Table 3: Results for a model of the VAR in Table 2. Estimation by 2SLS. Sample 1957(1) - 1985(4).

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Equation for: DLC
                Coefficient  Std.Error  t-value  t-prob
Constant          0.00789320   0.001093     7.22  0.0000
R               -0.000529542  0.0001648    -3.21  0.0017

sigma = 0.00520023

Equation for: DLRDI
                Coefficient  Std.Error  t-value  t-prob
Constant          0.00562295   0.002167     2.59  0.0107
DLC_1               0.679951     0.1982     3.43  0.0008
DLRDI_1            -0.194377     0.1073    -1.81  0.0726
R_1             -0.000482288  0.0002808    -1.72  0.0886

sigma = 0.00900076

Equation for: R
                Coefficient  Std.Error  t-value  t-prob
Constant           0.0561826     0.2214    0.254  0.8002
DLC_1                29.8290      20.25     1.47  0.1435
DLRDI_1              7.25586      10.96    0.662  0.5091
R_1                 0.966469    0.02868     33.7  0.0000

sigma = 0.919445

log-likelihood     699.540718
LR test of over-identifying restrictions: Chi^2(2) = 6.5238 [0.0383]*
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Table 4: Estimation results for a model of Dw_t conditional on Du_t and Dz_t.

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     Modelling Dw by OLS
                      Coefficient  Std.Error  t-value
Constant                0.163065    0.07467     2.18
Du                     -0.154668    0.07940    -1.95
Dz                     0.0707870    0.04984     1.42
w_1                    -0.277020    0.03310    -8.37
u_1                    -0.143024    0.04428    -3.23
z_1                     0.269691    0.03011     8.96

sigma                   0.493687  RSS        22.910312
R^2                     0.480392  F(5,94) =  17.38 [0.000]**
no. of observations          100  no. of parameters: 6

AR 1-2 test:      F(2,92)    =    2.6987 [0.0726]
ARCH 1-1 test:    F(1,98)    = 0.032887 [0.8565]
Normality test:   Chi^2(2)   =    3.0120 [0.2218]
Hetero test:      F(10,89)   =   0.56565 [0.8377]
Hetero-X test:    F(20,79)   =   0.81442 [0.6893]
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