Exam in: ECON 4160: Econometrics: Modelling and Systems Estimation—Postponed exam

Day of exam: 12 January 2022

Time of day: 09:00—14:00

This is a 5 hour home exam.

Guidelines:
In the grading, question A gets 60 % and B 40 %.

Question A (60 %)

Consider the deterministic dynamic model with two endogenous variables, Q_t and P_t:

$$Q_t = a_{q0} + a_{qp}P_t + a_{qq}Q_{t-1} \tag{1}$$
$$P_t = b_{p0} + b_{pq}Q_t + b_{pp}P_{t-1} \tag{2}$$

1. Show that the stationary solution, if it exists, is defined by the equation system:

$$\begin{pmatrix} (1-a_{qq}) & -a_{qp} \\ -b_{pq} & (1-b_{pp}) \end{pmatrix} \begin{pmatrix} Q^* \\ P^* \end{pmatrix} = \begin{pmatrix} a_{q0} \\ b_{p0} \end{pmatrix} \tag{3}$$

   where Q^* and P^* denote the stationary solution.

2. Show that the stationary solution can be expressed as:

$$\begin{pmatrix} Q^* \\ P^* \end{pmatrix} = \begin{pmatrix} \frac{(1-b_{pp})}{c} & \frac{a_{qp}}{c} \\ \frac{b_{pq}}{c} & \frac{(1-a_{qq})}{c} \end{pmatrix} \begin{pmatrix} a_{q0} \\ b_{p0} \end{pmatrix} \tag{4}$$

   where c is given as:

$$c = (1 - a_{qq} - b_{pp} - a_{qp}b_{pq} + a_{qq}b_{pp}).$$

3. Show that the reduced form of (1)-(2) can be written as:

$$\begin{pmatrix} Q_t \\ P_t \end{pmatrix} = \Phi \begin{pmatrix} Q_{t-1} \\ P_{t-1} \end{pmatrix} + \begin{pmatrix} -\frac{1}{a_{qp}b_{pq}-1} & -\frac{1}{a_{qp}b_{pq}-1}a_{qp} \\ -\frac{1}{a_{qp}b_{pq}-1}b_{pq} & -\frac{1}{a_{qp}b_{pq}-1} \end{pmatrix} \begin{pmatrix} a_{q0} \\ b_{p0} \end{pmatrix}, \tag{5}$$

   where the matrix $\mathbf{\Phi}$ is given as:

$$\mathbf{\Phi} = \begin{pmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{pmatrix} = \begin{pmatrix} -\frac{1}{a_{qp}b_{pq}-1}a_{qq} & -\frac{1}{a_{qp}b_{pq}-1}a_{qp}b_{pp} \\ -\frac{1}{a_{qp}b_{pq}-1}b_{pq}a_{qq} & -\frac{1}{a_{qp}b_{pq}-1}b_{pp} \end{pmatrix}. \tag{6}$$

   For reference, denote the vector with constants in (5) by $\mathbf{\Upsilon}$, i.e.:

$$\mathbf{\Upsilon} = \begin{pmatrix} -\frac{1}{a_{qp}b_{pq}-1} & -\frac{1}{a_{qp}b_{pq}-1}a_{qp} \\ -\frac{1}{a_{qp}b_{pq}-1}b_{pq} & -\frac{1}{a_{qp}b_{pq}-1} \end{pmatrix} \begin{pmatrix} a_{q0} \\ b_{p0} \end{pmatrix} \tag{7}$$

4. What is the the condition on the eigenvalues of $\mathbf{\Phi}$ (characteristic roots) that secures global asymptotic stability of the model given by (1) and (2)?

5. Consider the stochastic dynamic model:

$$Q_t = a_{q0} + a_{qp}P_t + a_{qq}Q_{t-1} + e_{qt}, \tag{8}$$
$$P_t = b_{p0} + b_{pq}Q_t + b_{pp}P_{t-1} + e_{pt}, \tag{9}$$

where e_{qt} and e_{pt} are two gaussian white-noise disturbances:

$$\begin{pmatrix} e_{qt} \\ e_{pt} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \omega_1^2 & \omega_{12} \\ \omega_{12} & \omega_2^2 \end{pmatrix}\right) \text{ for all } t. \tag{10}$$

What is the condition for weak stationarity of the time series variables defined by this model?

6. Assume weak stationarity. What is the expression for the expectation of the time series Q_t?

7. The reduced form of (8)-(9) can be written as the VAR

$$\boldsymbol{y}_t = \boldsymbol{\Phi}\boldsymbol{y}_{t-1} + \boldsymbol{\Upsilon} + \boldsymbol{\varepsilon}_t \tag{11}$$

where $\boldsymbol{y}_t$ is the $2x1$ vector with Q_t and P_t as elements, and $\boldsymbol{\varepsilon}_t$ is a $2x1$ vector with the gaussian white noise time series ε_{qt} and ε_{pt} as elements:

$$\boldsymbol{\varepsilon}_t = \begin{pmatrix} \varepsilon_{qt} \\ \varepsilon_{pt} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}\right) \text{ for all } t. \tag{12}$$

   (a) Explain why, in general, $\sigma_{12} \neq 0$.
   (b) Assume that $\omega_{12} = 0$ in (10). Give an algebraic expression for σ_{12} for this special case.

8. Discuss the identification of (8) and (9) under the assumption that all the coefficients of the model equations are different from zero.

Question B (40 %)

Assume that the Data Generating Process (DGP) is given by (11) and (12) in Question A and that the DGP is stationary. Consider the conditional model equation of Q_t:

$$Q_t = \phi_0 + \phi_1 Q_{t-1} + \beta_0 P_t + \beta_1 P_{t-1} + \epsilon_t \tag{13}$$

Assume that you have time series observations of Q_t and P_t, $t = 1, 2, ..., T$.

1. Explain why ϵ_t and ε_{pt} are uncorrelated.

2. Explain why the OLS estimator $\hat{\beta}_0$ has probability limit:

$$\text{plim } (\hat{\beta}_0) = \frac{\sigma_{12}}{\sigma_{22}}. \tag{14}$$

3. Assume instead that the DGP given by (11) and (12) is non stationary, and that Q_t and P_t are two cointegrated I(1) variables.

   (a) Explain how you can estimate the coefficients of the cointegration relationship by the estimation of a conditional model.
   (b) Explain the condition under which P_t is weakly exogenous with respect to the estimation of the coefficients of the cointegration relationship, and when it is not weakly exogenous.

4. Assume that the specification of the DGP is unknown, but that it can be assumed that the time series Q_t and P_t are either $I(1)$ or $I(0)$. We do not know if there is a long-run relationship between them.

   (a) Use the results in Table 1 to decide whether the time series variables are $I(1)$ or $I(0)$. Explain your reasoning.
   (b) Table 2 shows estimation results that can be used to test the null hypothesis of no long run relationship between Q_t and P_t. Give your conclusion and explain your reasoning.
   (In the table, DQ and DP denote the first differences of the two time series.)

Tables with estimation results and facimile of table with critical values for ECM-tests

Table 1: Dickey-Fuller tests of unit-root in Q_t and P_t.

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Unit-root tests
 The sample is: 4 - 201 (201 observations and 2 variables)

Q: ADF tests (T=198, Constant+Trend; 5%=-3.43 1%=-4.01)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob
 2      -3.110      0.92740  0.08692    0.4023  0.6879
 1      -3.090      0.92853  0.08673     1.634  0.1039
 0      -2.911      0.93284  0.08710

P: ADF tests (T=198, Constant+Trend; 5%=-3.43 1%=-4.01)
Dlag     t-adf      beta Y_1    sigma   t-DY_lag  t-prob
 2      -3.494*     0.90517   0.1023     1.116  0.2660
 1      -3.376      0.90909   0.1024    0.1970  0.8440
 0      -3.388      0.90979   0.1021
```

Table 2: Regression of DQ_t on DQ_{t-1}, DP_t, DP_{t-1}, Q_{t-1}, P_{t-1} and $Constant$.

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EQ(1) Modelling DQ by OLS
      The estimation sample is: 3 - 201

                 Coefficient  Std.Error  t-value  t-prob
DQ_1                0.117010    0.07153     1.64  0.1035
Constant          -0.0250875    0.02816   -0.891  0.3741
DP                 0.0758406    0.06093     1.24  0.2147
DP_1              -0.0184715    0.06120   -0.302  0.7631
Q_1               -0.0308654    0.01774    -1.74  0.0834
P_1                0.0305891    0.02175     1.41  0.1613

sigma              0.0884937  RSS              1.51140855
R^2                   0.0333  F(5,193) =      1.33 [0.253]
Adj.R^2           0.00825591  log-likelihood      203.217
no. of observations      199  no. of parameters         6
mean(DQ)         -0.00276963  se(DQ)            0.0888613

AR 1-2 test:      F(2,191)   =  0.52789 [0.5907]
ARCH 1-1 test:    F(1,197)   = 0.027473 [0.8685]
Normality test:   Chi^2(2)   = 0.043112 [0.9787]
Hetero test:      F(10,188) =  0.67027 [0.7511]
Hetero-X test:    F(20,178) =  0.78892 [0.7247]
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Table 3: Facsimile from article by Ericsson and MacKinnon.

Table 3. Response surface estimates for critical values of the ECM test of cointegration $\kappa_c(k)$: with a constant term.

k	Size (%)	θ_∞	(s.e.)	θ_1	θ_2	θ_3	$\hat{\sigma}$
1	1	−3.4307	(0.0006)	−6.52	−4.7	−10	0.00790
	5	−2.8617	(0.0003)	−2.81	−3.2	37	0.00431
	10	−2.5668	(0.0003)	−1.56	2.1	−29	0.00332
2	1	−3.7948	(0.0006)	−7.87	−3.6	−28	0.00847
	5	−3.2145	(0.0003)	−3.21	−2.0	17	0.00438
	10	−2.9083	(0.0002)	−1.55	1.9	−25	0.00338
3	1	−4.0947	(0.0005)	−8.59	−2.0	−65	0.00857
	5	−3.5057	(0.0003)	−3.27	1.1	−34	0.00462
	10	−3.1924	(0.0002)	−1.23	2.1	−39	0.00364
	1	−4.3555	(0.0006)	−8.90	−6.7	−31	0.00959