UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: ECON4240 – Game theory and economics of information, spring 2005

Date of exam: Tuesday, May 31, 2005 ***Grades will be announced: Wednesday, June 15***

Time for exam: 9:00 a.m. – 12:00 noon

The problem set covers 3 pages

Resources allowed:
- No resources allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

The exam consists of four problems. They count as indicated. Start by reading through the whole exam, and make sure that you allocate time to answering questions you find easy. You can get a good grade even if there are parts of problems that you do not have time to solve.

Problem 1 (20 %)
True or false? For each of the statements, if true, try to explain why, and if false, provide a counter-example.

a) If a finite normal-form game has more than one Nash equilibrium, then one of these Nash equilibria cannot Pareto dominate all the other Nash equilibria.
b) If a finite normal-form game has only one rationalizable strategy for each player, then there cannot be more than one Nash equilibrium.
c) If a finite extensive-form game of perfect information has more than one subgame perfect Nash equilibrium, then at least one player is indifferent between two or more terminal nodes.
d) If a finitely repeated game has a stage game with more than one equilibrium, then there exists a unique subgame perfect Nash equilibrium of the finitely repeated game.
e) If an infinitely repeated game has a stage game with a unique Nash equilibrium, then there exists a unique subgame perfect Nash equilibrium of the infinitely repeated game.

Problem 2 (30 %)

Consider a strategic situation between an *employer* (E) and a *worker* (W). E can either *accept* (A) or *reject* (R) W. W can either become *skilled* (S) through education, or remain *unskilled* (U). W can be of two types; either he is inherently *high ability* (H) or he is inherently *low ability* (L). The players' payoffs depending on their actions and W's type is shown below.

H

	S	U
A	2, 3	-1, 2
R	0, 1	0, 0

L

	S	U
A	-1,-1	-3,-2
R	0, -3	0, 0

a) For each of these games, determine the set of (pure) rationalizable strategies for each player, and the set of pure-strategy Nash equilibria.

b) Assume next that only W knows his own type, while player E thinks that the two types of W are equally likely. Model this situation in an ex ante perspective by specifying the Bayesian normal form.

c) For the Bayesian normal form found in part (b), determine the set of (pure) rationalizable strategies for each player, and the set of pure-strategy and/or mixed-strategy Nash equilibria.

Problem 3 (20 %)

Consider again the strategic situation between an *employer* (E) and a *worker* (W) described in Problem 2. Assume (as in parts b and c) of Problem 2) that only W knows his own type, while player E thinks that the two types of W are equally likely.

a) (*Screening*) Assume now that E acts before W, and that E's choice of A or R can be observed by W before he makes his choice of S or U. Show that there is a unique subgame perfect Nash equilibrium.

b) (*Signaling*) Assume now that W acts before E, and that W's choice of S or U can be observed by E before she makes her choice of A or R. Show that there is a unique perfect Bayesian equilibrium.

Problem 4 (30 %)

Consider a relationship between a principal and an agent in which the value of the product, denoted x, can be either $x_1 = 50{,}000$ or $x_2 = 25{,}000$. The agent's effort, denoted e, takes on one of three possible values, $e_1 = 5$, $e_2 = 20$, or $e_3 = 40$. The probability of the good result, x_1, is 0.25 for effort level e_1, 0.5 for effort level e_2, and 0.75 for effort level e_3.

Assume that the principal is risk neutral, so that the principal's utility is given by x - t, where t is the transfer from the principal to the agent. The agent is risk averse with utility function

$$U(t,e) = \sqrt{t} - e$$

The agent has a reservation utility of 120.

(a) Formulate the optimal contract under symmetric information for each level of effort. What is the principal's expected profit in each case? Which effort level does the principal prefer?

(b) Formulate the optimal contract when there is a moral hazard problem. As usual, we assume that the principal has all bargaining power an formulates the contract. Which effort level will be induced? How does the moral hazard problem have its influence?

(c) Discuss briefly what might happen if both sides have some bargaining power.

In order that you not use time on trivial calculations, you can use the following:

$100^2 = 10,000$

$110^2 = 12,100$

$125^2 = 15,625$

$140^2 = 19,600$

$160^2 = 25,600$

$170^2 = 28,900$

$180^2 = 32,400$

$0.25 \cdot 50,000 + 0.75 \cdot 25,000 = 31,250$

$0.50 \cdot 50,000 + 0.50 \cdot 25,000 = 37,500$

$0.75 \cdot 50,000 + 0.25 \cdot 25,000 = 43,750$

$0.50 \cdot 28,900 + 0.50 \cdot 12,100 = 20,500$

$0.75 \cdot 32,400 + 0.25 \cdot 10,000 = 26,800$