Exam in: ECON 4160: Econometrics: Modelling and Systems Estimation

Day of exam: 3 December 2021

Time of day: 09:00—14:00

This is a 5 hour home exam.

Guidelines:
In the grading, question A gets 50 %, B 50 %.

Question A (50 %)

Consider the dynamic simultaneous equations model (SEM):

$$
\begin{aligned}
rw_t - a_2 u_t &= a_0 + a_1 rw_{t-1} + a_3 z_{wt} + e_{wt}, && (1)\\
-b_1 rw_t + u_t &= b_0 + b_2 rw_{t-1} + b_3 u_{t-1} + b_4 z_{ut} + e_{ut}. && (2)
\end{aligned}
$$

Definitions of the variables:
rw_t: Log of the real wage.
u_t: Log of the unemployment rate.
z_{wt}: Strictly exogenous variable that affects wage formation
z_{ut}: Strictly exogenous variable that affects unemployment.
e_{wt} and e_{ut} are two gaussian white-noise disturbances:

$$
\begin{pmatrix} e_{wt} \\ e_{ut} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \omega_1^2 & \omega_{12} \\ \omega_{12} & \omega_2^2 \end{pmatrix} \right) \text{ for all } t.
$$

1. Show that the matrix with autoregressive coefficients of the reduced form of the SEM (1)-(2) is:
$$
\mathbf{\Phi} = \begin{pmatrix} \frac{a_1 + a_2 b_2}{1 - a_2 b_1} & \frac{a_2 b_3}{1 - a_2 b_1} \\ \frac{b_1 a_1 + b_2}{1 - a_2 b_1} & \frac{b_3}{1 - a_2 b_1} \end{pmatrix}. \quad (3)
$$
For reference, the expression of the reduced form is:
$$
\begin{pmatrix} rw_t \\ u_t \end{pmatrix} = \mathbf{\Phi} \begin{pmatrix} rw_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \frac{1}{1-a_2 b_1} \left[(a_0 + a_3 z_{wt} + e_{wt}) + a_2 (b_0 + b_4 z_{ut} + e_{ut}) \right] \\ \frac{1}{1-a_2 b_1} \left[b_1 (a_0 + a_3 z_{wt} + e_{wt}) + (b_0 + b_4 z_{ut} + e_{ut}) \right] \end{pmatrix}. \quad (4)
$$
2. Assume that z_{ut} and z_{wt} are I(0) variables and that the eigenvalues of $\mathbf{\Phi}$ are both less than one in magnitude. Explain what this implies for the stationarity of rw_t and u_t.
3. Assume stationarity. Derive the expressions for the expectations of rw_t and u_t.
4. Assume that $E(u_t) = 1.39$, and hence that the equilibrium rate of unemployment, U^*, is $U^* = e^{1.39} \approx 4.0$ percent. If the unemployment rate today is 6 percent, what does the model predict about the future development of the rate of unemployment?
5. Denote the error-terms of the reduced form (aka VAR disturbances) by ε_{wt} and ε_{ut}. Show that $Cov(\varepsilon_{wt}, \varepsilon_{ut}) \neq 0$ in general.
6. Assume $a_3 \neq 0$ and $b_4 \neq 0$. Explain why (1) and (2) are identified.
7. Explain why the OLS estimators of the coefficients of the model are inconsistent.
8. Explain why there is no simultaneity bias if 2SLS is used to estimate the two equations.

9. The conditional equation (5) and the marginal equation (6) represent a model of the VAR defined by the reduced form (4).

$$rw_t = \phi_0 + \phi_1 rw_{t-1} + \beta_0 u_t + \beta_1 u_{t-1} + \beta_3 z_{wt} + \beta_4 z_{ut} + \epsilon_t, \tag{5}$$
$$u_t = \varphi_{20} + \varphi_{21} rw_{t-1} + \varphi_{22} u_{t-1} + \gamma_{23} z_{wt} + \gamma_{24} z_{ut} + \varepsilon_{ut}. \tag{6}$$

   (a) What is the expression of β_0 ?
   (b) What are the properties of the OLS estimator of β_0?
   (c) Explain why $Cov(\epsilon_t, \varepsilon_{ut}) = 0$.
   (d) Explain what would be a test of strong exogeneity of u_t in this model.

Question B (50 %)

We have annual data series of the nominal wage level in Mainland Norway manufacturing, $W1_t$, measured as wage compensation in kroner per hour worked. Another time series, $SCOPE1_t$, measures value added per hour worked in Mainland Norway manufacturing. The log of the two time series are denoted $LW1_t$, and $LSCOPE1_t$.

1. Use the results in Table 1 to test the null hypothesis of a unit-root in $LW1_t$. Test the same hypothesis for $LSCOPE1_t$.

   In the following you can take as granted that the first difference of $LW1_t$ and $LSCOPE_t$, denoted by $DLW1_t$, and $DLSCOPE1_t$, are $I(0)$.

2. Table 2 shows results for the regression of $DLW1_t$ on $DLW1_{t-1}$, $LW1_{t-1}$ $LSCOPE1_{t-1}$, $DLSCOPE_t$, $DLSCOPE_{t-1}$ and $Constant$. Explain how you can use the ECM-test to test the null hypothesis of no cointegration between $LW1_t$ and $LSCOPE1_t$. Use a significance level of 5 %.

3. Conditional on the rejection of no cointegration, show that the estimated long-run relationship between $LW1_t$ and $LSCOPE1_t$ is:

$$LW1_t = 0.926 LSCOPE1_t - 0.033. \tag{7}$$

4. It can be shown that the estimated standard error of 0.926 is 0.04. Use this information to construct a test of the hypothesis that the long-run elasticity of $LSCOPE_t$ is equal to 1.

5. Table 3 shows results for the conditional ECM of $DLW1_t$ when the long-run coefficient of +1 has been imposed in the form of the variable

$$LW1ec_t = LW1_t - LSCOPE1_t. \tag{8}$$

   The results for the marginal model equation for $DLSCOPE$ is shown in Table 3, as EQ(3). Explain why the results support weak exogeneity of $LSCOPE1$ with respect to the parameters of the cointegration relationship between $LW1_t$ and $LSCOPE1_1$.

6. Table 4 shows the result for a restricted version of the wage equation in Table 3. Show that the exclusion restrictions on $DLSCOPE1$ and $DLSCOPE1\_1$ cannot be rejected at the 5 % level, neither individually nor jointly.(Hint: The 5 % critical value of the $\chi^2(2)$ distribution is 6. In the $F(2, 42)$ distribution, the 5 % critical value is 3.2).

7. A critique of the model in Table 4 is that it omits two variables that are regarded to be important factors in nominal wage formation: namely the rate of unemployment and the increase in cost of living. In order to accommodate the critique, we include the log of the unemployment rate, LU_t, and the inflation rate, INF_t .
EQ(5) in Table 5 shows the OLS estimation results for the augmented wage model equation. EQ(6) shows the results of Instrumental variables estimation.

   (a) Do the results support the empirical relevance of the two "new" explanatory variables, LU_t and/or INF_t?

   (b) Discuss briefly the potential weak exogeneity of LU_t and/or INF_t In this context it can be noted that if we add the relevant fitted values of LU_t and INF_t to the model in EQ(5), we obtain $RSS = 0.008197$. You may use this information to calculate the Durbin-Wu-Hausman (DWH) test.

   (c) What is the interpretation of the *Specification test* reported with EQ(6) in Table 5?

Tables with estimation results and facimile of table with critical values for ECM-test

Table 1: Dickey-Fuller tests of unit-root in $LW1_t$, $LW2_t$ and $LSCOPE1_t$.

```
Unit-root tests
The sample is: 1973 - 2018 (49 observations)

LW1: ADF tests (T=46, Constant+Trend; 5%=-3.51 1%=-4.17)
D-lag    t-adf       beta Y_1    sigma   t-DY_lag  t-prob
 2     -3.064       0.94222  0.01886     -1.833  0.0741
 1     -3.337       0.94469  0.01769      3.530  0.0010
 0     -4.432**     0.92315  0.01991

LSCOPE1: ADF tests (T=46, Constant+Trend; 5%=-3.51 1%=-4.17)
D-lag    t-adf       beta Y_1    sigma   t-DY_lag  t-prob
 2     -2.017       0.93277  0.03659   -0.06286  0.9502
 1     -2.040       0.93287  0.03615    -0.8152  0.4195
 0     -1.967       0.93597  0.03601
```

Table 2: Regression of $DLW1_t$ on $DLW1_{t-1}$ and $LW1_{t-1}$, $LSCOPE1_{t-1}$, $DLSCOPE_t$, $DLSCOPE_{t-1}$ and $Constant$.

```
EQ(1) Modelling DLW1 by OLS
      The estimation sample is: 1972 - 2018

                 Coefficient  Std.Error  t-value  t-prob
DLW1_1              0.509657     0.1156     4.41  0.0001
LW1_1              -0.213457    0.06173    -3.46  0.0013
LSCOPE1_1           0.197702    0.06190     3.19  0.0027
DLSCOPE1           0.0402308    0.07362    0.546  0.5877
DLSCOPE1_1       -0.00817773    0.07797   -0.105  0.9170
Constant         -0.00702748    0.05193   -0.135  0.8930

sigma              0.0169509  RSS             0.0117806301
R^2                 0.817897  F(5,41) =     36.83 [0.000]**
Adj.R^2              0.79569  log-likelihood         128.159
no. of observations       47  no. of parameters            6
mean(DLW1)         0.0639172  se(DLW1)             0.0375014

AR 1-2 test:      F(2,39)   =  0.80804 [0.4531]
ARCH 1-1 test:    F(1,45)   =   3.5161 [0.0673]
Normality test:   Chi^2(2)  =   2.9995 [0.2232]
Hetero test:      F(10,36)  =   1.4698 [0.1910]
Hetero-X test:    F(20,26)  =   1.8495 [0.0704]
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Table 3: Conditional and marginal model equations for $DLW1_t$ and $DLSCOPE1_t$.

EQ(2) Modelling DLW1 by OLS
The estimation sample is: 1972 - 2018

	Coefficient	Std.Error	t-value	t-prob
DLW1_1	0.698737	0.09705	7.20	0.0000
LW1ec_1	-0.210106	0.06597	-3.18	0.0027
DLSCOPE1	0.117478	0.07222	1.63	0.1113
DLSCOPE1_1	0.0603874	0.07859	0.768	0.4466
Constant	-0.110441	0.03649	-3.03	0.0042

sigma	0.0181185	RSS	0.0137877334
R^2	0.786872	F(4,42) =	38.77 [0.000]**
Adj.R^2	0.766574	log-likelihood	124.462
no. of observations	47	no. of parameters	5

EQ(3) Modelling DLSCOPE1 by OLS
The estimation sample is: 1972 - 2018

	Coefficient	Std.Error	t-value	t-prob
DLSCOPE1_1	0.156195	0.1642	0.951	0.3469
LW1ec_1	0.226758	0.1349	1.68	0.1001
DLW1_1	0.639059	0.1803	3.55	0.0010
Constant	0.136131	0.07420	1.83	0.0735

sigma	0.0382581	RSS	0.0629383203
R^2	0.382407	F(3,43) =	8.875 [0.000]**
Adj.R^2	0.339319	log-likelihood	88.78
no. of observations	47	no. of parameters	4

Table 4: Restricted conditional model equation for $DLW1_t$.

EQ(4) Modelling DLW1 by OLS
The estimation sample is: 1972 - 2018

	Coefficient	Std.Error	t-value	t-prob
DLW1_1	0.822094	0.07214	11.4	0.0000
LW1ec_1	-0.212169	0.05835	-3.64	0.0007
Constant	-0.108584	0.03285	-3.31	0.0019

sigma	0.0184593	RSS	0.0149927774
R^2	0.768244	F(2,44) =	72.93 [0.000]**
Adj.R^2	0.75771	log-likelihood	122.493
no. of observations	47	no. of parameters	3

Table 5: Model equation for $DLW1_t$ augmented by LU_t and INF_t

```
EQ(5) Modelling DLW1 by OLS
       The estimation sample is: 1972 - 2018

                 Coefficient  Std.Error  t-value  t-prob
DLW1_1              0.415205    0.09156     4.53  0.0000
LW1ec_1            -0.196059    0.04629    -4.24  0.0001
LU               -0.00868154   0.008268    -1.05  0.2997
INF                 0.521642     0.1100     4.74  0.0000
Constant           -0.125657    0.03070    -4.09  0.0002

sigma              0.0141218  RSS             0.00837587449

EQ(6) Modelling DLW1 by IVE
       The estimation sample is: 1972 - 2018

                 Coefficient  Std.Error  t-value  t-prob
LU           Y   -0.00318601    0.01162   -0.274  0.7854
INF          Y      0.524703     0.1625     3.23  0.0024
DLW1_1              0.453587     0.1072     4.23  0.0001
LW1ec_1            -0.204337    0.04803    -4.25  0.0001
Constant           -0.114005    0.03546    -3.21  0.0025

sigma              0.0142078  RSS             0.00847817298
no. endogenous variables   3  no. of instruments         6
no. of observations       47  no. of parameters          5
mean(DLW1)         0.0639172  se(DLW1)           0.0375014
Additional instruments:
DLSCOPE1_1
INF_1
LU_1

Specification test: Chi^2(1)  = 0.030707 [0.8609]
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Table 6: Facsimile from article by Ericsson and MacKinnon.

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Table 3. Response surface estimates for critical values of the ECM test of cointegration $\kappa_c(k)$: with a constant term.

k	Size (%)	θ_∞	(s.e.)	θ_1	θ_2	θ_3	$\hat{\sigma}$
1	1	−3.4307	(0.0006)	−6.52	−4.7	−10	0.00790
	5	−2.8617	(0.0003)	−2.81	−3.2	37	0.00431
	10	−2.5668	(0.0003)	−1.56	2.1	−29	0.00332
2	1	−3.7948	(0.0006)	−7.87	−3.6	−28	0.00847
	5	−3.2145	(0.0003)	−3.21	−2.0	17	0.00438
	10	−2.9083	(0.0002)	−1.55	1.9	−25	0.00338
3	1	−4.0947	(0.0005)	−8.59	−2.0	−65	0.00857
	5	−3.5057	(0.0003)	−3.27	1.1	−34	0.00462
	10	−3.1924	(0.0002)	−1.23	2.1	−39	0.00364
	1	−4.3555	(0.0006)	−8.90	−6.7	−31	[illegible]