ECON 4160: Econometrics: Modelling and Systems Estimation

Question A (40 %)

Consider the ADL-model equation:

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \beta_0 X_t + \beta_1 X_{t-1} + \epsilon_t, t = 1, 2, ..., T, \tag{1}$$

where X_t is a stationary time series which is integrated of order zero, $X_t \sim I(0)$. ϵ_t denotes an error-term which is white-noise and linearly uncorrelated with X_t, X_{t-1}, Y_{t-1}, hence $E(\epsilon_t \mid X_t, X_{t-1}, Y_{t-1}) = 0$.

1. Explain how the following model equations can be obtained as special cases of (1):
   (a) AR model.
   (b) Distributed lag model.
   (c) Static model.
   (d) Model in differences.
2. Show how (1) can be written by the use of lag-operator notation.
3. Assume coefficients $\phi_1 = 0.5$, $\beta_0 = 0.8$, $\beta_1 = -0.3$ in (1). Calculate the impact multiplier and the three first dynamic multipliers of the dependent variable with respect to a one-period unit-change in the explanatory variable (an impulse of $+1$ to X).
4. (a) Explain what we mean by the long-run multiplier of Y with respect to X.
   (b) Show that for equation (1) and the coefficients given in question A3, the long-run multiplier is $+1$.
5. Explain why the validity of the above calculations of multipliers depends on X_t being a strongly exogenous variable.
6. Assume that Y_t and X_t are jointly generated by a VAR with first order dynamics and with error-terms distributed as:

   $$\begin{pmatrix} \varepsilon_{yt} \\ \varepsilon_{xt} \end{pmatrix} \sim IIN(0, \mathbf{\Sigma}),$$

   where the matrix $\mathbf{\Sigma}$ has the two variances and the covariance of the error-terms as elements.

   (a) Why is (1) the conditional model equation of Y_t given X_t?
   (b) Are the OLS estimators of the coefficients of (1) consistent estimators? Explain your answer.

Question B (60 %)

1. In Table 1 you find results for ADF tests for three variables that are relevant for modelling wage-price dynamics in the US economy:

   LW: The natural logarithm of compensation per hour worked (in the non-agricultural business sector of the economy)

   DLW: The one-period change in LW (also called the first-difference of LW, ie., DLW = LW - LW_1, where LW_1 denotes the lagged LW.

   1/U: The reciprocal of the unemployment rate in the U.S. economy.

   The time series are quarterly.
   Explain why, by using the information in the table, it is reasonable to base modelling on the assumptions that LW is an I(1) variable while 1/U is an I(0) variable.

2. In the following we make use of three additional variables:

   PCE: Deflator of private consumption expenditure (consumer price index).

   Q: Deflator of value added in the non-agricultural business sector of the economy (producer price index).

   Z: Value added (in real terms) per hour worked in the non-agricultural business sector of the economy (productivity).

   You can take as given that the natural logarithms of these variables, LPCE, LW and LZ, are I(1) variables.

   There is a tradition for estimation of Wage Phillips Curve Models (W-PCM) on US time series data. Table 2 shows results for a W-PCM. They are reported with a single mis-specification test. This is to save space, you can take as given that none of the omitted standard mis-specification test indicate statistical residual mis-specification.

   (a) Interpret the AR 1-5 test and explain its importance for the consistency of the OLS estimators of the coefficients, and for the validity of using the standard errors in the judgement of the coefficients statistical significance.

   (b) Explain why the results in Table 2 support a downward sloping wage Phillips curve.

   (c) The restricted model in Table 3 imposes two restrictions on the model in Table 2. Use information found in the two tables to calculate the following test statistic:

   $$F(2, 204) = 0.4,$$

   when one decimal point is used. The p-value is 0.6 (which you do not need to show). What is your conclusion about the statistical validity of the restrictions implied by the model in Table 3?

   (d) What is the value of the Likelihood-Ratio (LR) test statistic in this case? Which distribution would you use to judge the statistical significance of the LR-test?

3. Use the results in Table 4, together with the critical values of the ECM-test of cointegration found in Table 7, to test the null hypothesis that LW is not cointegrated with LZ and LQ at a conventional level of significance.

4. Conditional on cointegration:

   (a) What is the estimated long-run relationship for the wage variable LW?

   (b) Can you give an economic interpretation of the coefficients of the estimated equation?

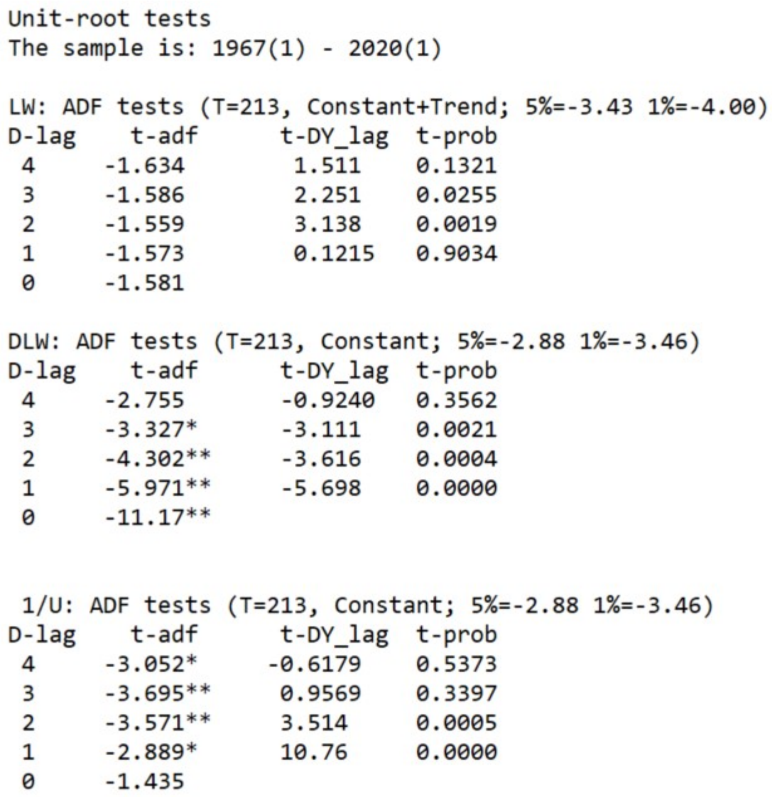
5. The ECM-test of cointegration used above assumes that LQ and LZ are weakly exogenous with respect to the cointegration coefficients. Can you explain in words the meaning of this assumption?

6. Table 5 contains results for the Johansen-method to cointegration analysis. The results were based on the estimation of a VAR with four endogenous variables: LW, LZ, LQ and (1/U). The VAR had second order dynamics, and it was not mis-specified. Explain why the output supports that the number of cointegration vectors (r) can be set to $r = 2$.

7. Table 6 shows the two vectors with cointegration coefficients (`beta` in the results).
   (a) Explain why the two vectors (and hence the two long-run relationships between the variables) are not identified without making further assumptions.
   (b) Given that the first vector can be identified, comment on the differences and similarities between this long-run relationship, and the long-run relationship that was estimated above (in QB4) by using the ECM-method to cointegration.

Tables with estimation results and facsimile of table with critical values for ECM-test

Table 1: Dickey-Fuller tests of unit-root. LW_t, DLW_t and $1/U_t$.



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Unit-root tests
The sample is: 1967(1) - 2020(1)

LW: ADF tests (T=213, Constant+Trend; 5%=-3.43 1%=-4.00)
D-lag    t-adf      t-DY_lag  t-prob
 4     -1.634        1.511    0.1321
 3     -1.586        2.251    0.0255
 2     -1.559        3.138    0.0019
 1     -1.573        0.1215   0.9034
 0     -1.581

DLW: ADF tests (T=213, Constant; 5%=-2.88 1%=-3.46)
D-lag    t-adf      t-DY_lag  t-prob
 4     -2.755       -0.9240   0.3562
 3     -3.327*      -3.111    0.0021
 2     -4.302**     -3.616    0.0004
 1     -5.971**     -5.698    0.0000
 0     -11.17**


 1/U: ADF tests (T=213, Constant; 5%=-2.88 1%=-3.46)
D-lag    t-adf      t-DY_lag  t-prob
 4     -3.052*     -0.6179    0.5373
 3     -3.695**     0.9569    0.3397
 2     -3.571**     3.514     0.0005
 1     -2.889*      10.76     0.0000
 0     -1.435
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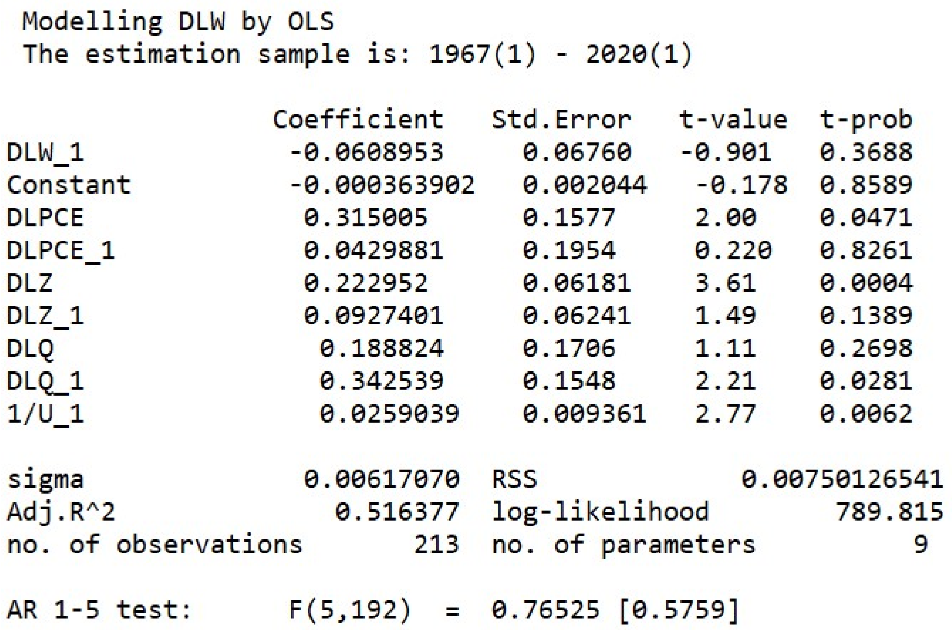
Table 2: Estimation results for a W-PCM.



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 Modelling DLW by OLS
 The estimation sample is: 1967(1) - 2020(1)

                Coefficient   Std.Error   t-value  t-prob
DLW_1            -0.0608953    0.06760    -0.901   0.3688
Constant         -0.000363902  0.002044    -0.178  0.8589
DLPCE             0.315005     0.1577      2.00    0.0471
DLPCE_1           0.0429881    0.1954      0.220   0.8261
DLZ               0.222952     0.06181     3.61    0.0004
DLZ_1             0.0927401    0.06241     1.49    0.1389
DLQ                0.188824    0.1706      1.11    0.2698
DLQ_1              0.342539    0.1548      2.21    0.0281
1/U_1              0.0259039   0.009361    2.77    0.0062

sigma             0.00617070  RSS              0.00750126541
Adj.R^2             0.516377  log-likelihood         789.815
no. of observations      213  no. of parameters            9

AR 1-5 test:      F(5,192)  =  0.76525 [0.5759]
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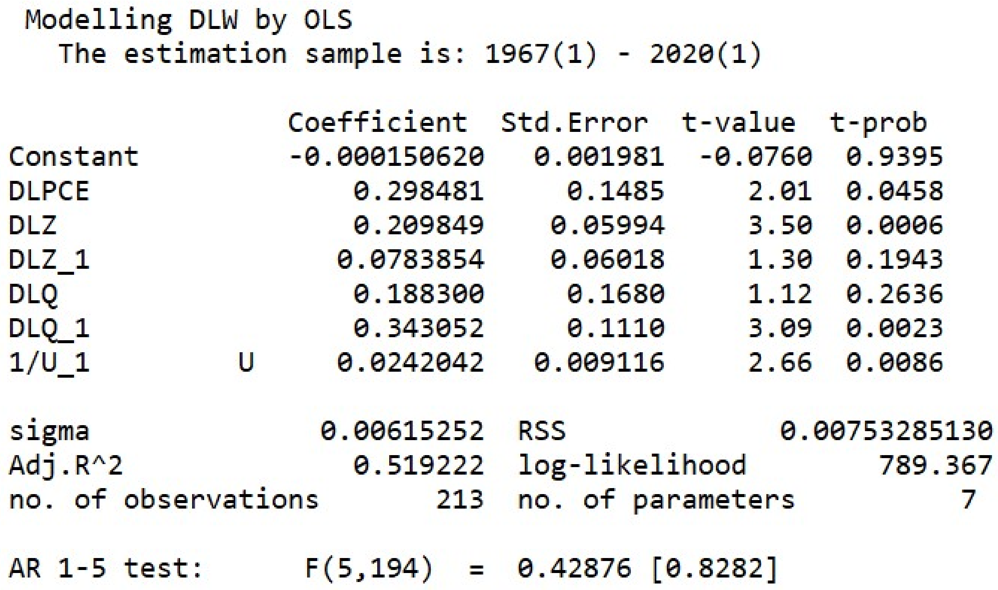
Table 3: Estimation results for a simplified W-PCM.



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 Modelling DLW by OLS
   The estimation sample is: 1967(1) - 2020(1)

                 Coefficient  Std.Error  t-value  t-prob
Constant        -0.000150620   0.001981  -0.0760  0.9395
DLPCE               0.298481     0.1485     2.01  0.0458
DLZ                 0.209849    0.05994     3.50  0.0006
DLZ_1              0.0783854    0.06018     1.30  0.1943
DLQ                 0.188300     0.1680     1.12  0.2636
DLQ_1               0.343052     0.1110     3.09  0.0023
1/U_1          U   0.0242042   0.009116     2.66  0.0086

sigma             0.00615252  RSS             0.00753285130
Adj.R^2             0.519222  log-likelihood        789.367
no. of observations      213  no. of parameters           7

AR 1-5 test:      F(5,194)  =  0.42876 [0.8282]
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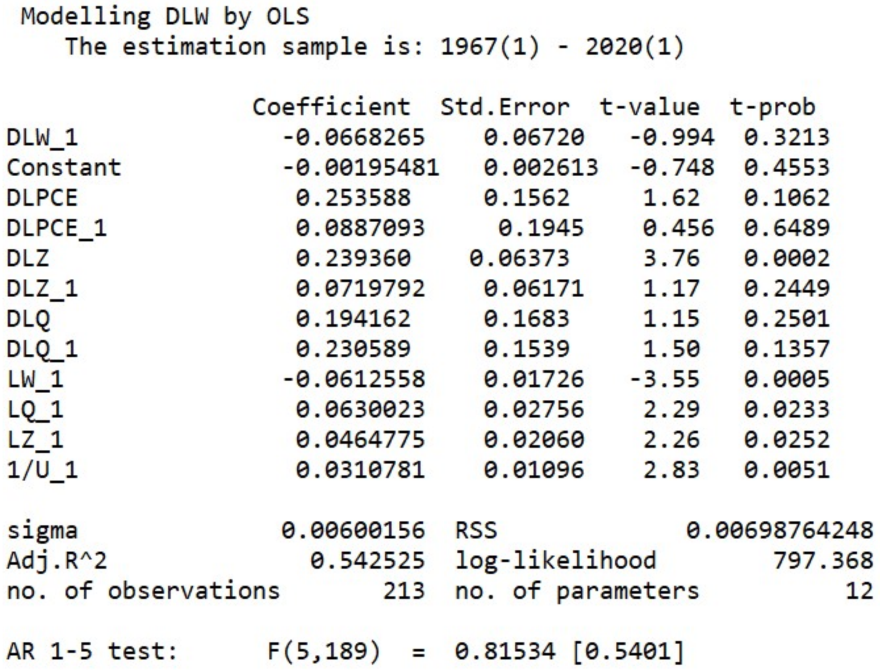
Table 4: Estimation results for a ECM equation for wages.



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 Modelling DLW by OLS
    The estimation sample is: 1967(1) - 2020(1)

                 Coefficient  Std.Error  t-value  t-prob
DLW_1              -0.0668265    0.06720   -0.994  0.3213
Constant           -0.00195481   0.002613  -0.748  0.4553
DLPCE               0.253588     0.1562     1.62   0.1062
DLPCE_1             0.0887093     0.1945    0.456  0.6489
DLZ                 0.239360    0.06373     3.76   0.0002
DLZ_1               0.0719792    0.06171    1.17   0.2449
DLQ                 0.194162     0.1683     1.15   0.2501
DLQ_1               0.230589     0.1539     1.50   0.1357
LW_1               -0.0612558    0.01726   -3.55   0.0005
LQ_1                0.0630023    0.02756    2.29   0.0233
LZ_1                0.0464775    0.02060    2.26   0.0252
1/U_1               0.0310781    0.01096    2.83   0.0051

sigma              0.00600156  RSS            0.00698764248
Adj.R^2              0.542525  log-likelihood       797.368
no. of observations       213  no. of parameters         12

AR 1-5 test:      F(5,189)  =  0.81534 [0.5401]
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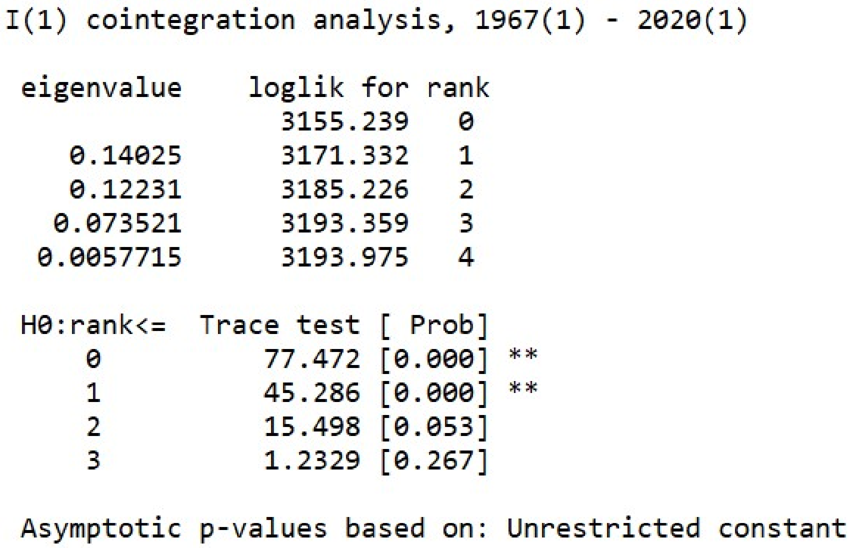
Table 5: Johansen method: Cointegration rank test results.



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I(1) cointegration analysis, 1967(1) - 2020(1)

 eigenvalue    loglik for rank
                 3155.239   0
   0.14025       3171.332   1
   0.12231       3185.226   2
  0.073521       3193.359   3
 0.0057715       3193.975   4

 H0:rank<=  Trace test [ Prob]
     0          77.472 [0.000] **
     1          45.286 [0.000] **
     2          15.498 [0.053]
     3          1.2329 [0.267]

 Asymptotic p-values based on: Unrestricted constant
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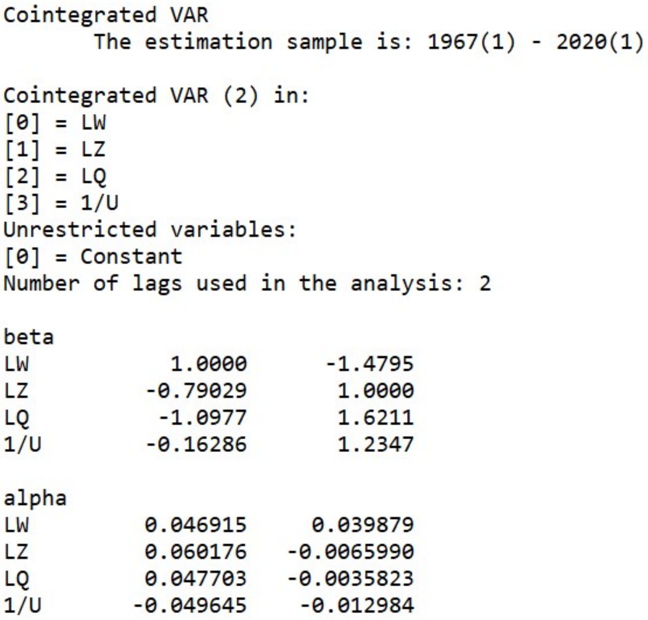
Table 6: Johansen method: Estimated $\boldsymbol{\beta}$-vectors.



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Cointegrated VAR
       The estimation sample is: 1967(1) - 2020(1)

Cointegrated VAR (2) in:
[0] = LW
[1] = LZ
[2] = LQ
[3] = 1/U
Unrestricted variables:
[0] = Constant
Number of lags used in the analysis: 2

beta
LW           1.0000      -1.4795
LZ         -0.79029       1.0000
LQ          -1.0977       1.6211
1/U        -0.16286       1.2347

alpha
LW         0.046915     0.039879
LZ         0.060176   -0.0065990
LQ         0.047703   -0.0035823
1/U       -0.049645    -0.012984
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Table 7: Facsimile from article by Ericsson and MacKinnon.

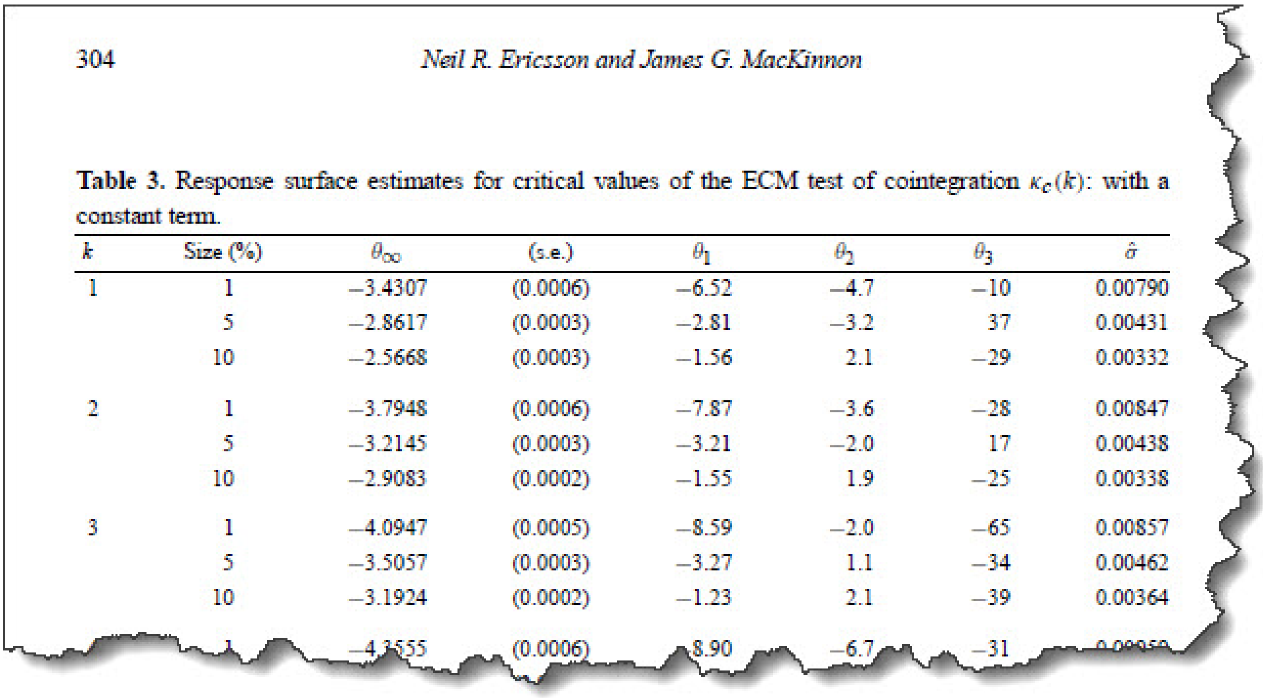

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Table 3. Response surface estimates for critical values of the ECM test of cointegration $\kappa_c(k)$: with a constant term.

k	Size (%)	θ_∞	(s.e.)	θ_1	θ_2	θ_3	$\hat{\sigma}$
1	1	–3.4307	(0.0006)	–6.52	–4.7	–10	0.00790
	5	–2.8617	(0.0003)	–2.81	–3.2	37	0.00431
	10	–2.5668	(0.0003)	–1.56	2.1	–29	0.00332
2	1	–3.7948	(0.0006)	–7.87	–3.6	–28	0.00847
	5	–3.2145	(0.0003)	–3.21	–2.0	17	0.00438
	10	–2.9083	(0.0002)	–1.55	1.9	–25	0.00338
3	1	–4.0947	(0.0005)	–8.59	–2.0	–65	0.00857
	5	–3.5057	(0.0003)	–3.27	1.1	–34	0.00462
	10	–3.1924	(0.0002)	–1.23	2.1	–39	0.00364
	1	–4.3555	(0.0006)	8.90	–6.7	–31	[illegible]