#2 The P–A model
– investment feasibility revisited

A risk-neutral principal wishes to

$$\max \underbrace{E[\, S(q) - t \,]}_{}$$

$$= \sum_i (S(q_i) - t_i) \underbrace{\Pr[\text{menu item } \#i \text{ is chosen}]}_{?}$$

For this to match

$$\sum (S(q_i) - t_i) \Pr[\text{agent is type } i]$$

we must arrange the menu items so that

<u>type i chooses menu item $\#i$</u>

I.e:

Type i prefers a_i to a_{1-i} , $i=0,1$
and to $(0,0)$, $i=0,1$

4 constraints
(+ two nonnegativity constraints $q_i \geq 0$)

This is only a * restriction on or "bookkeeping" ?

→ Consider any menu $\{b_1, b_2, b_3, \ldots\}$.

→ Let $a_i = \begin{cases} \text{the item chosen by type } i \\ \text{(if contract accepted)} \\ \text{or } (0,0) \text{ (if not accepted).} \end{cases}$

→ Then $\{a_0, a_1\}$ is an incentive feasible contract with the same outcome.

P's problem: If $\nu = Pr[A \text{ is type } 0]$

$$\max_{\{(q_0, t_0), (q_1, t_1)\}} \left[(S(q_0) - t_0)\, \nu + (S(q_1) - t_1)(1 - \nu) \right]$$

subject to

→ inc. feas. (4 constraints)

→ $q_i \geq 0$, both i (2 constraints)

Six constraints (... $t_i \geq 0$ will be automatically satisfied ...)

→ $2^6 = 32$ possible combinations of active/inactive constraints

Easy to eliminate quite a few:

* The constraints: Claim

Participation:

$$t_0 - C_0(q_0) \geq 0$$

$$t_1 - C_1(q_1) \geq 0$$

can be dropped (holds automatically)

active: $= 0$

Compatibility

$$t_0 - C_0(q_0) \geq t_1 - C_0(q_1)$$ active. $=$

$$t_1 - C_1(q_1) \geq t_0 - C_1(q_0)$$ simplifies to $\rightarrow q_0 \geq q_1$ under Spence-Mirrlees (to be made precise)

Positivity

$$q_0 \geq 0, \quad q_1 \geq 0$$ slightly modified by

... then yields 4 cases: $\{(0,0),(0,0)\}$, shutdown of type 1, pooling, and interior solution

Arguments to follow:

We have:

$$\underbrace{t_0 - C_0(q_0)}_{\substack{\text{type 0's profit} \\ =: U_0}} \geq t_1 - C_0(q_1) \qquad \text{(inv. comp)}$$

$$= \underbrace{t_1 - C_1(q_1)}_{\substack{\text{type 1's profit} \\ =: U_1}} + \underbrace{\left[C_1 - C_0\right]_{q=q_1}}_{=: \Delta C(q_1) \geq 0}$$

$$U_0 \geq U_1 + \Delta C(q_1)$$

so $U_0 \geq 0$ follows automatically. ✓

Furthermore: If $U_0 \geq U_1 > 0$, then reduce both t_i by U_1;
Then inv. comp is unaffected, profits are still nonnegative, the q_i the same – and P better off! So $\underline{\underline{U_1 = 0}}$ ✓

Similarly: $U_0 = U_1 + \Delta C(q_1)$ ✓

$$= \Delta C(q_1)$$

(otherwise: offer less transfer, constraints unaffected.)

Last claim now reads

$$C_1(q_0) \geq t_0 \qquad (\geq !)$$

Now $\quad t_0 = C_0(q_0) + \Delta C(q_1)$

So

$$C_1(q_0) - C_0(q_0) \geq \Delta C(q_1)$$

$$\Delta C(q_0) \geq \Delta C(q_1)$$

In the linear case, we saw last time: $\Delta C = (\theta_1 - \theta_0)\, q$.

$$\cancel{(\theta_1 - \theta_0)}\, q_0 \geq \cancel{(\theta_1 - \theta_0)}\, q_1$$

Then ~~...~~ feasibility holds if $q_0 \geq q_1$

(*) The more general Spence - Mirrlees condition $\dfrac{d C_1}{d q} \geq \dfrac{d C_0}{d q}$ grants the same. Interpret!

the book's $C''_{\theta q}$ ...

Even more generally,

the Constant sign condition

$$\frac{d}{dq}\left(C_1 - C_0\right) \begin{cases} \geq 0 & \text{always} \\ & \text{or} \\ \leq 0 & \text{always} \end{cases}$$

gives a similar feasibility condition: in the $\leq$ case, it suffices that [redacted] $q_0 \leq q_1$ (rather than $\geq$)

See Remark @ end of 2.10.1.

The constraints boil down to:

$$t_1 = C_1(q_1) \qquad \text{(type 1 breaks eve}$$

$$t_0 = C_0(q_0) + \underbrace{\Delta C(q_1)}_{= U_0,\ \text{type 0's profit}}$$

and under Spence-Mirrlees:

$$q_0 \underset{\text{S-M}}{\geq} q_1 \underbrace{\geq 0}_{\text{from the setup.}}$$

~~Then~~

These latter ineq's give 4 cases:

$q_0 = q_1 = 0$ (no trade)

$q_0 > q_1 = 0$ (shutdown)

$q_0 = q_1 > 0$ (pooling)

$q_0 > q_1 > 0$

... as claimed.

Rewrite the problem:

→ Insert for t_i from eq. constraints

→ Gather q_0-terms and q_1-terms separately

Problem becomes

$$\max_{q_0, q_1} \Big\{ (S(q_0) - C_0(q_0))\nu + (S(q_1) - C_1(q_1))(1-\nu) - \nu \Delta C(q_1) \Big\}$$

subject to

$q_0 \geq q_1 \geq 0$ under Spence-Mirrlees

max wrt. q_0:

→ $q_0 = 0$ if $S'(0) \leq C_0'(0)$
(recall $S - C_0$ concave by assumption)
yields $q_1 = 0$ too.

→ If $S'(0) > C_0'(0)$:

$$S'(q_0) = C_0'(q_0)$$

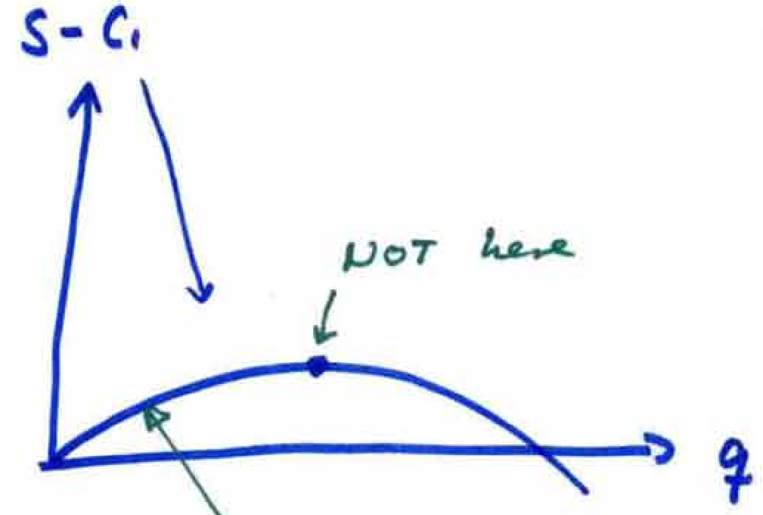
Both as in the full information case!

What about q_1?

1^{st} o.c.:

linear case: $\theta_1 - \theta_0$

$$S'(q_1) - C_1'(q_1) = \frac{\nu}{1-\nu} \cdot \underbrace{\frac{d(C_1 - C_0)}{dq}(q_1)}_{\geq 0 \text{ under S-M.}}$$



but somewhere here?

Reduced q_1 under S-M

Or: Maybe 0.

If $S'(0) - C_1'(0) - \frac{\nu}{1-\nu}\left(C_1'(0) - C_0'(0)\right) \leq 0$

then choose $q_1 = 0$.

Note: the criterion in the full information case is

$S'(0) - C_1'(0) \leq 0$.

Generalized marginal cost

Interpreting marginal utility

$S'(q_i)$ as a "price",

we get for type 1:

$$p = \theta_1 + \underbrace{\frac{\nu}{1-\nu}\underbrace{\Delta\theta}_{=\theta_1-\theta_0}}_{\text{information cost}}$$

m.c. under full information

Compared to full information case,

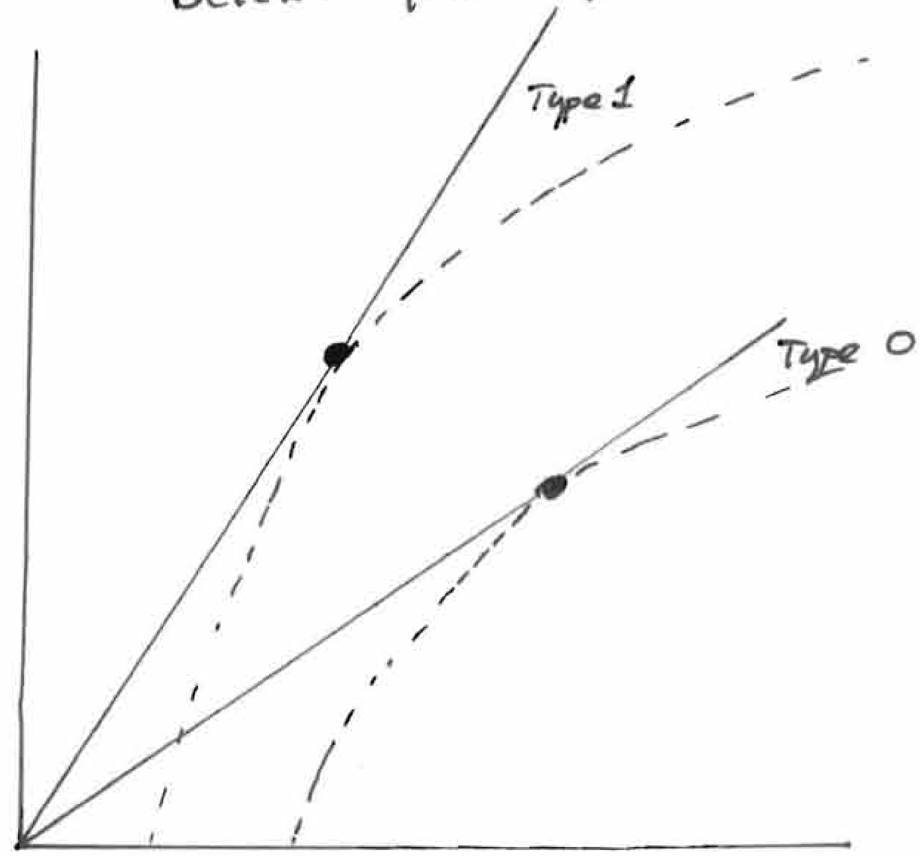
→ q_0 is kept constant,
but t_0 increased
(except under shutdown)

→ type 1's <u>profit</u> kept at 0
but – under Spence – Mirrlees –
q is reduced in order for
type 0 <u>not</u> to prefer (q_1, t_1)

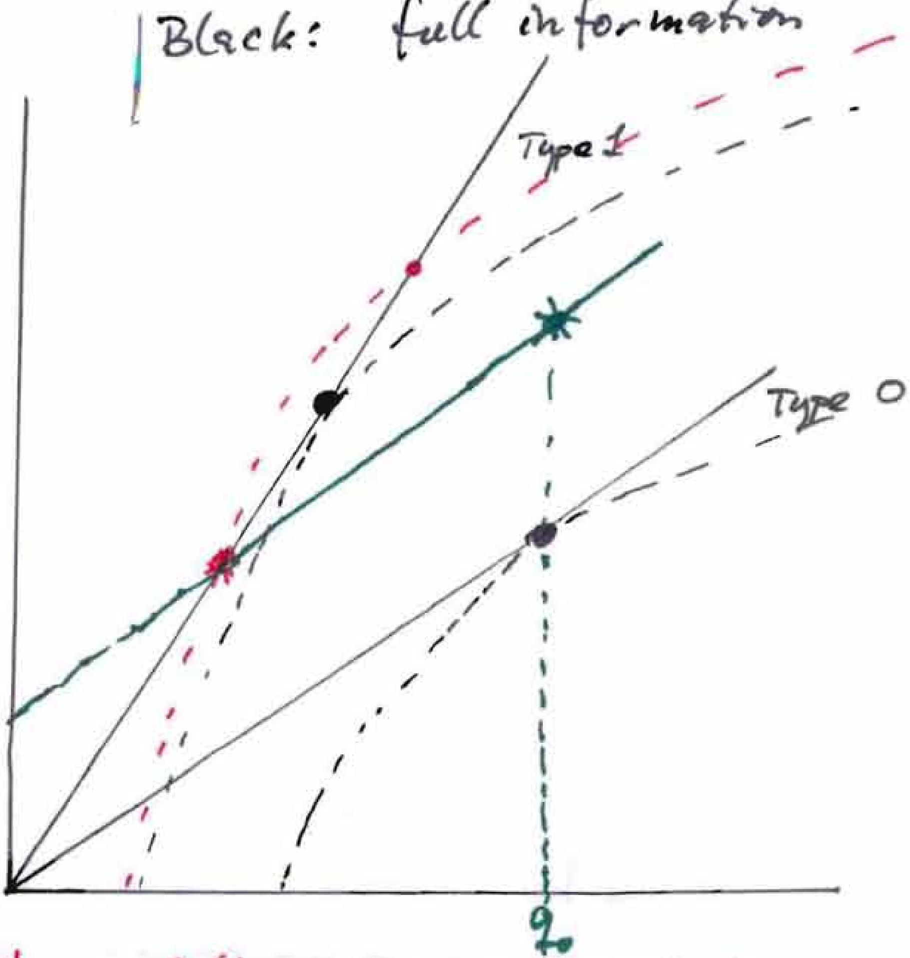
→ Total welfare <u>not</u> maximized
(except special cases)
⇝ policy implications for
authorities? Hardly! (Discuss?)

Sketch, case $C_i(q) = \theta_i\, q$

Black: full information



Sketch, case $C_i(q) = \theta_i q$



Red: P's indifference curve shifted up (bad) and (– by S-M –) we choose * not •

Green: The Type 0 curve(s) shifted up – parallel – until hits *.

The revelation principle

For our case, incentive feasibility will (truthfully!) reveal agent's type except when pooling.

More formally:

Let Θ be the set of possible types.

Def: A <u>direct revelation mechanism</u> is a mapping

$$g(\theta) = (q(\theta), t(\theta)), \quad \text{all } \theta \in \Theta$$

Interpretation: Reveal your type θ, and I'll offer the contract $g(\theta)$.

Def: ~~*~~ Such a g is <u>truthful</u> if it is incentive compatible for <u>any</u> agent to announce his/her true type.

Given a more general set $\mathcal{M}$
of messages, and a mechanism
of the form

"state to me an $m \in \mathcal{M}$
and I'll offer you the contract
$(\tilde{q}(m), \tilde{t}(m))$ "

– can we obtain results
we could not ~~by~~ obtain by
~~=~~ restricting to $\mathcal{M} = \Theta$?

<u>The revelation principle</u>:

The answer is "no".
Furthermore, <u>truthful</u> direct
revelation mechanisms suffice.

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So: just as incentive feasibility
only restricts "bookkeeping",
so does ~~a~~ this apparent restriction
of messages.