Department of Economics / NCF / 2010–03–13

ECON4240 spring 2010 – problems for seminar #4

Two problems, two pages.

Problem 1

Consider the following variant of the adverse selection model (the rent extraction-efficiency trade-off), with two types of agents:

- The agent ("A") produces an amount q of a good and receives a transfer t from the principal ("P").
- A is of one of two types, efficient or inefficient.
- The efficient agent's utility is $t - q^2$ while the inefficient agent's utility is $t - \theta q^2$, where $\theta > 1$. The agent's reservation utility is 0 regardless of type.
- The type is known to the agent. In the asymmetric information case, P only knows the *ex ante* probability of the agent being efficient, denoted p.
- The principal is risk neutral in profit, which is equal to $kq - t$ for a constant $k > 0$.
- A contract is a pair (t, q). The principal formulates a contract or a set of contracts; the agent accepts one of the proposed contracts or rejects them all.

(a) Formulate and solve the principal's problem if there is *perfect information* about the agent's type. Which contracts are offered and what is the principal's profit?

(b) Formulate the problem when an adverse selection problem is present (i.e. P only knows p).

(c) Characterize the solution to question (b). Which constraints are and which are not active (i.e. binding)?

(d) Compare the cases of symmetric and asymmetric information.

(e) Now assume $\theta = 2$, $p = ½$ and $k = 1$. Solve the asymmetric information problem explicitly in this case. Find and compare the principal's expected profit in the cases of symmetric and asymmetric information.

(f) If the principal only wants to contract with the efficient type of agent, which contract should then be offered? What is the expected profit for the parameter values of question (e)?

Problem 2

Consider a regulated monopoly that sells its product in an industry in which the consumers are of two different types.

- The firm's production costs are *public information*, and equal to $C(Q) = F + cQ$.
- The regulation establishes the amount of the payment that the consumer makes to the monopoly if a product is bought (T), the quantity that the monopoly can sell (Q), and the amount of the subsidy that the government transfers to the monopoly (S).
- There is a deadweight loss in the collection of taxes; hence the total social cost of the subsidy is $(1+g)S$, where $g > 0$.

(a) Assume that there is only one representative consumer, whose utility function is known to be $U(Q)$. Formulate the problem that the government must solve, bearing in mind that the monopolist's profits could be negative and that the consumer may decide not to buy. Characterize the optimal regulation policy.

(b) Assume that there is asymmetric information with respect to the consumers. Let $U(Q)$ and $V(Q)$ be the utility of consumers of type "G" and "B"from the consumption of Q units of the good, where $U(Q) > V(Q)$ and $U'(Q) > V'(Q)$. Let q be the proportion of type-G consumers. Formulate the problem that the regulator must solve (maximize social welfare under the conditions of participation of the different consumer types, incentive compatibility to reveal the true characteristic, and participation for the firm for both possible consumer types).

(c) In the above problem, it is easy to calculate the optimal levels for the subsidy in function of the other parameters of the problem (from the first-order conditions with respect to the subsidy). Hence formulate the problem only in terms of $\{(T_G, Q_G), (T_B, Q_B)\}$.

(d) Using the first-order conditions, prove that the incentive compatibility constraint of the consumer who least values the good does not bind. Show that the optimal contract must satisfy: $U'(Q) = c$ and $V'(Q) > c$. Discuss these results.