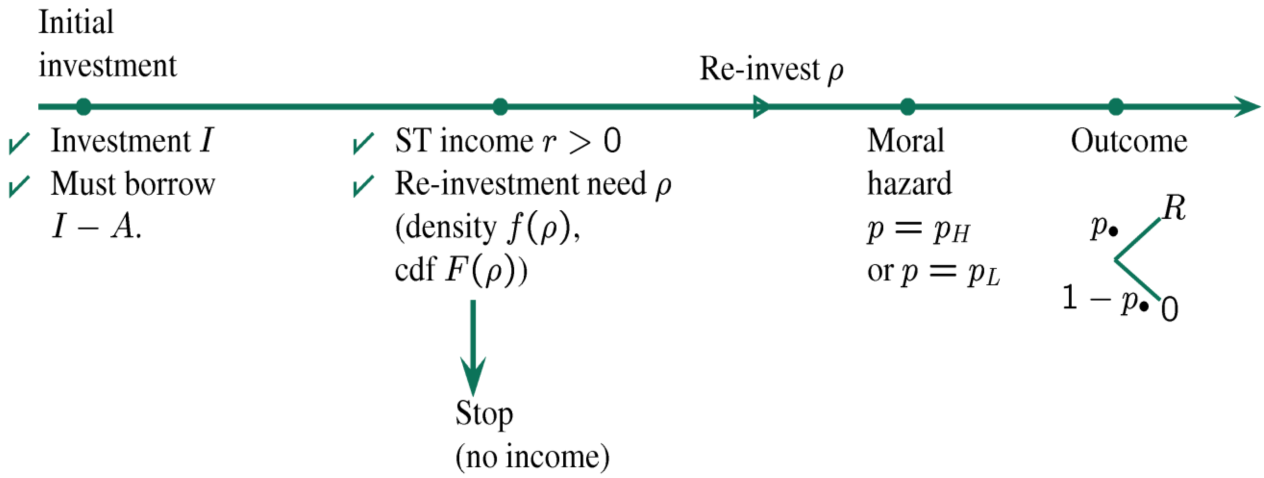
Liquidity management

- Multistage financing
- An intermediate date between the financing stage and the realization of the project outcome.
- Following up on the discussion of the liquidity/accountability tradeoff in chapter 4.
- The borrower needs to prepare for a liquidity shock.
- The borrower should hoard reserves.
    - Holding liquid securities
    - Credit line
    - Retensions
- Hoarding of reserves is an insurance mechanism
    - True even if borrower is risk neutral
    - Value of funds higher in bad states than in good states, because of credit rationing.
    - Borrower wants to transfer wealth from good states to bad states. This is what an insurance contract does.

Basic model

- Fixed investment, with a stochastic need for reinvestment at an intermediate date.



- Date 0: Investment I, own assets A, borrowing need $I - A$.
- Date 1 – the intermediate date:
  - Investment yields a short-term return r; deterministic and verifiable.
  - Continuation requires a *reinvestment* of size $\rho \geq 0$, *ex ante* unknown: probability distribution $F(\rho)$, density $f(\rho)$.
  - The value of ρ becomes known at date 1.
  - No reinvestment means liquidation of the firm, liquidation value 0.
- Date 2 – in case of reinvestment at date 1: Investment returns R if success, 0 if failure. Success probability p depends on borrower's effort: $p = p_H$ if she behaves, $p = p_L < p_H$ if not.
- Risk neutrality. Limited liability. Competition among lenders.
- Contract: $\{r_b, R_b, \rho^*\}$
  - r_b and R_b – what borrower receives at dates 1 and 2.
  - ρ^* – the cutoff reinvestment requirement: continue if and only if $\rho \leq \rho^*$.
- Borrower's net utility equals net present value of the project:

$$U_b(\rho^*) = [r + F(\rho^*)p_H R] - \left[I + \int_0^{\rho^*} \rho f(\rho) d\rho\right]$$

  - Second term: expected total investment
- Borrower's incentive constraint:

$$R_b \geq \frac{B}{\Delta p}$$

- Borrower receives 0 at date 1: $r_b = 0$.
  - All of r is paid out to outside investors.
  - Zero r_b increases R_b and alleviates the incentive problem at date 2.
- Expected pledgeable income:

$$\mathscr{P}(\rho^*) = r + F(\rho^*)p_H \left[R - \frac{B}{\Delta p}\right] - \int_0^{\rho^*} \rho f(\rho) d\rho$$

  - Investors must cover all the reinvestment

- NPV is maximized at $\rho^* = p_H R = \rho_1$.
    - $U_b'(\rho^*) = f(\rho^*)p_H R - \rho^* f(\rho^*)$.
    - For $\rho^* < \rho_1$, the expected gain from rescuing the project is larger than the cost.
- Pledgeable income is maximized at $\rho^* = p_H\left[R - \frac{B}{\Delta p}\right] = \rho_0$.
    - For $\rho^* > \rho_0$, the cost to the investors from continuing is larger than what they expect to get in return.

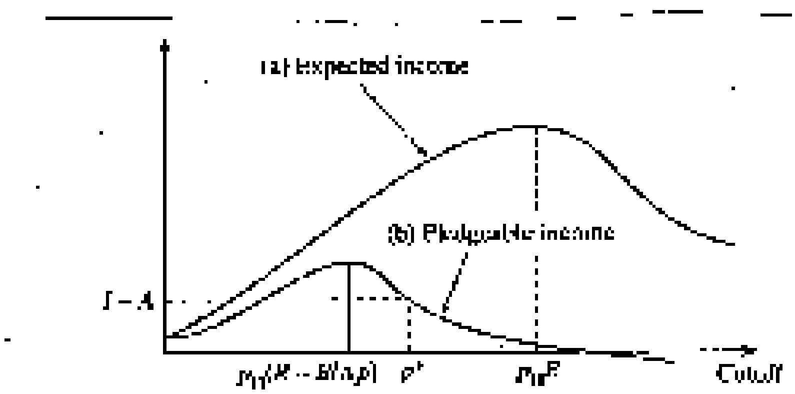


Figure 5.2, p. 204

- Three cases
    - Efficient cutoff: $\mathscr{P}(\rho_1) \geq I - A$.
        - The NPV-maximizing cutoff leaves enough for the investors: $\rho^* = \rho_1$.
    - Too much liquidation: $\mathscr{P}(\rho_1) < I - A \leq \mathscr{P}(\rho_0)$
        - $r_b = 0$, $R_b = B/\Delta p$, and

          $\rho^* \in [\rho_0, \rho_1)$ solves $\mathscr{P}(\rho) = I - A$
        - Credit rationing at date 1: In order to secure funds at date 0, the borrower accepts a reduced reinvestment cutoff at date 1.
    - No funding: $I - A > \mathscr{P}(\rho_0)$
        - Even maximizing pledgeable income is not enough.

Maturity at a cash rich firm

- *Cash rich firm*: $r > \rho^*$; high short-term returns.
- Implementing the optimal contract
  - Short-term debt: $d = r - \rho^*$.
  - Long-term debt: $D = R - \frac{B}{\Delta p}$ (to be paid if continuation)
- A theory of *maturity structure* of debt
  - Stronger firms have larger A, and subsequently (weakly) higher ρ^* and therefore less short-term debt.
  - The more current debt a firm has, the lower is its A, and the more short-term its future debt will be.
- Short-term debt vs dividend.

Credit lines for cash poor firms

- *Cash poor firm*: $r < \rho^*$. The extreme case: $r = 0$.
- With $r = 0$, there are no short-term returns to cover (in part) the liquidity needs at the intermediate date.
- Can a wait-and-see strategy work?
  - At date 1, the value of ρ is known. But the outside investors are not able to supply more funds than what the firm is worth to them, so the firm will only get funding if

$$\rho \le p_H\left[R - \frac{B}{\Delta p}\right] = \rho_0.$$

  - This is not optimal, since $\rho^* \in [\rho_0, \rho_1]$.
- It is better to *hoard reserves* at date 0 to face the liquidity shock at date 1.
  - *Liquidity management* is necessary.

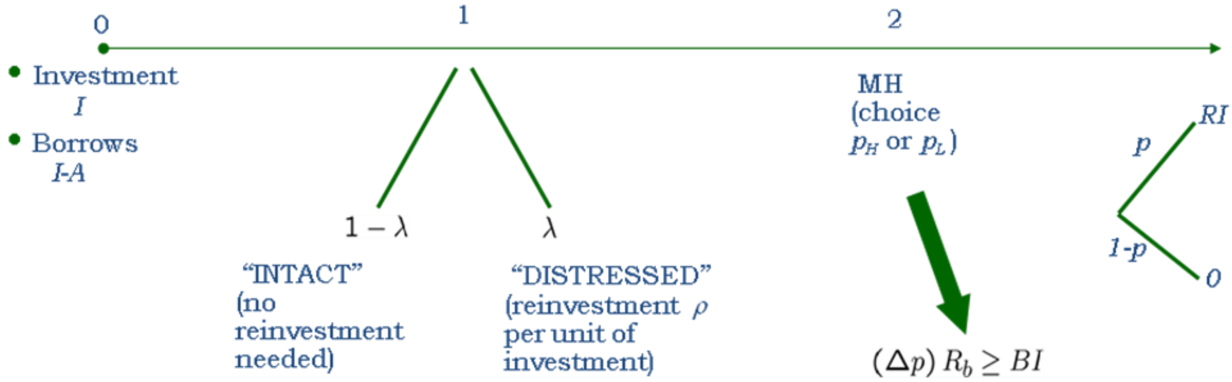
- Two ways to hoard reserves:
  - Borrowing $I + \rho^*$ at date 0, with a covenant that no further claims be issued at date 1, so that initial claimholders are not diluted.
  - Securing a *line of credit* equal to $\rho^* - \rho_0$, with a right to dilute initial claimholders in order to get ρ_0 in new funds at date 1.
    - A line of credit is an agreement providing credit up to a certain amount.
  - The line of credit must be *non-revokable*; otherwise, the lender would not want to abide with the agreement in cases where $\rho \in (\rho_0, \rho^*)$.

<u>Growth opportunities</u>

- An alternative scenario: if you do not reinvest at the intermediate date, you don't have to close down; but if you do reinvest, you increase the prospects of your project.
  - Reinvestment increases probabilities of success from p_H and p_L (depending on borrower efforts) to $p_H + \tau$ and $p_L + \tau$, where $0 < \tau < 1 - p_H$.
- Better growth opportunities (higher τ) call for longer maturities, that is, less short-term debt.

The liquidity-scale tradeoff

- Liquidity management with a variable investment.
- The entrepreneur now faces a choice between a larger investment and more liquidity.
- Variable-investment model.
- First a simple version – two values of the per-unit liquidity shock
    - 0, with probability $1 - \lambda$: the firm is *intact*.
    - ρ, with probability λ: the firm is *in distress*.



- Initial investment I. Continuation, which requires a reinvestment ρI if the firm is in distress at date 1, is subject to moral hazard.
- Project yields RI at date 2 if success, 0 otherwise.
- Success probability p_H or p_L.
- Private benefit from misbehaving BI.
- Assumption: $\rho_0 < c < \rho_1$, where $c \equiv \min\left\{1+\lambda\rho, \frac{1}{1-\lambda}\right\}$.
    - No liquidity shock: $\lambda = 0$, and so $c = 1$.
- Borrower receives R_b if success, 0 otherwise, where $R_b \geq \frac{B}{\Delta p}$.
- If distress: abandon or pursue the project?

- Abandon project if distress
  - Investors' breakeven constraint

    $(1 - \lambda)\rho_0 I = I - A$
  - Entrepreneur's net utility = NPV

    $$U_b^0 = [(1 - \lambda)\rho_1 - 1]I = \frac{(1-\lambda)\rho_1 - 1}{1-(1-\lambda)\rho_0} A = \frac{\rho_1 - \frac{1}{1-\lambda}}{\frac{1}{1-\lambda} - \rho_0} A$$
  - Compare with case without liquidity shock: $\lambda = 0$.
- Pursue project if distress
  - Investors' breakeven constraint

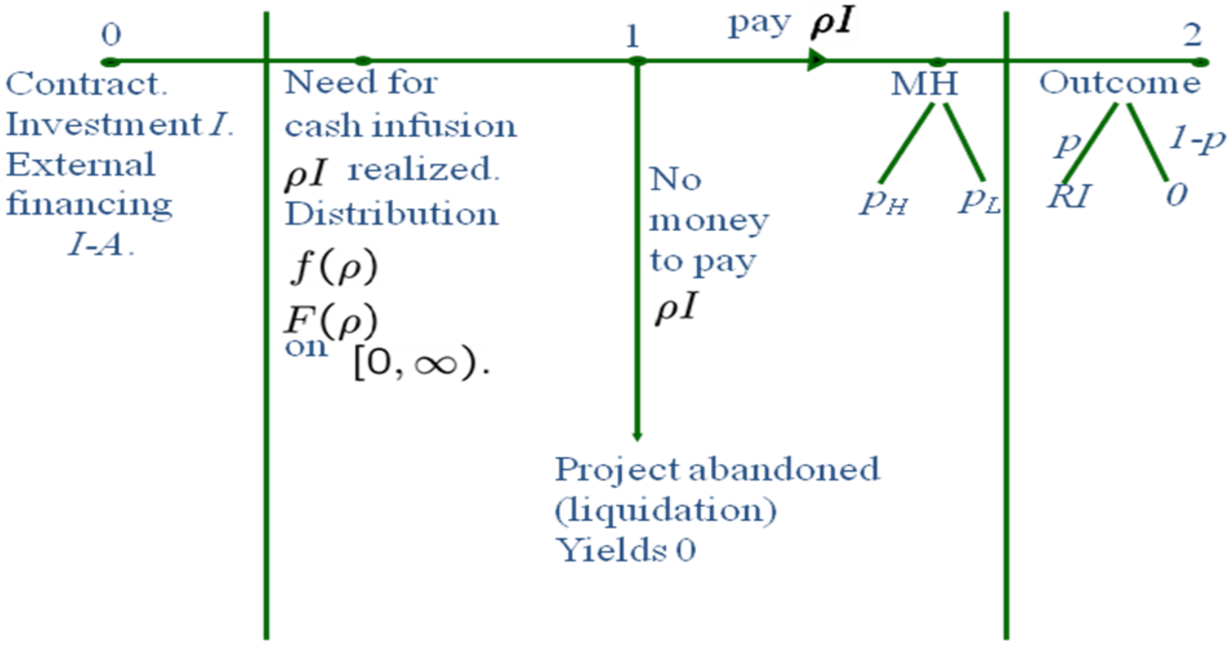
    $\rho_0 I = (1 + \lambda\rho)I - A$
  - Entrepreneur's net utility = NPV

    $$U_b^1 = [\rho_1 - (1 + \lambda\rho)]I = \frac{\rho_1 - (1+\lambda\rho)}{(1+\lambda\rho) - \rho_0} A$$
- Pursuing the project in case of distress at date 1 is better than abandoning it if:

  $$U_b^1 \geq U_b^0 \Leftrightarrow 1 + \lambda\rho \leq \frac{1}{1-\lambda} \Leftrightarrow \rho \leq \frac{1}{1-\lambda}$$
- Withstanding the liquidity shock is optimal if it is
  - low: ρ is low
  - likely: λ is high.
- If $\rho_0 < \rho \leq \frac{1}{1-\lambda}$, then *liquidity management* is required.
  - For example: a credit line.

A continuum of liquidity shocks

- Continuous investment, continuous shock.
- At date 1, continuation requires a reinvestment ρI, where $\rho \geq 0$.
  - Per-unit-of-investment cost overruns.
  - Probability distribution $F(\rho)$, density $f(\rho)$.



- NPV($\tilde{\rho}$) – net present value for a given cutoff $\tilde{\rho}$.

$$\text{NPV}(\tilde{\rho}) = \{F(\tilde{\rho})p_H R - [1 + \int_0^{\tilde{\rho}} \rho f(\rho) d\rho]\}I$$

- Assumption: There exists some $\tilde{\rho}$ such that NPV($\tilde{\rho}$) > 0.
- Question: What is the optimal cutoff rule ρ^*?
- Incentive constraint if continuation: $R_b \geq \dfrac{BI}{\Delta p}$
- Breakeven constraint with cutoff at ρ^*:

$$F(\rho^*)p_H(RI - R_b) \geq I - A + \int_0^{\rho^*} \rho I f(\rho) d\rho$$

- Borrowing capacity:

$$I \le k(\rho^*)A = \frac{1}{1+\int_0^{\rho^*}\rho f(\rho)d\rho - \rho_0 F(\rho^*)}A$$

- Recall the equity multiplier without liquidity shock: $k = \frac{1}{1-\rho_0}$

- Liquidity shocks reduce the equity multiplier: $k(\rho^*) < \frac{1}{1-\rho_0}$.

- Due to competition among creditors, borrower obtains NPV(ρ*).

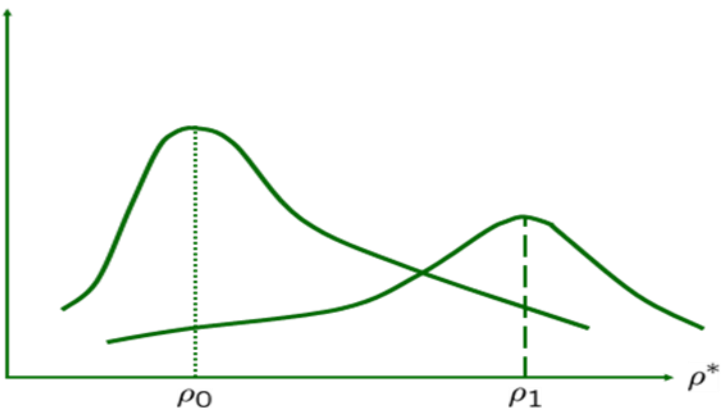
$$U_b = \{F(\rho^*)\rho_1 - [1+\int_0^{\rho^*}\rho f(\rho)d\rho]\}I \Leftrightarrow$$

$$U_b = m(\rho^*)k(\rho^*)A,$$

where

$$m(\rho^*) = F(\rho^*)\rho_1 - 1 - \int_0^{\rho^*}\rho f(\rho)d\rho$$

- The *margin* per unit of investment: $m(\rho^*)$
- The borrower must trade off the margin and the equity multiplier
  - Maximizing $m(\rho^*)$ would maximize profit and yield $\rho^* = \rho_1$. But $k'(\rho_1) < 0$.
  - Maximizing $k(\rho^*)$ would maximize pledgeable income and yield ρ_0. But $m'(\rho_0) > 0$.



- Write the borrower's net utility as

$$U_b = \frac{\rho_1 - c(\rho^*)}{c(\rho^*)-\rho_0}A, \text{ where: } c(\rho^*) = \frac{1+\int_0^{\rho^*}\rho f(\rho)d\rho}{F(\rho^*)}$$

- Note: $F(\rho^*)c(\rho^*) = 1+\int_0^{\rho^*}\rho f(\rho)d\rho$
  - $c(\rho^*)$ is the *expected cost per unit of effective investment*
- Maximizing U_b is tantamount to minimizing $c(\rho^*)$.

- Minimizing $c(\rho^*)$:

$$c'(\rho^*) = \frac{\rho^* f(\rho^*)F(\rho^*) - [1 + \int_0^{\rho^*} \rho f(\rho)d\rho] f(\rho^*)}{[F(\rho^*)]^2}$$

$$c'(\rho^*) = \frac{f(\rho^*)}{F(\rho^*)}[\rho^* - c(\rho^*)].$$

- The optimal cutoff is implicitly defined by:

$$\rho^* = c(\rho^*)$$

- In equilibrium, the borrower's net utility is

$$U_b = \frac{\rho_1 - \rho^*}{\rho^* - \rho_0} A$$

- The optimum cutoff lies between the expected per-unit-of-investment pledgeable income and income:

$$\rho_0 < \rho^* < \rho_1$$

  - *Trading off size and liquidity*: Increasing the cutoff above ρ^* would be good for profit but would also increase the demand for liquidity.

Risk management

- Suppose there is some residual uncertainty ε in the reinvestment requirement at date 1, such that $E(\varepsilon \mid \rho) = 0$.
- Consequences are adverse if liquidity falls short of a reinvestment
- Calls for buying insurance even if the entrepreneur is risk neutral.
- Tirole, Sec. 5.4

<u>Endogenous liquidity shocks</u>

- The entrepreneur may incur efforts to reduce – or even eliminate – the need for reinvestments. How to provide her with incentives to do this?
- A simple situation:
  - Before date 1, the borrower can incur effort costs *c* that will eliminate reinvestment needs completely: $\rho = 0$ with probability 1. If not, then ρ is drawn from the distribution $F(\rho)$ as before.
  - If the firm is cash poor – little or no income *r* at date 1 – the optimal contract has a covenant that no more funds shall be reinvested. But is this credible?
  - If the borrower does *not* incur costs *c* and the liquidity needs turn out to be $0 \leq \rho \leq \rho_0$, then it is in both lender's and borrower's interest to renegotiate the original contract.
  - This scope for renegotiation reduces the borrower's incentives to incur the effort costs *c*.
  - *Soft budget constraint*.
- More generally: Suppose the borrower can act at date 0 in a way that would improve the project, and that information arrives at date 1 that indicates whether or not she did so.
  - Moral hazard at both dates 0 and 1 (with respect to outcomes at dates 1 and 2).
  - Examples
    - Short-term income *r* stochastic *and* dependent on date-0 efforts
    - The project, if abandoned at date 1, has a liquidation value *L* that is stochastic and dependent on date 0 efforts
    - The project's date-2 return can be improved through efforts at date 0, and information about these improvements may be available before the reinvestment decision is made.
- Here: short-term income affected stochastically by date-0 efforts.

Endogenous intermediate income

- Variable-investment model.
- The usual stochastic return *RI* at date 2, subject to date-1 moral hazard.
- An investment of *I* at date 0 returns *rI* at date 1, where *r* is verifiable, and $r \in [0, r^+]$.
- Exerting effort affects the probability distribution of *r*.
- If the entrepreneur works at date 0, then *r* is distributed according to *G*(*r*), with density *g*(*r*). If the entrepreneur shirks at date 0, then *r* is distributed according to $\tilde{G}(r)$, with density $\tilde{g}(r)$.
- The likelihood ratio

$$l(r) = \frac{g(r) - \tilde{g}(r)}{g(r)}$$

- *The monotone likelihood ratio property* (MLRP): $l'(r) \geq 0$.
    - Implying that the distribution of *r* improves if the entrepreneur works: $G(r) \leq \tilde{G}(r)$, $\forall\, r$.
- Private benefit at date 0 if entrepreneur shirks: $B_0 I$.
- Benchmark: Credibility is not an issue – the "no soft budget constraint" (NSBC) case.
- Contract: $\{\rho^*(r), \Delta(r)\}$, where
    - $\rho^*(r)$ is the state-contingent cutoff
    - $\Delta(r) \geq 0$ is the borrower's state-contingent "extra rent" per unit of investment:
        - If continuation,

$$\Delta(r) = p_H\left(R_b - \frac{BI}{\Delta p}\right),$$

          what the borrower receives over and above the minimum required to preserve date-1 incentives.
        - If liquidation, $\Delta(r)$ is cash compensation.

- Lenders' breakeven constraint (IR_l):

$$\left\{\int_0^{r^+}\left[r+F\left(\rho^*(r)\right)\rho_0-\int_0^{\rho^*(r)}\rho f(\rho)d\rho-\Delta(r)\right]g(r)dr\right\} I\geq I-A$$

- Borrower's date-0 incentive constraint (IC_b):

$$\left\{\int_0^{r^+}\left[F\left(\rho^*(r)\right)\left(\rho_1-\rho_0\right)+\Delta(r)\right]\left[g(r)-\tilde{g}(r)\right]dr\right\} I\geq B_0 I \Leftrightarrow$$

$$\left\{\int_0^{r^+}\left[F\left(\rho^*(r)\right)\left(\rho_1-\rho_0\right)+\Delta(r)\right]l(r)g(r)dr\right\} I\geq B_0 I$$

- The optimal contract maximizes borrower's net utility subject to the two above constraints, with respect to $\{\rho^*(r), \Delta(r), I\}$. We ignore the choice of I for the moment.

$$U_b=\left\{\int_0^{r^+}\left[r+F\left(\rho^*(r)\right)\rho_1-\int_0^{\rho^*(r)}\rho f(\rho)d\rho-1\right]g(r)dr\right\} I$$

- Lagrangian multipliers: μ for IR_l and ν for IC_b.
- Pointwise maximization.
  - For each r, find the optimal pair $\{\rho^*(r), \Delta(r)\}$
- Fix r. First-order conditions with respect to $\rho^*(r)$ and $\Delta(r)$:

$$\{f(\rho^*)\rho_1-\rho^* f(\rho^*)+\mu[f(\rho^*)\rho_0-\rho^* f(\rho^*)]+\nu[f(\rho^*)(\rho_1-\rho_0)]l(r)\} \times g(r)I=0$$

$$\{-\mu+\nu l(r)\}g(r)I=0$$

$\Leftrightarrow$

$$\rho^*(r)=\frac{\rho_1+\mu\rho_0}{1+\mu}+\frac{\nu(\rho_1-\rho_0)}{1+\mu}l(r)$$

$$\mu=\nu l(r)$$

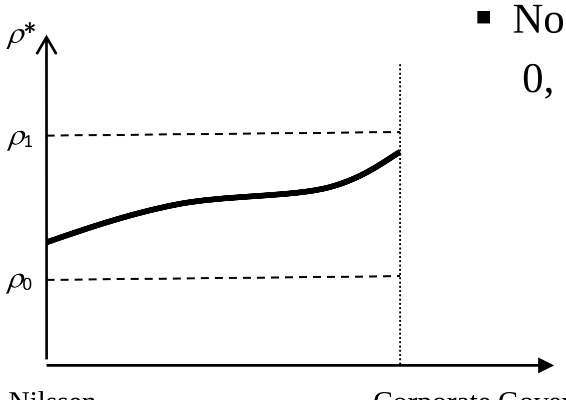
  - But the constraint $\Delta(r)\geq 0$ may be binding. Therefore,
    - either: $\Delta(r)>0 \Rightarrow \mu=\nu l(r) \Rightarrow \rho^*=\rho_1$,
    - or: $\Delta(r)=0 \Rightarrow -\mu+\nu l(r)\leq 0 \Rightarrow \rho^*\leq \rho_1$.

- $E_{G(\cdot)}[l(r)] = \int_0^{r^+} \frac{g(r)-\tilde{g}(r)}{g(r)} g(r)dr = \int_0^{r^+} g(r)dr - \int_0^{r^+} \tilde{g}(r)dr = 0$
- This implies: $E[\rho^*(r)] = \frac{\rho_1 + \mu\rho_0}{1+\mu}$
  - In expectation, the cutoff is a weighted average of ρ_1 and ρ_0, and $\rho_0 < E[\rho^*(r))] < \rho_1$; as in the case without date-0 moral hazard, the firm *trades off size and liquidity*.
- We can write:

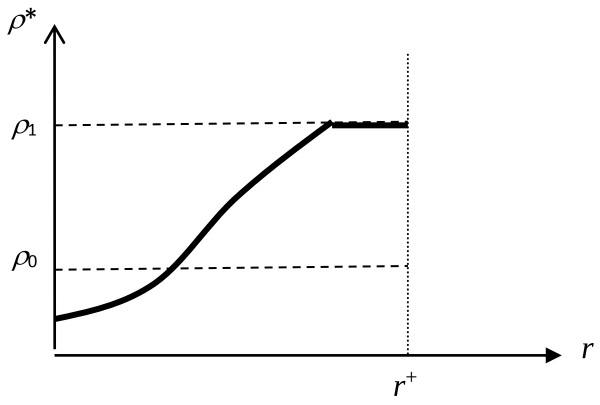
$$\rho^*(r) = E[\rho^*(r)] + \lambda l(r),$$

where: $\lambda = \frac{\nu}{1+\mu}(\rho_1 - \rho_0) > 0.$

- By assumption (MLRP): $l'(r) \geq 0$. Therefore: $\frac{d\rho^*}{dr} \geq 0$.
- The continuation rule is more lenient, the higher is the date-1 income r.
- Two possibilities:
  - $\rho^*(r)$ increases moderately
    - because the date-0 incentive problem is small
      - date-0 private benefits B_0 not very high, so that the borrower's date-0 incentive constraint is not very restrictive, making ν low;
      - date-0 liquidity shocks being mainly outside the borrower's control, so that $l(r)$ stays close to 0.
    - or because the date-1 incentive problem is small
      - date-1 private benefits B small, or $\Delta p/p_H$ large, again making ν low.
        - No extra rent to the borrower: $\Delta(r) = 0, \forall r$.

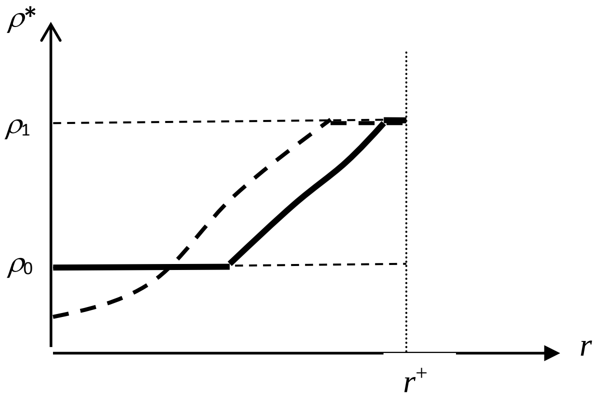


- $\rho^*(r)$ increases steeply
  - because one or both of the two moral hazard problems are more serious
  - When intermediate income is high, first-best can be reached: $\rho^* = \rho_1$.
  - Extra rent to the borrower at high r: When intermediate income is high, she gets to keep some of it.
  - At a low intermediate income, we may even have $\rho^* < \rho_0$.



- *Soft budget constraint*: $\rho^* < \rho_0$ is not credible.
  - The parties will renegotiate a contract whenever r is realized and $\rho^*(r) < \rho_0$.
  - Formally, same problem as before, with an added constraint: $\rho^* \geq \rho_0$.
  - When incentive problems are small, so that there is only a moderate increase in $\rho^*(r)$ in the NSBC case, there is no change in the optimal contract.
  - When incentive problems are greater, the constraint $\rho^* \geq \rho_0$ binds for small values of r.

- Increasing ρ^* in order to satisfy the credibility constraint at low values of r calls for decreasing it for higher values of r, in order to keep satisfying the lenders' breakeven constraint.



- Credibility problems at low values of r decreases continuation – and reduces efficiency – at larger values.

Free cash flow

- Tirole, Sec. 5.6.
- If the firm has more cash than it needs, there are incentives for *overinvestment*. It has been argued that debt may mitigate this problem.
- Back to the discussion of the liquidity-scale tradeoff.
- But now there is a deterministic short-term income rI, which is fully pledgeable.
- Lenders' breakeven constraint with cutoff at ρ^*:

$$rI + F(\rho^*)p_H(RI - R_b) \geq I - A + \int_0^{\rho^*} \rho I\, f(\rho)\, d\rho$$

- Everything as if the unit investment cost is $(1 - r)$ rather than 1.
- Cutoff implicitly given by:

$$\rho^* = c(\rho^*) = \frac{1 - r + \int_0^{\rho^*} \rho f(\rho)\, d\rho}{F(\rho^*)}$$

  - Cutoff ρ^* is now *decreasing* in the short-term income r.
    - A high r makes it possible to reduce continuation in order to increase the borrowing capacity.
- The *free-cash-flow* assumption: $r > \rho^*$.
  - The entrepreneur would like to commit herself not to reinvest the amount $(r - \rho^*)I$.
  - This calls for *short-term debt*, that is, debt to be paid at the intermediate date.
  - In more general settings, short-term debt may not fully resolve the free-cash-flow problem.