## Credit rationing

- Loan markets are special
- Personalized
- Clearing through both quantities and prices
- This is because of private information among borrowers
- Adverse selection: There are both good and bad firms out there, and banks cannot tell who is who.
- Moral hazard: Banks cannot observe actions taken by firms.
- Increasing the interest rate makes the borrower care less about the project that is being financed.
- Lower borrower's income in the absence of bankruptcy
- No effect on her income in case of bankruptcy
- Moral hazard: a reduced stake reduces incentives
- Adverse selection: an increased interest rate attracts low-quality borrowers
- In equilibrium, borrowers may be rationed.
- Rationed - not getting the funds you need even though you are willing to accept the terms that are required.
- Prices alone do not clear markets.
- In order to get outside financing, you may need own funds.


## A simple model: Fixed investment

Project costs $I$.
Entrepreneur has equity $A<I$; borrows $I-A$; is protected by LL.

Moral hazard

|  | $\operatorname{Pr}$ (success) | Private <br> benefit |
| :--- | :---: | :---: |
| Behaves | $p_{H}$ | 0 |
| Misbehaves | $p_{L}$ | $B$ |$\quad$| $R$ | $p$ |
| :--- | :--- |
| 0 | $1-p$ |

- A risk neutral entrepreneur has a project requiring a fixed investment $I$.
- If success: project return is $R$. If failure, return is 0 .
- The entrepreneur has own funds $A<I$.
- $A=$ net worth, or cash on hand.
- She needs to borrow $I-A$ to carry out the project.
- Project is risky, and success depends on entrepreneur's effort.
- Misbehaving lowers the success probability of this project $\left(p_{L}<p_{H}\right)$, but creates private benefits $B$ to the entrepreneur.
- $\Delta p=p_{H}-p_{L}$.
- Assume project is viable if and only if entrepreneur behaves
- Net present value (NPV) if she behaves: $p_{H} R-I>0$.
- NPV if not: $p_{L} R-I+B<0$.
- In combination: $\frac{p_{H} R}{I}>1>\frac{p_{L} R}{I-B}$
- No loan will be granted that induces misbehavior.
- Loan contract: If success, borrower gets $R_{b}$, lender $R_{l}=R-R_{b}$.
- Limited liability: If failure, both receive 0 .
- Lenders are risk neutral and behave competitively.
- Competition among lenders implies $p_{H} R_{l}=I-A \Rightarrow R_{l}=\frac{I-A}{p_{H}}$.
- The interest rate is given by: $R_{l}=(1+\imath)(I-A) \Rightarrow 1+t=\frac{1}{p_{H}}$.
- For $p_{H}<1$, there is a default premium: $l>0$.
- Are lenders interested at these terms? - Credit analysis.
- Need to preserve borrower a sufficient stake in order to induce incentives
- The incentive compatibility constraint

$$
p_{H} R_{b} \geq p_{L} R_{b}+B \Rightarrow R_{b} \geq \frac{B}{\Delta p}
$$

- What the borrower gets from behaving must be more than what she gets from misbehaving
- There is a lower limit on the borrower's return
- Increasing in the private benefits $B$.
- Decreasing in the effect of behaving $\Delta p$.
- The maximum income that can be pledged to lenders without inducing misbehavior is

$$
R-\frac{B}{\Delta p}
$$

- Expected pledgeable income is therefore

$$
P=p_{H}\left(R-\frac{B}{\Delta p}\right)
$$

- Lenders' individual rationality constraint

$$
p_{H}\left(R-\frac{B}{\Delta p}\right) \geq I-A
$$

- Expected pledgeable income must exceed lenders’ initial expenses
- Other names
- breakeven constraint
- participation constraint
- A necessary and sufficient condition for financing of the entrepreneur's project
- Minimum level of own funds in order to get outside financing

$$
A \geq \bar{A}=p_{H} \frac{B}{\Delta p}-\left(p_{H} R-I\right)
$$

- Assumption:

$$
\begin{equation*}
\bar{A}>0 \Leftrightarrow(0<) p_{H} R-I<p_{H} \frac{B}{\Delta p} \tag{*}
\end{equation*}
$$

- Otherwise, even a borrower without any wealth of her own would get credit
- NPV of project is less than the minimum that must be left to the borrower in order to ensure incentives.
- A project may have NPV $>0$, and still not get funded
- This happens in cases where $A<\bar{A}$.
- "One only lends to the rich".
- The agency rent: what must be left to the borrower to ensure incentives

$$
p_{H} \frac{B}{\Delta p}
$$

- The condition $A \geq \bar{A}$ says that agency rent net of borrower's own input must be less than the project's NPV

$$
p_{H} \frac{B}{\Delta p}-A \leq p_{H} R-I
$$

- The borrower's net utility

$$
\begin{aligned}
U_{b} & =0, & \text { if } A<\bar{A} ; \\
& =p_{H} R_{b}-A=p_{H}\left(R-R_{l}\right)-A=p_{H} R-I, & \text { if } A \geq \bar{A} .
\end{aligned}
$$

- The borrower gets the entire net present value, if only she can get the project funded.
- Determinants of credit rationing
- Little cash on hand (low A)
- High agency costs (high $p_{H} \frac{B}{\Delta p}$ ).
- Moral hazard determined by two factors
- The extent of private benefits from misbehavior: $B$
- The extent to which the verifiable final outcome reveals misbehavior
- Informativeness measured by the likelihood ratio

$$
\frac{\Delta p}{p_{H}}=\frac{p_{H}-p_{L}}{p_{H}}
$$

- Is this debt or equity?
- Debt: Entrepreneur owes $R_{l}$ and must pay this or go bankrupt
- Equity: Entrepreneur and investor own $R_{b} / R$ and $R_{l} / R$ each in the firm.
- A few "dynamic" considerations

○ A second investment (sec. 3.2.4)

- Dilution of initial lenders' claim
- Overinvestment
- Argument for a negative debt covenant prohibiting further debt
- Reputational capital (sec. 3.2.5)
- The borrower would gain by a lowering of private benefits $B$.

$$
b<B \Rightarrow \bar{A}(b)<\bar{A}(B)
$$

- A more reliable borrower is more likely to get loan
- Two benefits of successful projects today
- Increased retained earnings: A higher
- Improved reputation: (lenders' perception of) $B$ lower


## Relative performance evaluation

- Making agents accountable for events they have no control over weakens incentives in general
- One should always try to make use of the most precise measurement of the agent's performance - the sufficient statistic (Holmström, 1979).
- Benchmarking
- Reinterpreting the model in terms of benchmarking
- Three states of nature
- Favorable state (probability $p_{L}$ ): Project will succeed whatever the entrepreneur does.
- Unfavorable state (probability $1-p_{H}$ ): Project will fail whatever the entrepreneur does.
- Intermediate state (probability $\Delta p=p_{H}-p_{L}$ ): Success not guaranteed but will result if entrepreneur behaves.
- No-one knows the true state. But lenders can - say, by looking at other firms in the same industry - learn whether or not the state is favorable.
- Contract: Entrepreneur receives nothing in the favorable state; otherwise, she receives $R_{b}$ if success.
- Incentive compatibility constraint is the same: $R_{b} \geq \frac{B}{\Delta p}$
- But pledgeable income is increased, since entrepreneur is not paid for being lucky: $p_{H} R-\Delta p \frac{B}{\Delta p}=p_{H} R-B$.


## Debt overhang

- Project is profitable, but entrepreneur is unable to raise funds because of previously incurred debt
- Two interpretations
- Previous investors have collateral claims that reduce net worth $A$ to below the threshold level $\bar{A}$.
- Previous debt needs to be renegotiated in order to enable new investments.


## Previous debt reduces net worth

- Suppose the entrepreneur has $A$ in cash but owes $D$ to the initial investors.
- Initial investors insisted on a covenant specifying that further loans require their consent.
- The assets $A$ are pledged as collateral to initial investors in case of default.
- Let $A>\bar{A}>A-D \geq 0$.
- The new project would have been undertaken in absence of previous debt but is not undertaken, because the investors (old and new together) cannot recoup their expenses $(I-A)$ plus the previous debt $(D)$, since $A-D<\bar{A}$, but they can get $D$ by seizing the collateral, since $A \geq D$.


## Lack of renegotiation with previous lenders

- Suppose the borrower has no cash: $A=0$
- But $\bar{A}<0$ : the project would be able to attract funds even without any net worth for the borrower.
- The borrower has already a long-term debt $D$, which is due later.
- The problem cannot be overcome by the (expected) profitability of the new project: The slack in pledgeable income, $-\bar{A}$, is smaller than what has to be paid back to previous investors, $p_{H} D$, if the project is funded:

$$
p_{H} D>-\bar{A} \Leftrightarrow \bar{A}+p_{H} D>0
$$

- Initial investors may want to put in more funds, since they get nothing in case of bankruptcy now $(A=0)$.
- But what if initial investors have no funds available? Are new investors willing? The problem is that old debt is senior, and that the borrower needs to keep a minimum stake in the project to ensure incentives; so expected pledgeable income is

$$
p_{H}\left(R-\frac{B}{\Delta p}-D\right)
$$

- New investors are willing to fund if and only if:

$$
p_{H}\left(R-\frac{B}{\Delta p}-D\right) \geq I \Leftrightarrow \bar{A}+p_{H} D \leq 0
$$

- This contradicts the assumption above.
- It is impossible to raise funds from new investors unless some debt forgiveness is renegotiated with old investors.


## Borrowing capacity: a variable-investment model



- Constant returns to scale in investment: Investing $I \geq 0$ yields a return $R I$ if success, 0 if failure, with $R>0$.
- Borrower's private benefit from misbehaving: $B I$, with $B>0$.
- Borrower can choose to behave or not.
- Borrower's cash: $A$; must borrow $I-A$ to invest $I$.
- Loan contract: $\left\{R_{b}, R_{l}\right\}$, where $R_{b}+R_{l}=R I$.
- Assume project is profitable if and only if borrower behaves

$$
p_{H} R>1>p_{L} R+B
$$

- ... but that NPV per unit of investment is less than agency costs per unit

$$
p_{H} R-1<\frac{p_{H} B}{\Delta p}
$$

- Equivalent to the $\bar{A}>0$ assumption in the fixedinvestment model
- Needed here to ensure equilibrium investment being finite, because of the constant-returns-to-scale technology.
- Lenders behave competitively
- Lenders' credit analysis
- Incentive compatibility: $R_{b} \geq \frac{B I}{\Delta p}$
- Breakeven: $p_{H}\left(R I-R_{b}\right) \geq I-A$
- Borrower's net utility: $U_{b}=\left(p_{H} R-1\right) I$
- The borrower would like as much funding as possible.
- The equity multiplier
- Determined by incentive compatibility and breakeven constraints. Combining them, we get

$$
\begin{aligned}
& I \leq k A, \text { where } \\
& k=\frac{1}{1-p_{H}\left(R-\frac{B}{\Delta p}\right)}>1 .
\end{aligned}
$$

- The borrower can lever her wealth, with the equity multiplier $k$.
- The equity multiplier is smaller, the higher is the private benefit $B$, and the lower is the likelihood ratio $\Delta p / p_{H}-$ our two measures of agency cost.
- The entrepreneur's borrowing capacity.
- Outside financing capacity; debt capacity
- It is possible for the borrower to invest $k$ times her cash $A$, that is, to borrow $d=k-1$ times her cash, where

$$
d=\frac{p_{H}\left(R-\frac{B}{\Delta p}\right)}{1-p_{H}\left(R-\frac{B}{\Delta p}\right)} .
$$

- The maximum loan, $d A$, is the borrowing capacity.
- The borrowing capacity
- increases with per-unit return $R$
- decreases with the extent of the agency problem
- The shadow value of equity
- Borrower's gross utility: $U_{b}^{g}=A+U_{b}$
- Combine $U_{b}=\left(p_{H} R-1\right) I$ and $I=k A$ to get:

$$
U_{b}^{g}=v A, \text { where } v=\frac{p_{H} \frac{B}{\Delta p}}{1-p_{H}\left(R-\frac{B}{\Delta p}\right)}>1
$$

- The shadow value of equity $v$
- increases in the per-unit return $R$
- decreases in the extent of the agency problem
- Useful notation
- Expected payoff per unit of investment: $\rho_{1}=p_{H} R$
- Expected pledgeable income per unit of investment:

$$
\rho_{0}=p_{H}\left(R-\frac{B}{\Delta p}\right)
$$

- Earlier assumptions imply: $\rho_{1}>1>\rho_{0}$.
- The equity multiplier: $k=\frac{1}{1-\rho_{0}}$
- The borrowing capacity per unit of net worth: $d=\frac{\rho_{0}}{1-\rho_{0}}$
- The shadow value of equity: $v=\frac{\rho_{1}-\rho_{0}}{1-\rho_{0}}$
- Borrower's net utility: $U_{b}=(v-1) A=\left(\rho_{1}-1\right) I$.
- Note: Firms with a low agency cost has a greater sensitivity of investment to cash flow.

$$
\frac{\partial^{2} I}{\partial A \partial \rho_{0}}=\frac{\partial k}{\partial \rho_{0}}=\frac{1}{\left(1-\rho_{o}\right)^{2}}>0
$$

## The maximal incentives principle

- Resolving the debt vs equity question.
- Salvage value of assets
- Investing $I \geq 0$ yields a return $R^{S} I$ if success, $R^{F} I$ if failure, with $R^{S}>R^{F}>0$.
- Define $R I=\left(R^{S}-R^{F}\right) I$ as the profit increase following success.
- When secondary asset markets perform better, we should expect $R^{F}$ to be higher.
- Generalizing $\rho_{1}>1>\rho_{0}$ :

$$
p_{H} R+R^{F}>1>p_{H}\left(R-\frac{B}{\Delta p}\right)+R^{F}
$$

- Contract: $\left\{R_{b}^{S}, R_{b}^{F}, I\right\}$ - how much to invest, and how much of the returns generated that the borrower should have following success and failure.
- The optimal contract maximizes the entrepreneur's net utility,

$$
p_{H} R_{b}^{S}+\left(1-p_{H}\right) R_{b}^{F}-A,
$$

subject to two constraints:

- the entrepreneur's incentive compatibility constraint:

$$
R_{b}^{S}-R_{b}^{F} \geq \frac{B I}{\Delta p}
$$

- the investors' breakeven constraint:

$$
p_{H}\left(R^{S} I-R_{b}^{S}\right)+\left(1-p_{H}\right)\left(R^{F} I-R_{b}^{F}\right) \geq I-A
$$

- In equilibrium, both constraints will be binding.
- As before, the entrepreneur receives all the NPV:

$$
U_{b}=\left(p_{H} R+R^{F}-1\right) I
$$

- In equilibrium, the entrepreneur receives nothing following failure: $R_{b}^{F}=0$.
- Suppose instead $R_{b}^{F}>0$. Then one can reduce it, and increase $R_{b}^{S}$, at a rate $\frac{\Delta R_{b}^{F}}{\Delta R_{b}^{S}}=-\frac{p_{H}}{1-p_{H}}$, keeping the breakeven constraint binding and the entrepreneur's utility unchanged; but this would make the incentive compatibility constraint slack - a contradiction.
- An all-equity firm is not optimal
- With no debt, the entrepreneur would, after a failure, receive her share of $R^{F} I$ corresponding to her share of the firm's stocks.
- Outside investors must hold debt $D \geq R^{F} I$.
- Borrowing capacity: $I=k A$, and so $D=I-A=d A=(k-1) A$, where now

$$
k=\frac{1}{1-\left[p_{H}\left(R-\frac{B}{\Delta p}\right)+R^{F}\right]} .
$$

- Firms borrow more
- the lower agency costs are;
- the more liquid assets are.
- Incentives are maximized when outside investors hold a combination of debt and equity.


## Extensions of the analysis

- Supplementary sections to chapter 3
- A continuum of effort levels, disutility of effort $g(e)$
- A continuum of outcomes, probability of outcome $R$ with effort level $e$ is $p(R \mid e)$.
- Linking effort and outcome: higher effort tends to increase income - the monotone likelihood ratio property (MLRP)

$$
\frac{\partial}{\partial R}\left[\frac{\frac{\partial p(R \mid e)}{\partial e}}{p(R \mid e)}\right]>0
$$

- Essentially same result: A standard debt contract - making entrepreneur a residual claimant for the marginal income above the debt repayment level
- Risk aversion - brings in another problem: the insurance/incentives tradeoff.
- Providing incentives means making the risk averse entrepreneur take part in the lottery.
- A solution exists if effort can be verified after contracts are signed, but before outcome is realized, so that contracts can be renegotiated. This makes it possible to separate the insurance and incentives problems.
- Semi-verifiable outcome
- Outcome from investment not verifiable, unless outside investors incur an audit cost.
- The incentive problem is related to hiding income, rather than to enjoying private benefits or reducing effort.
- Outcome is reported by entrepreneur: $\hat{R}$.
- The problem for outsiders is to induce truthful reporting.
- Contract now includes a probability $y(\hat{R})$ of no audit for each report $\hat{R}$.
- Again, a standard debt contract.
- Non-verifiable outcome
- Not even an audit can verify outcome.
- Repayment is the result of threats of termination or nonfinancing of future projects.

