Exam in: ECON 4160: Econometrics: Modelling and Systems Estimation—**"Fasit"**

Day of exam: 3 December 2021

Time of day: 09:00—14:00

This is a 5 hour home exam.

Guidelines:
In the grading, question A gets 50 %, B 50 %.

Question A (50 %)

Consider the dynamic simultaneous equations model (SEM):

$$\begin{aligned} rw_t - a_2 u_t &= a_0 + a_1 rw_{t-1} + a_3 z_{w,t} + e_{wt}, & (1)\\ -b_1 rw_t + u_t &= b_0 + b_2 rw_{t-1} + b_3 u_{t-1} + b_4 z_{ut} + e_{ut}. & (2) \end{aligned}$$

Definitions of the variables:
rw_t: Log of the real wage.
u_t: Log of the unemployment rate.
z_{wt}: Strictly exogenous variable that affects wage formation
z_{ut}: Strictly exogenous variable that affects unemployment.
e_{wt} and $e_{u,t}$ are two Gaussian white-noise disturbances:

$$\begin{pmatrix} e_{wt} \\ e_{ut} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \omega_1^2 & \omega_{12} \\ \omega_{12} & \omega_2^2 \end{pmatrix} \right) \text{ for all } t.$$

1. Show that the matrix with autoregressive coefficients of the reduced form of the SEM (1)-(2) is:

$$\mathbf{\Phi} = \begin{pmatrix} \frac{a_1 + a_2 b_2}{1 - a_2 b_1} & \frac{a_2 b_3}{1 - a_2 b_1} \\ \frac{b_1 a_2 + b_2}{1 - a_2 b_1} & \frac{b_3}{1 - a_2 b_1} \end{pmatrix}. \quad (3)$$

Trykkfeil. Skulle være a_1.
Det ble gitt 30 min ekstra pga mulig tidstap

For reference, the expression of the reduced form is:

$$\begin{pmatrix} rw_t \\ u_t \end{pmatrix} = \mathbf{\Phi} \begin{pmatrix} rw_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \frac{1}{1-a_2 b_1} \left[(a_0 + a_3 z_{wt} + e_{wt}) + a_2 (b_0 + b_4 z_{ut} + e_{ut}) \right] \\ \frac{1}{1-a_2 b_1} \left[b_1 (a_0 + a_3 z_{wt} + e_{wt}) + (b_0 + b_4 z_{ut} + e_{ut}) \right] \end{pmatrix}. \quad (4)$$

Answer: Inverse of matrix with contemporaneous coefficients:

$$\begin{pmatrix} 1 & -a_2 \\ -b_1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{1-a_2 b_1} & \frac{a_2}{1-a_2 b_1} \\ \frac{b_1}{1-a_2 b_1} & \frac{1}{1-a_2 b_1} \end{pmatrix}$$

Therefore, the matrix with autoregressive coefficients is:

$$\Phi = \begin{pmatrix} \frac{1}{1-a_2 b_1} & \frac{a_2}{1-a_2 b_1} \\ \frac{b_1}{1-a_2 b_1} & \frac{1}{1-a_2 b_1} \end{pmatrix} \begin{pmatrix} a_1 & 0 \\ b_2 & b_3 \end{pmatrix} = \begin{pmatrix} \frac{a_1 + a_2 b_2}{1-a_2 b_1} & \frac{a_2 b_3}{1-a_2 b_1} \\ \frac{a_1 b_1 + b_2}{1-a_2 b_1} & \frac{b_3}{1-a_2 b_1} \end{pmatrix}$$

Hence the RF is:

$$\begin{pmatrix} rw_t) \\ u_t \end{pmatrix} = \begin{pmatrix} \frac{1}{1-a_2 b_1} & \frac{a_2}{1-a_2 b_1} \\ \frac{b_1}{1-a_2 b_1} & \frac{1}{1-a_2 b_1} \end{pmatrix} * \begin{pmatrix} a_1 & 0 \\ b_2 & b_3 \end{pmatrix} \begin{pmatrix} rw_{t-1}) \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \frac{1}{1-a_2 b_1} & \frac{a_2}{1-a_2 b_1} \\ \frac{b_1}{1-a_2 b_1} & \frac{1}{1-a_2 b_1} \end{pmatrix} \begin{pmatrix} a_0 + a_3 z_{wt} + e_{wt} \\ b_0 + b_4 z_{ut} + e_{ut} \end{pmatrix}$$

$$\begin{pmatrix} rw_t) \\ u_t \end{pmatrix} = \Phi \begin{pmatrix} rw_{t-1}) \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \frac{1}{1-a_2 b_1} [(a_0 + a_3 z_{w,t} + e_{wt}) + a_2 (b_0 + b_4 z_{ut} + e_{ut})] \\ \frac{1}{1-a_2 b_1} [b_1 (a_0 + a_3 z_{wt} + e_{w,t}) + (b_0 + b_4 z_{ut} + e_{ut})] \end{pmatrix}$$

2. Assume that z_{ut} and z_{wt} are I(0) variables and that the eigenvalues of $\mathbf{\Phi}$ are both less than one in magnitude.Explain what this implies for the stationarity of rw_t and u_t.
Answer: If one of the eigenvalues of $\mathbf{\Phi}$ ± 1 the both rw_t and u_t is I(1) in general. (Can think of special cases where only one of them is I(1), by imposing $\varphi_{21} = 0$ and orthogonal VAR disturbances, but then the system is practice partitioned, into one I(1) series and one I(0) series).

3. Assume stationarity. Derive the expressions for the expectations of rw_t and u_t.
Answer: Stationary solution:

$$\begin{pmatrix} E(rw_t) \\ E(u_t) \end{pmatrix} = \Phi \begin{pmatrix} E(rw_t) \\ E(u_t) \end{pmatrix} + \begin{pmatrix} \frac{1}{1-a_2b_1}\left[(a_0 + a_3E(z_{wt})) + a_2\left(b_0 + b_4E(z_{ut})\right)\right] \\ \frac{1}{1-a_2b_1}\left[(b_0 + b_4E(z_{ut})) + b_1\left(a_0 + a_3E(z_{wt})\right)\right] \end{pmatrix}$$

$$\begin{pmatrix} E(rw_t) \\ E(u_t) \end{pmatrix} = (I - \Phi)^{-1} \begin{pmatrix} \frac{1}{1-a_2b_1}\left[(a_0 + a_3E(z_{wt})) + a_2\left(b_0 + b_4E(z_{ut})\right)\right] \\ \frac{1}{1-a_2b_1}\left[(b_0 + b_4E(z_{ut})) + b_1\left(a_0 + a_3E(z_{wt})\right)\right] \end{pmatrix}$$

4. Assume that $E(u_t) = 1.39$, and hence that the equilibrium rate of unemployment, U^*, is $U^* = e^{1.39} \approx 4.0$ percent. If the unemployment rate today is 6 percent, what does the model predict about the future development of the rate of unemployment: A development towards lower unemployment rate, a relative constant rate of 6 percent, or a higher rate?
Answer: Because of stationarity, the models predicts that there will be a development towards 4.0 % unemployment, because $U^* = 4.0$ is the stationary equilibrium. (With reference to the solution of the homogenous solution, the development can be monotonous, both eigenvalues are positive real numbers. Negative roots/eigenvalues (or imaginary roots) can imply non-monotonous development).

5. Denote the error-terms of the reduced form (aka VAR disturbances) by ε_{wt} and ε_{ut}. Show that $Cov(\varepsilon_{wt}, \varepsilon_{ut}) \neq 0$ in general.
Answer:

$$\varepsilon_{wt} = \frac{1}{1 - a_2b_1}[e_{wt} + a_2e_{ut}]$$
$$\varepsilon_{ut} = \frac{1}{1 - a_2b_1}[e_{u,t} + b_1e_{wt}]$$

$$\begin{aligned} E(\varepsilon_{wt}\varepsilon_{ut}) &= \left(\frac{1}{1 - a_2b_1}\right)^2 E[(e_{wt} + a_2e_{ut}) * (e_{ut} + b_1e_{wt})] \\ &= \left(\frac{1}{1 - a_2b_1}\right)^2 E[b_1e_{wt}^2 + e_{wt}e_{ut} + a_2e_{ut}^2 + a_2b_1e_{wt}e_{ut}] \\ &= \left(\frac{1}{1 - a_2b_1}\right)^2 [b_1Var(e_{wt}) + (1 + a_2b_1)Cov(e_{wt}, e_{ut}) + b_1Var(e_{ut})] \end{aligned}$$

Hence, if both $a_2 \neq 0$ and $b_1 \neq 0$ $Cov(\varepsilon_{wt}\varepsilon_{ut}) \neq 0$, even in the case where the two SEM-disturbances are uncorrelated.
It is possible that some answers mention recursive model here as a case of uncorrelated RF residuals, i.e., $\omega_12 = 0$ and $a_2 \neq 0$ or $b_1 \neq 0$, which deserves a positive note.

6. Assume $a_3 \neq 0$ and $b_4 \neq 0$. Explain why (1) and (2) are identified.
Answer: (1) and (2)are two simultaneous equations, $n = 2$. With reference to DEEMM and the lectures, (1)is exactly identified on the order condition if excludes $2 - 1 = 1$ of the variables of the model. It excludes 2: u_{t-1} and z_{ut}, and is therefore over-identified on the (necessary) order condition . It is also identified on the rank condition, $b_4 \neq 0$ secures that. If $b_3 \neq 0$, it is over-identified on the rank-condition. A symmetric argument applies to (2): It is exactly identified as it excludes one variable, z_{ut}, which has non-zero coefficient $a_3 \neq 0$ in (1).

7. Explain why the OLS estimators of the coefficients of the model are inconsistent.
Answer: Due to simultaneity, (1) is not a conditional model equation and therefore u_t is correlated with the SEM error term $e_w e$. This creates the simultaneity of the OLS estimator.

8. Explain why there is no simultaneity bias if 2SLS is used to estimate the two equations.
Answer: With 2SLS, the first stage is to estimate the reduce form equations by OLS. Without loss of generality we can consider (1). In the second stage of 2SLS (1) is estimated by OLS using $\hat{u}_t$ from the first stage in the place of u_t. $\hat{u}_t$ is uncorrelated with $e_w t$ in ((1)) because is has been obtained by setting the RF error-term to zero. Hence there is no bias in the 2SLS estimator. It can be mentioned in a favourably way that 2SLS is equivalent to the GIV estimator, meaning that is the IV estimator the uses the optimal linear combination of the available (valid) instruments. (The weights are the coefficients estimated at the first stage).

9. The conditional equation (5)and the marginal equation (6) represent a model of the VAR defined by the reduced form (4).

$$rw_t = \phi_0 + \phi_1 rw_t + \beta_0 u_t + \beta_1 u_{t-1} + \beta_3 z_{wt} + \beta_4 z_{ut} + \epsilon_t \quad (5)$$
$$u_t = \varphi_{20} + \varphi_{21} rw_{t-1} + \varphi_{22} u_{t-1} + \gamma_{23} z_{wt} + \gamma_{24} z_{ut} + \varepsilon_{ut} \quad (6)$$

> Trykkfeil, skulle vært rw_t-1 selvsagt. Det ble opplyst om denne også. Men ble ikke gitt tillegg i tid siden ikke påvirker svarene av til (a)-(d)

(a) What is the expression of β_0 ? **Answer:**

$$\hat{\beta}_0 = \frac{Cov(\varepsilon_{wt}, \varepsilon_{ut})}{Var(\varepsilon_{ut}}$$

(b) What are the properties of the OLS estimator of β_0?
Answer: With reference to the gaussian error terms of the SEM that was specified at the start of QA, the OLS estimator is consistent. In finite samples there is a bias, because rw_{t-1} is pre-determined rather than strictly exogenous.

(c) Explain why $Cov(\epsilon_t, \varepsilon_{ut}) = 0$.
Answer: (5) conditions on u_t, which means that the error-term ϵ_t is uncorrelated with the disturbance ε_{ut} in the marginal equation for u_t.

(d) Explain what would be a test of strong exogeneity of u_t in this model? tested.
Answer: Given that u_t is weakly exogenous, a t-test of $H_0 \varphi_{21} = 0$ (absence of joint Granger causality).

Question B (50 %)

We have annual data series of the nominal wage level in Mainland Norway manufacturing, $W1_t$, measured as wage compensation in kroner per hour worked. Another time series, $SCOPE1_t$, measures value added per hour worked in Mainland Norway manufacturing. The log of the two time series are denoted $LW1_t$, and $LSCOPE1_t$.

1. Use the results in Table 1 to test the null hypothesis of a unit-root in $LW1_t$. Test the same hypothesis for $LSCOPE1_t$.
Answer: LW1: The ADF test with `D-lag 1` is the correct one to use for testing the H_0 of I(1). It does not reject. (The ADF with `D-lag 2` is also valid. It is only a little over-parameterized). LSCOPE1: All the ADF are valid tests, none of them rejects.
In the following you can take as granted that the first difference of $LW1_t$ and $LSCOPE_t$, denoted by $DLW1_t$, and $DLSCOPE1_t$, are $I(0)$.

2. Table 2 shows results for the regression of $DLW1_t$ on $DLW1_{t-1}$, $LW1_{t-1}$ $LSCOPE1_{t-1}$, $DLSCOPE_t$, $DLSCOPE_{t-1}$ and $Constant$. Explain how you can use the ECM-test to test the null hypothesis of no cointegration between $LW1_t$ and $LSCOPE1_t$. Use

a significance level of 5 %.
Answer: The relevant critical values to use are from Table in the article by Ericsson and MacKinnon paper. The H_0 of no cointegration is rejecrted at the 5 % level as the critical value is -3.21.

3. Conditional on the rejection of no cointegration, show that the estimated long-run relationship between $LW1_t$ and $LSCOPE1_t$ is:

$$LW1_t = 0.926LSCOPE1_t - 0.033. \tag{7}$$

Answer: Let $\hat{\beta}_1$ denote the estimated slope coefficient of the cointegration relationship. It becomes:

$$\hat{\beta}_1 = \frac{0.197702}{0.213457} = 0.93619$$
$$\hat{\beta}_0 = \frac{-0.00702748}{0.213457} = -0.003\,292\,2$$

4. It can be shown that the estimated standard error of 0.926 is 0.04. Use this information to construct a test of the hypothesis that the long-run elasticity of $LSCOPE_t$ is equal to 1.
Answer: $H_0 : \beta_1 = 1 \Leftrightarrow H_0 : \beta_1 - 1 = 0$, t-value:

$$\frac{\hat{\beta}_1 - 1}{\sqrt{Var(\hat{\beta}_1)}} = \frac{0.93619 - 1}{0.04} = -1.59$$

which is not significant at the usual significance level of 5 %.

5. Table 3 shows results for the conditional ECM of $DLW1_t$ when the long-run coefficient of +1 has been imposed in the form of the variable

$$LW1ec_t = LW1_t - LSCOPE1_t. \tag{8}$$

The results for the marginal model equation for $DLSCOPE$ is shown in Table 3, as EQ(3). Explain why the results support weak exogeneity of $LSCOPE1$ with respect to the parameters of the cointegration relationship between $LW1_t$ and $LSCOPE1_1$.
Answer: The relevant test statistic is the t-value (1.68) of LW1ec_1 in EQ(3). A the 5 % level, the hypothesis of WE of LSCOPE1 with respect to the cointegration parameters cannot be rejected.

6. Table 4 shows the result for a restricted version of the wage equation in Table 3. Show that the exclusion restrictions on $DLSCOPE1$ and $DLSCOPE1\_1$ cannot be rejected at the 5 % level, neither individually nor jointly. (Hint: The 5 % critical value of the $\chi^2(2)$ distribution is 6. In the $F(2, 42)$ distribution, the 5 % critical value is 3.2).
Answer: Using LR test:

$$\chi^2(2) = -2 * (122.493 - 124.462) = 3.298$$
$$F(2, 42) = \frac{(0.0149927774 - 0.0137877334)}{0.0137877334} * \left(\frac{47 - 5}{2}\right) = 1.8354$$

Neither of two are significant at the 5 % level.

7. A critique of the model in Table 4 is that it omits two variables that are regarded to be important factors in nominal wage formation: namely the rate of unemployment and the increase in cost of living. In order to accommodate the critique, we include the log of the unemployment rate, LU_t, and the inflation rate, INF_t.
EQ(5) in Table 5 shows the OLS estimation results for the augmented wage model equation. EQ(6) shows the results of Instrumental variables estimation.

(a) Do the results support the empirical relevance of the two "new" explanatory variables, LU_t and/or INF_t?

Answer: EQ(5), OLS estimation: The t-values show that the relevance of INF is supported by the test, while the t-value of LU is insignificant. EQ(6) IV estimation. Same conclusions.

(b) Discuss briefly the potential weak exogeneity of LU_t and/or INF_t In this context it can be noted that if we add the relevant fitted values of LU_t and INF_t to the model in EQ(5), we obtain $RSS = 0.008197$. You may use this information to calculate the Durbin-Wu-Hausman (DWH) test.

Answer: There is very little difference between the OLS and IV estimates, which is informal evidence of weak exogeneity. The DWH test gives an formal test by testing the joint significance of the two fitted values from the reduced form model equations of LU_t and/or INF_t on Constant, DLW1_1, LW1ec_1, DLSCOPE1_1, INF_1, LU_m øqw|xbnm, ++s|ghjkl,1. We can denote those fitted values by $\hat{LU}_t$ and $\hat{INF}_t$. The questions gives as information that when $\hat{LU}_t$ and $\hat{INF}_t$ are added to EQ(5) we get $RSS = 0.008197$. The test of joint insignificance is therefore:

$$F(2,42) = \frac{(0.0083787449 - 0.008197)}{0.008197} * \left(\frac{47-5}{2}\right) = 0.465\,61$$

and we do not reject the hypothesis of Weak exogeneity.

(c) What is the interpretation of the *Specification test* reported with EQ(6) in Table 5?

Answer: In eq(6) we have two explanatory variables that are endogenous (in the implied SEM model). Hence exact identification rests on 2 instrumental variables. In EQ(6) there are 3 IVs and the Specification test is interpretable as a test of the validity of the over-identifying instruments (i.e., uncorrelated with the structural disturbance). Another interpretation is that the over-identifying instrument does not have any explanatory power for DLWC1, beyond the relevance that has as an instrument for LU_t and/or INF_t, hence the name *Specification test*.

Tables with estimation results and facimile of table with critical values for ECM-test

Table 1: Dickey-Fuller tests of unit-root in $LW1_t$, $LW2_t$ and $LSCOPE1_t$.

```
Unit-root tests
The sample is: 1973 - 2018 (49 observations)

LW1: ADF tests (T=46, Constant+Trend; 5%=-3.51 1%=-4.17)
D-lag    t-adf       beta Y_1    sigma   t-DY_lag  t-prob
 2     -3.064        0.94222  0.01886     -1.833  0.0741
 1     -3.337        0.94469  0.01769      3.530  0.0010
 0     -4.432**      0.92315  0.01991

LSCOPE1: ADF tests (T=46, Constant+Trend; 5%=-3.51 1%=-4.17)
D-lag    t-adf       beta Y_1    sigma   t-DY_lag  t-prob
 2     -2.017        0.93277  0.03659   -0.06286  0.9502
 1     -2.040        0.93287  0.03615    -0.8152  0.4195
 0     -1.967        0.93597  0.03601
```

Table 2: Regression of $DLW1_t$ on $DLW1_{t-1}$ and $LW1_{t-1}$, $LSCOPE1_{t-1}$, $DLSCOPE_t$, $DLSCOPE_{t-1}$ and $Constant$.

```
EQ(1) Modelling DLW1 by OLS
      The estimation sample is: 1972 - 2018

                 Coefficient  Std.Error  t-value  t-prob
DLW1_1              0.509657     0.1156     4.41  0.0001
LW1_1              -0.213457    0.06173    -3.46  0.0013
LSCOPE1_1           0.197702    0.06190     3.19  0.0027
DLSCOPE1           0.0402308    0.07362    0.546  0.5877
DLSCOPE1_1       -0.00817773    0.07797   -0.105  0.9170
Constant         -0.00702748    0.05193   -0.135  0.8930

sigma              0.0169509  RSS            0.0117806301
R^2                 0.817897  F(5,41) =     36.83 [0.000]**
Adj.R^2              0.79569  log-likelihood      128.159
no. of observations       47  no. of parameters         6
mean(DLW1)         0.0639172  se(DLW1)          0.0375014

AR 1-2 test:      F(2,39)   =  0.80804 [0.4531]
ARCH 1-1 test:    F(1,45)   =   3.5161 [0.0673]
Normality test:   Chi^2(2)  =   2.9995 [0.2232]
Hetero test:      F(10,36)  =   1.4698 [0.1910]
Hetero-X test:    F(20,26)  =   1.8495 [0.0704]
```

Table 3: Conditional and marginal model equations for $DLW1_t$ and $DLSCOPE1_t$.

EQ(2) Modelling DLW1 by OLS
The estimation sample is: 1972 - 2018

	Coefficient	Std.Error	t-value	t-prob
DLW1_1	0.698737	0.09705	7.20	0.0000
LW1ec_1	-0.210106	0.06597	-3.18	0.0027
DLSCOPE1	0.117478	0.07222	1.63	0.1113
DLSCOPE1_1	0.0603874	0.07859	0.768	0.4466
Constant	-0.110441	0.03649	-3.03	0.0042

sigma	0.0181185	RSS	0.0137877334
R^2	0.786872	F(4,42) =	38.77 [0.000]**
Adj.R^2	0.766574	log-likelihood	124.462
no. of observations	47	no. of parameters	5

EQ(3) Modelling DLSCOPE1 by OLS
The estimation sample is: 1972 - 2018

	Coefficient	Std.Error	t-value	t-prob
DLSCOPE1_1	0.156195	0.1642	0.951	0.3469
LW1ec_1	0.226758	0.1349	1.68	0.1001
DLW1_1	0.639059	0.1803	3.55	0.0010
Constant	0.136131	0.07420	1.83	0.0735

sigma	0.0382581	RSS	0.0629383203
R^2	0.382407	F(3,43) =	8.875 [0.000]**
Adj.R^2	0.339319	log-likelihood	88.78
no. of observations	47	no. of parameters	4

Table 4: Restricted conditional model equation for $DLW1_t$.

EQ(4) Modelling DLW1 by OLS
The estimation sample is: 1972 - 2018

	Coefficient	Std.Error	t-value	t-prob
DLW1_1	0.822094	0.07214	11.4	0.0000
LW1ec_1	-0.212169	0.05835	-3.64	0.0007
Constant	-0.108584	0.03285	-3.31	0.0019

sigma	0.0184593	RSS	0.0149927774
R^2	0.768244	F(2,44) =	72.93 [0.000]**
Adj.R^2	0.75771	log-likelihood	122.493
no. of observations	47	no. of parameters	3

Table 5: Model equation for $DLW1_t$ augmented by LU_t and INF_t

```
EQ(5) Modelling DLW1 by OLS
        The estimation sample is: 1972 - 2018

                 Coefficient  Std.Error  t-value  t-prob
DLW1_1              0.415205    0.09156     4.53  0.0000
LW1ec_1            -0.196059    0.04629    -4.24  0.0001
LU               -0.00868154   0.008268    -1.05  0.2997
INF                 0.521642     0.1100     4.74  0.0000
Constant           -0.125657    0.03070    -4.09  0.0002

sigma              0.0141218  RSS              0.00837587449

EQ(6) Modelling DLW1 by IVE
       The estimation sample is: 1972 - 2018

                 Coefficient  Std.Error  t-value  t-prob
LU            Y  -0.00318601    0.01162   -0.274  0.7854
INF           Y     0.524703     0.1625     3.23  0.0024
DLW1_1              0.453587     0.1072     4.23  0.0001
LW1ec_1            -0.204337    0.04803    -4.25  0.0001
Constant           -0.114005    0.03546    -3.21  0.0025

sigma              0.0142078  RSS              0.00847817298
no. endogenous variables   3  no. of instruments           6
no. of observations       47  no. of parameters            5
mean(DLW1)         0.0639172  se(DLW1)             0.0375014
Additional instruments:
DLSCOPE1_1
INF_1
LU_1

Specification test: Chi^2(1)  = 0.030707 [0.8609]
```

Table 6: Facsimile from article by Ericsson and MacKinnon.

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Table 3. Response surface estimates for critical values of the ECM test of cointegration $\kappa_c(k)$: with a constant term.

k	Size (%)	θ_∞	(s.e.)	θ_1	θ_2	θ_3	$\hat{\sigma}$
1	1	−3.4307	(0.0006)	−6.52	−4.7	−10	0.00790
	5	−2.8617	(0.0003)	−2.81	−3.2	37	0.00431
	10	−2.5668	(0.0003)	−1.56	2.1	−29	0.00332
2	1	−3.7948	(0.0006)	−7.87	−3.6	−28	0.00847
	5	−3.2145	(0.0003)	−3.21	−2.0	17	0.00438
	10	−2.9083	(0.0002)	−1.55	1.9	−25	0.00338
3	1	−4.0947	(0.0005)	−8.59	−2.0	−65	0.00857
	5	−3.5057	(0.0003)	−3.27	1.1	−34	0.00462
	10	−3.1924	(0.0002)	−1.23	2.1	−39	0.00364
	1	−4.3555	(0.0006)	−8.90	−6.7	−31	[illegible]