Guidelines:
In the grading, question A gets 33 %, B 33 % and C 33 %.

Question A (33 %)

1. Consider the cobweb model with Q_t (quantity) and P_t (price) as endogenous variables. Show that the autoregressive coefficient in the final form equation for P_t is the ratio between the slope coefficients of supply (c) and demand (a):

$$\phi_1 = \frac{c}{a}, \tag{1}$$

where we have used ϕ_1 to denote the autoregressive coefficient in the final form equation for P_t.

2. Consider more specifically the stochastic version of the cobweb model, with white-noise random disturbances in the two equations. What is the condition for weak stationary of the time series P_t and Q_t which are generated by the respective final form equations?

3. Consider the particular special case where the final form equation for P_t is:

$$P_t = -0.2P_{t-1} + 1.2 + \varepsilon_{Pt}, t = 0, 1, ..., T, \tag{2}$$

where ε_{Pt} is white-noise. Assume, abstracting from estimation, that you are given the task of forecasting P_t for the period $t = T+1, T+2, ..., T+H$ where H denotes the forecast horizon.

   (a) Draw (rough) sketches of the forecast function $E(P_{T+h} \mid \mathcal{I}_T)$, for $h = 1, 2, ..., H$ for two different initial conditions: $P_T = 0.5$ and $P_T = 1.5$.
   HINT: Since the absolute value of the autocorrelation coefficient is relatively small, setting H to for example 5 may be enough to get a clear picture.

   (b) What does the two forecasts have in common, and why?

4. Assume that in period $t = T+1$, ie., in the first period after you have produced your forecast, there is a structural break in the economy (ie., in the DGP) so that the final form equation becomes:

$$P_t = -0.2P_{t-1} + 2.4 + \varepsilon_{Pt}, t = T+1, ..., T+H, \tag{3}$$

with no change in the distribution of ε_{Pt}.
Explain how the break in the DGP will affect the forecast errors of the dynamic forecasts $E(P_{T+h} \mid \mathcal{I}_T)$, $h = 1, 2, ..., H$, for the case where the initialization price was $P_T = 0.5$.

5. Imagine that in period $T+1$, you are asked to update your forecast. Assume that you can observe $P_{T+1} = 2$, but that you do not yet know the coefficient values in the new final form equation (3). Can you suggest ways of robustifying the forecasts for $t = T+2, ..., T+H$ which are conditional on $P_{T+1} = 2$?

Question B (33 %)

Consider, as a generalization of the cobweb model, the partial equilibrium model of Q_t and P_t with X_{1t} and X_{2t} as strictly exogenous explanatory variables:

$$Q_t + a_{12,0}P_t = a_{10} + a_{11,1}P_{t-1} + a_{12,1}Q_{t-1} + \gamma_{11}X_{1t} + \gamma_{12}X_{2t} + \epsilon_{Qt}, \tag{4}$$

$$a_{21,0}Q_t + P_t = a_{20} + a_{21,1}P_{t-1} + a_{22,1}Q_{t-1} + \gamma_{21}X_{1t} + \gamma_{22}X_{2t} + \epsilon_{Pt}. \tag{5}$$

To complete the econometric model specification, it is assumed that the error-terms ϵ_{Qt} and ϵ_{Pt} are jointly normally distributed, conditional on Q_{t-1}, P_{t-1}, X_{1t} and X_{2t}. The variances of the error-terms are denoted ω_Q^2 and ω_P^2, and the covariance is denoted ω_{PQ}.

1. Give an example of a set of parameter restrictions that would define a recursive model, as a special case of the SEM we have specified.
2. Assume that the simultaneous equations (4)-(5) can be solved (ie., we do *not* consider a recursive model). The reduced form is a VAR. Denote the variances and the covariance of the VAR error-terms by σ_Q^2, σ_P^2, and σ_{PQ}. The VAR has an equivalent statistical representation which consists of a conditional model equation and a marginal model equation. Describe the structure of those two model equations for the case where we are interested in the conditional model of Q_t given P_t.
NOTE: With the exception of the coefficient of P_t, call it β_1, you are not expected to give algebraic expressions for the parameters of the conditional and marginal model equations. It is enough to explain verbally which variables are on the right hand side of each of the two model equations.
3. P_t is an endogenous explanatory variable in the conditional model equation. Does this imply that the OLS estimator of β_1 is inconsistent? Explain you answer.
4. We now return to the SEM we specified at the start of this question and discuss identification. In this question it assumed that the variances and the covariance of the SEM error-terms are unrestricted. Discuss identification in the following situations:
   (a) $\gamma_{12} = \gamma_{22} = 0$; and all other coefficients non-zero.
   (b) $\gamma_{12} = 0$; and all other coefficients non-zero.
   (c) $a_{12,1} = \gamma_{11} = \gamma_{22} = 0$; and all other coefficients non-zero.
5. It is custom to say about the 2SLS estimator that it makes use of "optimal instrumental variables". Can you give an explanation of the meaning of that statement?

Question C (33 %)

We have a quarterly temperature time series, TEMP_t.[1] In line with standard notation, the quarterly difference is denoted DTEMP_t, ie., $\text{DTEMP}_t = \text{TEMP}_1 - \text{TEMP}_{t-1}$.

1. Explain how the information in Table 1 can be used to conclude, based on the use of formal statistical tests, that $\text{TEMP}_t \sim I(1)$.
2. We also have a quarterly time series of CO_2 in the atmosphere on Mauna Loa (Hawaii) in the northern Pacific Ocean. Denote the natural logarithm of this variable by $LCO2$. Assume that $LCO2_t \sim I(1)$. Explain how the information in Table 2 can be used to conclude, based on the use of formal statistical tests, that TEMP_t and LCO2_t are cointegrated.

[1] Western-hemisphere land-ocean temperature in degrees Celsius as a deviation from the mean temperature in the period 1951-1980.

3. Explain why the implied ECM variable is:

$$\text{ecmTEMPLCO2}_t = \text{TEMP}_t - 4.8\text{LCO2}_t, \tag{6}$$

where the cointegration parameter is given with one decimal point.

4. Table 3 shows the estimation results for the model of $DTEMP_t$ conditional on $TEMP_t$ and $LCO2_t$ being cointegrated, while Table 4 shows the results for a marginal equation for $DLCO2$.

   (a) Logically speaking, why cannot both TEMP_t and LCO2_t be weakly exogenous with respect to the cointegration parameter?

   (b) How can you use Table 3 and Table 4 to assess the possible weak exogeneity of one of the variables TEMP_t and LCO2_t with respect to the cointegration parameter, and what is your conclusion?

5. Table 5 shows results obtained by using the Johansen-method for cointegration analysis. Are these results (by and large) in support of the results obtained earlier in this question?

Tables with estimation results and facimile of table with critical values for ECM-test

Table 1: Dickey-Fuller tests of unit-root. $TEMP_t$ and $DTEMP_t$.

The sample is: 1959(2) - 2022(3) (254 observations)

TEMP: ADF tests (T=251, Constant; 5%=-2.87 1%=-3.46)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob
4	-1.207	0.97079	0.1622	1.012	0.3126
3	-1.092	0.97378	0.1623	-2.968	0.0033
2	-1.511	0.96352	0.1648	-5.655	0.0000
1	-2.448	0.93839	0.1748	-2.981	0.0032
0	-3.092*	0.92270	0.1775		

DTEMP: ADF tests (T=251, Constant; 5%=-2.87 1%=-3.46)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob
4	-9.104**	-1.0123	0.1623	1.197	0.2325
3	-10.03**	-0.86981	0.1624	-0.8702	0.3851
2	-14.40**	-0.97936	0.1623	3.157	0.0018
1	-17.85**	-0.65434	0.1652	6.021	0.0000
0	-19.71**	-0.21869	0.1765		

Table 2: Estimation results for a model of $DTEMP_t$

EQ(1) Modelling DTEMP by OLS
The estimation sample is: 1959(2) - 2022(3)

	Coefficient	Std.Error	t-value	t-prob
DTEMP_1	-0.131801	0.07731	-1.70	0.0895
DTEMP_2	-0.142193	0.07502	-1.90	0.0592
DTEMP_3	-0.00838194	0.06940	-0.121	0.9040
DTEMP_4	-0.00779189	0.06356	-0.123	0.9025
Constant	-8.63916	1.961	-4.41	0.0000
TEMP_1	-0.329043	0.06562	-5.01	0.0000
DLCO2	23.7800	10.16	2.34	0.0201
DLCO2_1	32.8985	10.01	3.29	0.0012
DLCO2_2	-20.7048	10.25	-2.02	0.0446
DLCO2_3	1.40087	10.17	0.138	0.8905
DLCO2_4	-9.45658	10.10	-0.936	0.3502
LCO2_1	1.57595	0.3367	4.68	0.0000
Seasonal	-0.696419	0.1955	-3.56	0.0004
Seasonal_1	-0.932327	0.2572	-3.63	0.0004
Seasonal_2	-0.424001	0.1993	-2.13	0.0344

sigma	0.140492	RSS	4.71737253
R^2	0.422551	F(14,239) =	12.49 [0.000]**
Adj.R^2	0.388725	log-likelihood	145.822
no. of observations	254	no. of parameters	15
mean(DTEMP)	0.00346457	se(DTEMP)	0.179694

AR 1-5 test: F(5,234) = 1.2239 [0.2986]

Table 3: Estimation results for a model of $DTEMP_t$, conditional on cointegration.

EQ(2) Modelling DTEMP by OLS
The estimation sample is: 1959(2) - 2022(3)

	Coefficient	Std.Error	t-value	t-prob
DTEMP_1	-0.131776	0.07714	-1.71	0.0889
DTEMP_2	-0.142090	0.07475	-1.90	0.0585
DTEMP_3	-0.00815509	0.06867	-0.119	0.9056
DTEMP_4	-0.00757659	0.06286	-0.121	0.9042
Constant	-8.65646	1.833	-4.72	0.0000
ecmTEMPLCO2_1	-0.328954	0.06539	-5.03	0.0000
DLCO2	23.7087	9.736	2.44	0.0156
DLCO2_1	32.8042	9.268	3.54	0.0005
DLCO2_2	-20.8142	9.271	-2.25	0.0257
DLCO2_3	1.30000	9.328	0.139	0.8893
DLCO2_4	-9.52581	9.703	-0.982	0.3272
Seasonal	-0.696195	0.1949	-3.57	0.0004
Seasonal_1	-0.932209	0.2566	-3.63	0.0003
Seasonal_2	-0.424476	0.1980	-2.14	0.0330

sigma	0.140199	RSS	4.71738508
R^2	0.422549	F(13,240) =	13.51 [0.000]**
Adj.R^2	0.391271	log-likelihood	145.822
no. of observations	254	no. of parameters	14
mean(DTEMP)	0.00346457	se(DTEMP)	0.179694

AR 1-5 test: F(5,235) = 1.2157 [0.3024]

Table 4: Estimation results for a marginal model of $DLCO2_t$, conditional on cointegration.

EQ(3) Modelling DLCO2 by OLS
The estimation sample is: 1959(2) - 2022(3)

	Coefficient	Std.Error	t-value	t-prob
DLCO2_1	-0.106905	0.06093	-1.75	0.0806
DLCO2_2	-0.129611	0.06077	-2.13	0.0340
DLCO2_3	0.0494536	0.06163	0.802	0.4231
DLCO2_4	0.304635	0.06112	4.98	0.0000
Constant	-0.0162792	0.01208	-1.35	0.1790
DTEMP_1	0.00151583	0.0005009	3.03	0.0027
DTEMP_2	0.00126003	0.0004879	2.58	0.0104
DTEMP_3	0.00141311	0.0004451	3.17	0.0017
DTEMP_4	0.00125382	0.0004080	3.07	0.0024
ecmTEMPLCO2_1	-0.000536080	0.0004312	-1.24	0.2150
Seasonal	0.00590183	0.001232	4.79	0.0000
Seasonal_1	0.00716308	0.001634	4.38	0.0000
Seasonal_2	-0.00369491	0.001288	-2.87	0.0045

sigma	0.00092758	RSS	0.000207357403
R^2	0.985325	F(12,241) =	1348 [0.000]**
Adj.R^2	0.984594	log-likelihood	1419.93
no. of observations	254	no. of parameters	13
mean(DLCO2)	0.00109223	se(DLCO2)	0.00747327

AR 1-5 test: F(5,236) = 1.0565 [0.3787]

Table 5: Results for the Johansen-method for cointegration analysis.

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I(1) cointegration analysis, 1959(2) - 2022(3)

 eigenvalue    loglik for rank
                 1513.841   0
    0.13991      1532.982   1
  0.0056153      1533.697   2

 H0:rank<=  Trace test [ Prob]
     0         39.712 [0.000] **
     1         1.4303 [0.232]

Asymptotic p-values based on: Unrestricted trend and constant
Unrestricted variables:
Constant
Trend
Number of lags used in the analysis: 5

beta (scaled on diagonal; cointegrating vectors in columns)
TEMP         1.0000    -0.010200
LCO2        -5.896       1.0000

alpha
TEMP       -0.54616     -0.30961
LCO2        0.00023     -0.010983

Standard errors of alpha, conditional om r=1
TEMP       0.088758
LCO2       0.000647
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Table 6: Facsimile from article by Ericsson and MacKinnon.

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Table 3. Response surface estimates for critical values of the ECM test of cointegration $\kappa_c(k)$: with a constant term.

k	Size (%)	θ_∞	(s.e.)	θ_1	θ_2	θ_3	$\hat{\sigma}$
1	1	−3.4307	(0.0006)	−6.52	−4.7	−10	0.00790
	5	−2.8617	(0.0003)	−2.81	−3.2	37	0.00431
	10	−2.5668	(0.0003)	−1.56	2.1	−29	0.00332
2	1	−3.7948	(0.0006)	−7.87	−3.6	−28	0.00847
	5	−3.2145	(0.0003)	−3.21	−2.0	17	0.00438
	10	−2.9083	(0.0002)	−1.55	1.9	−25	0.00338
3	1	−4.0947	(0.0005)	−8.59	−2.0	−65	0.00857
	5	−3.5057	(0.0003)	−3.27	1.1	−34	0.00462
	10	−3.1924	(0.0002)	−1.23	2.1	−39	0.00364
	1	−4.3555	(0.0006)	8.90	−6.7	−31	[illegible]