Question A (50 %)

1. Consider the linear deterministic difference equation:

$$y_t = 1.0 + 1.2y_{t-1} - 0.3y_{t-2} \tag{1}$$

What is the condition for global asymptotical stability of this equation?

2. Assume that $y_{-1} = 0$ and $y_0 = 1$. Solve the equation for $t = 1$, $t = 2$ and $t = 3$.

3. Show that the condition is satisfied for equation (1).

4. What is the number, denoted by y^*, that the solution approaches as $t \to \infty$?

5. Consider the linear stochastic difference equation;

$$Y_t = 1.0 + 1.2Y_{t-1} - 0.3Y_{t-2} + \epsilon_t \tag{2}$$

where the time series ϵ_t , $t = 0, \pm 1, \pm 2, \pm 3, ...$, is white-noise. Is the time series Y_t generated by (2) weakly (covariance) stationary? Explain briefly.

6. Assuming stationarity, show that $E(Y_t) = y^*$.

7. Calculate the responses of Y_t, Y_{t+1}, Y_{t+2} and Y_{t+3} with respect to a small change (so called shock) in ϵ_t.

8. Equation (2) is a special case of the 2nd order difference equation:

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \epsilon_t, \tag{3}$$

where ϵ_t , $t = 0, \pm 1, \pm 2, \pm 3, ...$, is an exogenous stationary time series (not necessarily white-noise). Explain why (3) can be interpreted as a final form equation which is derived from a VAR with two endogenous variables and with first-order dynamics.

Question B (50 %)

Consider the SEM:

$$w_t - a_{12,0}q_t - a_{13,0}p_t = a_{10} + a_{11,1}w_{t-1} + a_{12,1}q_{t-1} + a_{13,1}p_{t-1} + \gamma_{11}z_t + \gamma_{12}u_t + \epsilon_{wt}, \tag{4}$$

$$-a_{21,0}w_t + q_t = a_{20} + a_{21,1}w_{t-1} + a_{22,1}q_{t-1} + \gamma_{21}z_t + \gamma_{22}u_t + \epsilon_{qt}, \tag{5}$$

$$p_t = 0.6q_t + 0.4z_t \tag{6}$$

To aid interpretation, we can define w_t as wage compensation per hour and let q_t and p_t denote two domestic price indices: q_t is a producer price index . p_t denotes a consumer price index (cost of living index). w_t, q_t and p_t are the endogenous variables of the model. Finally, let z_t denote a price index for foreign goods and services, and let u_t denote a measure of the rate of unemployment (in the domestic economy). Both z_t and u_t are pre-determined variables. The two error-terms are Gaussian multivariate white-noise:

$$\begin{pmatrix} \epsilon_{wt} \\ \epsilon_{qt} \end{pmatrix} \sim IIN(\mathbf{0}, \mathbf{\Sigma}) \tag{7}$$

where the elements of the covariance matrix $\mathbf{\Sigma}$ are σ_w^2, σ_q^2 and σ_{wq}.

1. Discuss the identification of the model equations in the following cases:

    (a) There are no linear restrictions on the coefficients of the model (remember that a zero-restriction on an individual coefficient is a special case of a linear restriction).

(b) There are four restrictions on the coefficients of the model, namely $a_{12,0} = 0$, $a_{13,0} + a_{13,1} = 0$, $\gamma_{11} = 0$ and $\gamma_{22} = 0$.

The SEM corresponding to the case in Question B1(b) can be expressed as:

$$w_t - a_{13,0}\Delta p_t = a_{10} + a_{11,1}w_{t-1} + a_{12,1}q_{t-1} + \gamma_{11}z_t + \gamma_{12}u_t + \epsilon_{wt}, \quad (8)$$

$$-a_{21,0}w_t + q_t = a_{20} + a_{21,1}w_{t-1} + a_{22,1}q_{t-1} + \gamma_{21}z_{1t} + \gamma_{22}z_t + \epsilon_{qt}, \quad (9)$$

$$p_t = 0.6q_t + 0.4z_t \quad (10)$$

with (7) holding as before. We have time series data for the variables of the model. Table 1 and Table 2 show estimation results for equation (8) using OLS and IV/2SLS. estimation.

2. Explain why the OLS estimation results are affected by simultaneity bias.

3. Explain why the IV/2SLS results are not affected by simultaneity bias.

4. What is the interpretation of the Specification test reported with the IV/2SLS estimation results?

5. Table 3 shows the results of ADF tests for the individual times series (in level form). Explain how you can use that information to test the null hypothesis of I(1)-ness for each of the variables, and give your conclusion.
In order to save space we do not show the ADF tests for the differences of the variables. In the following you can take as granted that none of the first differences are I(1), and in particular that $Dp \sim I(0)$.

6. Imagine that you share the data set with another student who has been given the task of testing the hypothesis of a relationship between the wage,w, the foreign price index, z, and the unemployment measure, u. Imagine also that the student has produced the model shown in Table 4 and now asks you to confirm that it represents formally correct evidence in support of the existence of a long-run relationship. What would your answer be?

7. Based on the results shown in Table 5, how can you test the null hypothesis of no relationship between w, z,and u, and what is your conclusion?

Tables with estimation results and facsimile of table with critical values for ECM-test

Table 1: Estimation results for equation(8) using OLS

```
EQ( 1) Modelling w by OLS
      The dataset is: Postponed_QB_d.in7
      The estimation sample is: 2 - 101

                  Coefficient  Std.Error  t-value  t-prob
w_1                 0.836864    0.03373     24.8  0.0000
Constant            0.115875     0.2706    0.428  0.6694
Dp                  0.987819    0.05421     18.2  0.0000
q_1                 0.168720    0.04052     4.16  0.0001
u                  -0.468979    0.04383    -10.7  0.0000

sigma               0.469422  RSS               20.933921
R^2                  0.99891  F(4,95) =    2.176e+04 [0.000]**
Adj.R^2             0.998864  log-likelihood      -63.7039
no. of observations      100  no. of parameters          5
mean(w)             -22.0388  se(w)                13.9251

AR 1-2 test:      F(2,93)   =  0.57631 [0.5640]
ARCH 1-1 test:    F(1,98)   =  0.69016 [0.4081]
Normality test:   Chi^2(2)  =   3.0483 [0.2178]
Hetero test:      F(8,91)   =  0.89443 [0.5246]
Hetero-X test:    F(14,85)  =   1.1678 [0.3147]
```

Table 2: Estimation results for equation (8) using IV/2SLS

```
EQ( 2) Modelling w by IVE
      The dataset is: Postponed_QB_d.in7
      The estimation sample is: 2 - 101

                  Coefficient  Std.Error  t-value  t-prob
Dp             Y    0.921447    0.05686     16.2  0.0000
w_1                 0.850084    0.03414     24.9  0.0000
Constant            0.285678     0.2757     1.04  0.3027
q_1                 0.152169    0.04103     3.71  0.0004
u                  -0.482328    0.04429    -10.9  0.0000

sigma               0.473112  RSS              21.2642994
Reduced-form sigma   0.61276
no. endogenous variables   2  no. of instruments         6
no. of observations      100  no. of parameters          5
mean(w)             -22.0388  se(w)                13.9251
Additional instruments:
z
p_1

Specification test: Chi^2(1)  =  0.14915 [0.6993]
Testing beta = 0:   Chi^2(4)  =   85606. [0.0000]**

AR 1-2 test:      F(2,93)   =  0.70988 [0.4943]
ARCH 1-1 test:    F(1,98)   =  0.24801 [0.6196]
Normality test:   Chi^2(2)  =   2.7876 [0.2481]
Hetero test:      F(8,91)   =  0.68729 [0.7017]
Hetero-X test:    F(14,85)  =   1.1145 [0.3574]
```

Table 3: Dickey-Fuller tests of unit-root.

Unit-root tests
The dataset is:Postponed_QB_d.in7
The sample is: 4 - 101 (101 observations and 5 variables)

w: ADF tests (T=98, Constant; 5%=-2.89 1%=-3.50)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-0.4456	0.99580	1.267	0.8020	0.4246	0.5132	
1	-0.3367	0.99686	1.265	3.588	0.0005	0.4996	0.4246
0	0.1189	1.0012	1.340			0.6063	0.0018

q: ADF tests (T=98, Constant; 5%=-2.89 1%=-3.50)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-0.4638	0.99670	0.7687	0.9848	0.3272	-0.4862	
1	-0.2956	0.99793	0.7685	5.967	0.0000	-0.4964	0.3272
0	0.5927	1.0048	0.8964			-0.1985	0.0000

p: ADF tests (T=98, Constant; 5%=-2.89 1%=-3.50)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-0.4547	0.99527	0.9760	1.248	0.2150	-0.008677	
1	-0.2772	0.99714	0.9788	3.448	0.0008	-0.01264	0.2150
0	0.2058	1.0022	1.033			0.08487	0.0018

u: ADF tests (T=98, Constant; 5%=-2.89 1%=-3.50)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-2.550	0.81146	0.9150	-1.371	0.1735	-0.1376	
1	-3.122*	0.77974	0.9193	-0.1256	0.9003	-0.1382	0.1735
0	-3.392*	0.77667	0.9145			-0.1585	0.3911

z: ADF tests (T=98, Constant; 5%=-2.89 1%=-3.50)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-0.9085	0.98174	1.490	1.035	0.3033	0.8377	
1	-0.7803	0.98445	1.491	1.299	0.1972	0.8286	0.3033
0	-0.6045	0.98803	1.496			0.8258	0.2568

Table 4: Estimation results for a model of w_t

EQ(3) Modelling w by OLS
The dataset is: Postponed_QB_d.in7
The estimation sample is: 2 - 101

	Coefficient	Std.Error	t-value	t-prob
Constant	-30.4086	1.629	-18.7	0.0000
z	1.53420	0.06069	25.3	0.0000
u	-2.34139	0.3313	-7.07	0.0000

sigma	4.47054	RSS	1938.61701
R^2	0.899014	F(2,97) =	431.8 [0.000]**
Adj.R^2	0.896932	log-likelihood	-290.122
no. of observations	100	no. of parameters	3
mean(w)	-22.0388	se(w)	13.9251

AR 1-2 test:	F(2,95)	=	95.783 [0.0000]**
ARCH 1-1 test:	F(1,98)	=	30.999 [0.0000]**
Normality test:	Chi^2(2)	=	6.9785 [0.0305]*
Hetero test:	F(4,95)	=	2.6189 [0.0397]*
Hetero-X test:	F(5,94)	=	2.0759 [0.0752]

Table 5: Estimation results for a model of Dw_t

```
EQ(4) Modelling Dw by OLS
      The dataset is: Postponed_QB_d.in7
      The estimation sample is: 2 - 101

                  Coefficient  Std.Error  t-value  t-prob
Constant             -0.590574     0.7121   -0.829  0.4089
w_1                  -0.105831    0.01889    -5.60  0.0000
z                     0.137667    0.03199     4.30  0.0000
u                    -0.806881    0.07488    -10.8  0.0000

sigma                 0.910926  RSS                79.6595231
R^2                   0.550122  F(3,96) =     39.13 [0.000]**
Adj.R^2               0.536063  log-likelihood       -130.523
no. of observations        100  no. of parameters           4
mean(Dw)              0.306275  se(Dw)                1.33738

AR 1-2 test:      F(2,94)    =   4.5560 [0.0129]*
ARCH 1-1 test:    F(1,98)    =   1.5151 [0.2213]
Normality test:   Chi^2(2)   =   4.1852 [0.1234]
Hetero test:      F(6,93)    =   1.0033 [0.4281]
Hetero-X test:    F(9,90)    =   1.6465 [0.1141]
```

Table 6: Facsimile from article by Ericsson and MacKinnon.

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Table 3. Response surface estimates for critical values of the ECM test of cointegration $\kappa_c(k)$: with a constant term.

k	Size (%)	θ_∞	(s.e.)	θ_1	θ_2	θ_3	$\hat{\sigma}$
1	1	−3.4307	(0.0006)	−6.52	−4.7	−10	0.00790
	5	−2.8617	(0.0003)	−2.81	−3.2	37	0.00431
	10	−2.5668	(0.0003)	−1.56	2.1	−29	0.00332
2	1	−3.7948	(0.0006)	−7.87	−3.6	−28	0.00847
	5	−3.2145	(0.0003)	−3.21	−2.0	17	0.00438
	10	−2.9083	(0.0002)	−1.55	1.9	−25	0.00338
3	1	−4.0947	(0.0005)	−8.59	−2.0	−65	0.00857
	5	−3.5057	(0.0003)	−3.27	1.1	−34	0.00462
	10	−3.1924	(0.0002)	−1.23	2.1	−39	0.00364
	1	−4.3555	(0.0006)	−8.90	−6.7	−31	[illegible]