Postponed exam in: ECON 4160: Econometrics: Modelling and Systems Estimation

Day of exam: 11 January 2023

Time of day: 09:00—14:00

This is a 5 hour home exam.

Guidelines:
In the grading, question A gets 50 % and B 50.

Question A (50 %)

1. Consider the linear deterministic difference equation

$$y_t = 1.0 + 1.2y_{t-1} - 0.3y_{t-2} \tag{1}$$

What is the condition for global asymptotical stability of this equation?
A: The definition of globally asymptotically stability of the equation is that any solution of the homogenous equation:

$$y_t^h - 1.2y_{t-1}^h + 0.3y_{t-2}^h = 0$$

approaches zero as $t \to \infty$. The condition for (global asymptotical) stability is that the two roots of the associated characteristic equation

$$\lambda^2 - 1.2\lambda + 0.3 = 0$$

have modulus less that one.

2. Assume that $y_{-1} = 0$ and $y_0 = 1$. Solve the equation for $t = 1$, $t = 2$ and $t = 3$.
A:

$$\begin{aligned} y_1 &= 1.0 + 1.2y_0 - 0.3y_{-1} = 1.0 + 1.2 = 2.2 \\ y_2 &= 1.0 + 1.2y_1 - 0.3y_0 = 1.0 + 1.2 * 2.2 - 0.3 * 1 = 3.34 \\ y_3 &= 1.0 + 1.2y_2 - 0.3y_1 = 1.0 + 1.2 * 3.34 - 0.3 * 2.2 = 4.348 \end{aligned}$$

3. Show that the condition is satisfied for equation (1).
A: We have from mathematics, that the formula for the two characteristic roots is:

$$\begin{aligned} \lambda_{1,2} &= \frac{1.2 \pm \sqrt{1.2^2 - 4 * 0.3}}{2} \\ \text{which gives} &: \\ \lambda_1 &= \frac{1.2 + \sqrt{1.2^2 - 4 * 0.3}}{2} = 0.84495 \\ \lambda_2 &= \frac{1.2 - \sqrt{1.2^2 - 4 * 0.3}}{2} = 0.35505 \end{aligned}$$

As the two roots are real, positive, and less than one, the condition is satisfied.

4. What is the number, denoted by y^*, that the soltution approaches as $t \to \infty$?
A:The stationary solution value y^* does indeed exist, as the condition for (global asymptotical) stability is satisfied. It is found as:

$$\begin{aligned} y^* &= 1.0 + 1.2y^* - 0.3y^* \\ y^* &= \frac{1}{1 - 1.2 + 0.3} = 10 \end{aligned}$$

5. Consider the linear stochastic difference equation;

$$Y_t = 1.0 + 1.2Y_{t-1} - 0.3Y_{t-2} + \epsilon_t, \qquad (2)$$

where the time series ϵ_t , $t = 0, \pm1, \pm2, \pm3, ...$, is white-noise. Is the time series Y_t generated by (2) weakly (or covariance) stationary? Explain briefly.
A: (2) is a second order AR-process. The condition that it generates a stationary time series is that none of the characteristic roots of the associated homogenous equation has modulus equal to 1. When we consider the causal solution, the condition is that both roots have modulus less than one (the same condition as for global asymptotical stability of the (solution of) the difference equation).

6. Assuming stationarity, show that $E(Y_t) = y^*$.
A:

$$E(Y_t) = 1.0 + 1.2E(Y_{t-1}) - 0.3E(Y_{t-2})$$

and using that, by definition, the unconditional expectation of a stationary time series does not depend on time (as such):

$$\begin{aligned} E(Y_t) &= 1.0 + 1.2E(Y_t) + 0.3E(Y_t) \\ E(Y_t) &= \frac{1}{1 - 1.2 + 0.3} = 10 \equiv y^* \end{aligned}$$

7. Calculate the responses of Y_t, Y_{t+1}, Y_{t+2} and Y_{t+3} with respect to a small change (so called shock) in ϵ_t.
A: By direct reasoning, and making use of the fact that equation (2) holds in all time periods:

$$Y_{t+j} = 1.0 + 1.2Y_{t+j-1} - 0.3Y_{t+j-2} + \epsilon_{t+j},\ j = 0, 1, 2, 3.....$$

we get:

$$\begin{aligned} j &= 0 \Rightarrow \delta_0 = \frac{\partial Y_t}{\partial \epsilon_t} = 1 \\ j &= 1 \Rightarrow \delta_1 = \frac{\partial Y_{t+1}}{\partial \epsilon_t} = 1.2\frac{\partial Y_t}{\partial \epsilon_t} = 1.2 \\ j &= 2 \Rightarrow \delta_2 = \frac{\partial Y_{t+2}}{\partial \epsilon_t} = 1.2\frac{\partial Y_{t+1}}{\partial \epsilon_t} - 0.3\frac{\partial Y_t}{\partial \epsilon_t} = 1.14 \\ j &= 3 \Rightarrow \delta_3 = \frac{\partial Y_{t+3}}{\partial \epsilon_t} = 1.2\frac{\partial Y_{t+2}}{\partial \epsilon_t} - 0.3\frac{\partial Y_{t+1}}{\partial \epsilon_t} = 1.032 \end{aligned}$$

Another method uses lag-operator notation to express (2) as:

$$Y_t(1L^0 - 1.2L + 0.3L^2) = 1.0 + \epsilon_t$$

and note that the solution can be expressed as

$$Y_t = (\delta_0 L^0 + \delta_1 L + \delta_2 L^2 + ...)(1.0 + \epsilon_t)$$

where $(\delta_0 L^0 + \delta_1 L + \delta_2 L^2 + ...)$ denotes the inverse of the polynomial $(1 - 1.2L + 0.3L^2)$. Therefore:

$$(1 - 1.2L + 0.3L^2) \cdot (\delta_0 L^0 + \delta_1 L + \delta_2 L^2 + ...) = 1$$

by definition of the inverse. But, conveniently, we can write $1 \equiv 1L^0 + 0L + 0L^2 + ...$, hence:

$$(1 - 1.2L + 0.3L^2) \cdot (\delta_0 L^0 + \delta_1 L + \delta_2 L^2 + ...) = 1L^0 + 0L + 0L^2 + ...$$

from which the dynamic responses can be obtained (similar to the method on page 139 in DEEMM)

$$\begin{array}{llcl} L^0: & 1 \cdot \delta_0 = 1 & \Longrightarrow & \delta_0 = 1 \\ L^1: & (-1.2\delta_0 + \delta_1)L = 0 & \Longrightarrow & \delta_1 = 1.2 \\ L^2: & (-1.2\delta_0\delta_1 + 0.3\delta_0 + \delta_2)L^2 = 0 & \Longrightarrow & \delta_2 = 1.2 * 1.2 - 0.3 = 1.14 \\ L^3: & (0.3\delta_1 - 1.2\delta_2 + \delta_3)L^3 = 0 & & \delta_3 = 1.2 * 1.14 - 0.3 * 1.12 = 1.032 \end{array}$$

8. Equation (2) is a special case of the 2nd order difference equation

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \epsilon_t \tag{3}$$

where ϵ_t , $t = 0, \pm 1, \pm 2, \pm 3, ...$, is an exogenous stationary time series (not necessarily white-noise). Explain why (3) can be interpreted as a final form equation which is derived from a VAR with first-order dynamics.
A: By definition a final form equation for Y_t is an equation that only has lagged values of Y_t and exogenous variables on the right hand side. (3) is therefore interpretable as a final form equation for Y_t when Y_t is generated by a VAR with two endogenous variables, namely Y_t and (for example), X_t , and with first-order dynamics.

Question B (50 %)

Consider the SEM (4)-(6).

$$w_t - a_{12,0}q_t - a_{13,0}p_t = a_{10} + a_{11,1}w_{t-1} + a_{12,1}q_{t-1} + a_{13,1}p_{t-1} + \gamma_{11}z_t + \gamma_{12}u_t + \epsilon_{wt}, \tag{4}$$

$$-a_{21,0}w_t + q_t = a_{20} + a_{21,1}w_{t-1} + a_{22,1}q_{t-1} + \gamma_{21}z_t + \gamma_{22}u_t + \epsilon_{qt}. \tag{5}$$

$$p_t = 0.6q_t + 0.4z_t \tag{6}$$

To aid interpretation we can define w_t as wage compensation per hour and let q_t and p_t denote two different domestic price indices: q_t is a producer price index (for value added for example). p_t denotes a consumer price index (cost of living index). w_t, q_t and p_t are the endogenous variables of the model. Finally, let z_t denote a price index on foreign goods, and let u_t denote a measure of the rate of unemployment (in the domestic economy). Assume that both z_t and u_t are pre-determined variables.

The two error-terms are Gaussian multivariate white-noise:

$$\begin{pmatrix} \epsilon_{wt} \\ \epsilon_{qt} \end{pmatrix} \sim IIN(0, \Sigma) \tag{7}$$

where the elements of the covariance matrix $\mathbf{\Sigma}$ are σ_w^2, σ_q^2 and σ_{wq}.

1. Discuss the identification of the model equations in the following cases:

   (a) There are no linear restrictions on the coefficients of the model (remember that a zero-restriction on an individual coefficients is a special case of a linear restriction).
   A:Using the order condition, the requirement for exact identification of (4) is that the are 3-1=2 independent restrictions. This restriction is not satisfied, and therefore (4) is not identified. In equation (5) p_t and p_{t-1} are exclude, so the order condition for exact identification is satisfied for equation (5). For the rank condition to be satisfied the array of coefficients of p_t and p_{t-1} from (4) and (6)

$$\begin{pmatrix} a_{13,0} & a_{13,1} \\ 1 & 0 \end{pmatrix}$$

   has to have rank=2, which is indeed the case here. Hence (5) is identified.

   (b) There are restrictions on the coefficients of the model, namely $a_{12,0} = 0$ and $a_{13,0} + a_{13,1} = 0$ and $\gamma_{11} = 0$ and $\gamma_{22} = 0$.
   A: The first three restrictions apply to (4), which is therefore over-identified on the order-condition. The fourth is a restriction on (5), which makes that equation over-identified. Not shown, but the rank condition is clearly identified.

The SEM corresponding to the case in Question B1(b) can be expressed as:

$$w_t - a_{13,0}\Delta p_t = a_{10} + a_{11,1}w_{t-1} + a_{12,1}q_{t-1} + \gamma_{11}z_t + \gamma_{12}u_t + \epsilon_{wt}, \quad (8)$$

$$-a_{21,0}w_t + q_t = a_{20} + a_{21,1}w_{t-1} + a_{22,1}q_{t-1} + \gamma_{21}z_{1t} + \gamma_{22}z_t + \epsilon_{qt}, \quad (9)$$

$$p_t = 0.6q_t + 0.4z_t \quad (10)$$

with (7) holding as before. We have time series data for the variables of the model. Table 1 and Table 2 show estimation results for equation (8) using OLS and IV/2SLS estimation.

2. Explain why the OLS estimation results are affected by simultaneity bias.

3. Explain why the IV/2SLS results are not affected by simultaneity bias.

4. What is the interpretation of the Specification test reported with the IV/2SLS estimation results?

5. Table 3 shows the results of ADF tests for the individual times series (in level form). Explain how you can use the information to test the null hypotheses of I(1)-ness for each of the variables, and give your conclusion.
   In order to save space we do not show the ADF tests for the differences of the variables. In the following you can take as a granted that none of the first differences are I(1), and in particular that $Dp \sim (0)$.

6. Imagine that you share the data with another student who has been given the task of testing the hypothesis of a relationship between the wage,w, the foreign price index, z,and the unemployment measure, u. The student has produced the model shown in Table 4 and comes to you to get confirmation that Table 4 represents formally correct evidence of the existence a long-run relationship. What would your answer be?

7. Based on the results shown in Table 5 how can you test the null hypothesis of no relationship between w, z,and u, and what is your conclusion?

Tables with estimation results and facsimile of table with critical values for ECM-test

Table 1: Estimation results for (8) using OLS

```
EQ( 1) Modelling w by OLS
      The dataset is: Postponed_QB_d.in7
      The estimation sample is: 2 - 101

                 Coefficient  Std.Error  t-value  t-prob
w_1                 0.836864    0.03373     24.8  0.0000
Constant            0.115875     0.2706    0.428  0.6694
Dp                  0.987819    0.05421     18.2  0.0000
q_1                 0.168720    0.04052     4.16  0.0001
u                  -0.468979    0.04383    -10.7  0.0000

sigma               0.469422  RSS                20.933921
R^2                  0.99891  F(4,95) =    2.176e+04 [0.000]**
Adj.R^2             0.998864  log-likelihood       -63.7039
no. of observations      100  no. of parameters           5
mean(w)             -22.0388  se(w)                 13.9251

AR 1-2 test:      F(2,93)   =  0.57631 [0.5640]
ARCH 1-1 test:    F(1,98)   =  0.69016 [0.4081]
Normality test:   Chi^2(2)  =   3.0483 [0.2178]
Hetero test:      F(8,91)   =  0.89443 [0.5246]
Hetero-X test:    F(14,85)  =   1.1678 [0.3147]
```

Table 2: Estimation results for (8) using IV/2SLS

```
EQ( 2) Modelling w by IVE
      The dataset is: Postponed_QB_d.in7
      The estimation sample is: 2 - 101

                 Coefficient  Std.Error  t-value  t-prob
Dp             Y    0.921447    0.05686     16.2  0.0000
w_1                 0.850084    0.03414     24.9  0.0000
Constant            0.285678     0.2757     1.04  0.3027
q_1                 0.152169    0.04103     3.71  0.0004
u                  -0.482328    0.04429    -10.9  0.0000

sigma               0.473112  RSS                21.2642994
Reduced-form sigma    0.61276
no. endogenous variables    2  no. of instruments          6
no. of observations       100  no. of parameters           5
mean(w)             -22.0388  se(w)                 13.9251
Additional instruments:
z
p_1

Specification test: Chi^2(1)  =  0.14915 [0.6993]
Testing beta = 0:   Chi^2(4)  =   85606. [0.0000]**

AR 1-2 test:      F(2,93)   =  0.70988 [0.4943]
ARCH 1-1 test:    F(1,98)   =  0.24801 [0.6196]
Normality test:   Chi^2(2)  =   2.7876 [0.2481]
Hetero test:      F(8,91)   =  0.68729 [0.7017]
Hetero-X test:    F(14,85)  =   1.1145 [0.3574]
```

Table 3: Dickey-Fuller tests of unit-root.

```
Unit-root tests
The dataset is:Postponed_QB_d.in7
The sample is: 4 - 101 (101 observations and 5 variables)

w: ADF tests (T=98, Constant; 5%=-2.89 1%=-3.50)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob        AIC  F-prob
 2    -0.4456       0.99580     1.267     0.8020  0.4246     0.5132
 1    -0.3367       0.99686     1.265      3.588  0.0005     0.4996  0.4246
 0     0.1189        1.0012     1.340                        0.6063  0.0018

q: ADF tests (T=98, Constant; 5%=-2.89 1%=-3.50)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob        AIC  F-prob
 2    -0.4638       0.99670    0.7687     0.9848  0.3272    -0.4862
 1    -0.2956       0.99793    0.7685      5.967  0.0000    -0.4964  0.3272
 0     0.5927        1.0048    0.8964                       -0.1985  0.0000

p: ADF tests (T=98, Constant; 5%=-2.89 1%=-3.50)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob        AIC  F-prob
 2    -0.4547       0.99527    0.9760      1.248  0.2150 -0.008677
 1    -0.2772       0.99714    0.9788      3.448  0.0008  -0.01264  0.2150
 0     0.2058        1.0022     1.033                       0.08487  0.0018

u: ADF tests (T=98, Constant; 5%=-2.89 1%=-3.50)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob        AIC  F-prob
 2     -2.550       0.81146    0.9150     -1.371  0.1735    -0.1376
 1     -3.122*      0.77974    0.9193    -0.1256  0.9003    -0.1382  0.1735
 0     -3.392*      0.77667    0.9145                       -0.1585  0.3911

z: ADF tests (T=98, Constant; 5%=-2.89 1%=-3.50)
D-lag    t-adf      beta Y_1    sigma   t-DY_lag  t-prob        AIC  F-prob
 2    -0.9085       0.98174     1.490      1.035  0.3033     0.8377
 1    -0.7803       0.98445     1.491      1.299  0.1972     0.8286  0.3033
 0    -0.6045       0.98803     1.496                        0.8258  0.2568
```

Table 4: Estimation results for a model of w_t

```
EQ(3) Modelling w by OLS
      The dataset is: Postponed_QB_d.in7
      The estimation sample is: 2 - 101

                  Coefficient  Std.Error  t-value  t-prob
Constant             -30.4086      1.629    -18.7  0.0000
z                     1.53420    0.06069     25.3  0.0000
u                    -2.34139     0.3313    -7.07  0.0000

sigma                 4.47054  RSS                1938.61701
R^2                  0.899014  F(2,97) =      431.8 [0.000]**
Adj.R^2              0.896932  log-likelihood       -290.122
no. of observations       100  no. of parameters           3
mean(w)              -22.0388  se(w)                 13.9251

AR 1-2 test:       F(2,95)    =   95.783 [0.0000]**
ARCH 1-1 test:     F(1,98)    =   30.999 [0.0000]**
Normality test:    Chi^2(2)   =   6.9785 [0.0305]*
Hetero test:       F(4,95)    =   2.6189 [0.0397]*
Hetero-X test:     F(5,94)    =   2.0759 [0.0752]
```

Table 5: Estimation results for a model of Dw_t

```
EQ(4) Modelling Dw by OLS
      The dataset is: Postponed_QB_d.in7
      The estimation sample is: 2 - 101

                 Coefficient  Std.Error  t-value  t-prob
Constant            -0.590574     0.7121   -0.829  0.4089
w_1                 -0.105831    0.01889    -5.60  0.0000
z                    0.137667    0.03199     4.30  0.0000
u                   -0.806881    0.07488    -10.8  0.0000

sigma                0.910926  RSS                79.6595231
R^2                  0.550122  F(3,96) =     39.13 [0.000]**
Adj.R^2              0.536063  log-likelihood       -130.523
no. of observations       100  no. of parameters           4
mean(Dw)             0.306275  se(Dw)                1.33738

AR 1-2 test:      F(2,94)   =   4.5560 [0.0129]*
ARCH 1-1 test:    F(1,98)   =   1.5151 [0.2213]
Normality test:   Chi^2(2)  =   4.1852 [0.1234]
Hetero test:      F(6,93)   =   1.0033 [0.4281]
Hetero-X test:    F(9,90)   =   1.6465 [0.1141]
```

Table 6: Facsimile from article by Ericsson and MacKinnon.

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Table 3. Response surface estimates for critical values of the ECM test of cointegration $\kappa_c(k)$: with a constant term.

k	Size (%)	θ_∞	(s.e.)	θ_1	θ_2	θ_3	$\hat{\sigma}$
1	1	−3.4307	(0.0006)	−6.52	−4.7	−10	0.00790
	5	−2.8617	(0.0003)	−2.81	−3.2	37	0.00431
	10	−2.5668	(0.0003)	−1.56	2.1	−29	0.00332
2	1	−3.7948	(0.0006)	−7.87	−3.6	−28	0.00847
	5	−3.2145	(0.0003)	−3.21	−2.0	17	0.00438
	10	−2.9083	(0.0002)	−1.55	1.9	−25	0.00338
3	1	−4.0947	(0.0005)	−8.59	−2.0	−65	0.00857
	5	−3.5057	(0.0003)	−3.27	1.1	−34	0.00462
	10	−3.1924	(0.0002)	−1.23	2.1	−39	0.00364
	1	−4.3555	(0.0006)	8.90	−6.7	−31	[illegible]