### **Credit rationing**

- Loan markets are special
  - Personalized
  - o Clearing through both quantities and prices
- This is because of private information among borrowers
  - Adverse selection: There are both good and bad firms out there, and banks cannot tell who is who.
  - Moral hazard: Banks cannot observe actions taken by firms.
- Increasing the interest rate makes the borrower care less about the project that is being financed.
  - o Lower borrower's income in the absence of bankruptcy
  - No effect on her income in case of bankruptcy
- Moral hazard: a reduced stake reduces incentives
- Adverse selection: an increased interest rate attracts low-quality borrowers
- In equilibrium, borrowers may be *rationed*.
  - Rationed not getting the funds you need even though you are willing to accept the terms that are required.
  - Prices alone do not clear markets.
- In order to get outside financing, you may need own funds.

## A simple model: Fixed investment

Moral hazard Verifiable outcome Project costs I. Entrepreneur has equity A < I; borrows I - A; Private Pr (success) R pis protected by LL. benefit 0 Behaves  $p_H$  $\overline{B}$ Misbehaves  $p_L$ 

- A risk neutral entrepreneur has a project requiring a fixed investment *I*.
  - o If success: project return is R. If failure, return is 0.
- The entrepreneur has own funds A < I.
  - $\circ$  A = net worth, or cash on hand.
- She needs to borrow I A to carry out the project.
- Project is risky, and success depends on entrepreneur's effort.
  - O Misbehaving lowers the success probability of this project  $(p_L < p_H)$ , but creates private benefits B to the entrepreneur.
  - $\circ \ \Delta p = p_H p_L.$
- Assume project is viable if and only if entrepreneur behaves
  - *Net present value* (NPV) if she behaves:  $p_HR I > 0$ .
  - NPV if not:  $p_L R I + B < 0$ .
  - In combination:  $\frac{p_H R}{I} > 1 > \frac{p_L R}{I B}$
  - No loan will be granted that induces misbehavior.
- Loan contract: If success, borrower gets  $R_b$ , lender  $R_l = R R_b$ .
- Limited liability: If failure, both receive 0.

- Lenders are risk neutral and behave competitively.
- Competition among lenders implies  $p_H R_l = I A \Rightarrow R_l = \frac{I A}{p_H}$ .
- The interest rate is given by:  $R_l = (1 + \iota)(I A) \Rightarrow 1 + \iota = \frac{1}{p_H}$ .
  - For  $p_H$  < 1, there is a default premium: t > 0.
- Are lenders interested at these terms? Credit analysis.
  - Need to preserve borrower a sufficient stake in order to induce incentives
  - The incentive compatibility constraint

$$p_H R_b \ge p_L R_b + B \Longrightarrow R_b \ge \frac{B}{\Delta p}$$

- What the borrower gets from behaving must be more than what she gets from misbehaving
- There is a lower limit on the borrower's return
  - Increasing in the private benefits *B*.
  - Decreasing in the effect of behaving  $\Delta p$ .
- The maximum income that can be *pledged to lenders* without inducing misbehavior is

$$R - \frac{B}{\Delta p}$$

o Expected pledgeable income is therefore

$$P = p_H \left( R - \frac{B}{\Delta p} \right)$$

• Lenders' individual rationality constraint

$$p_H \left( R - \frac{B}{\Delta p} \right) \ge I - A$$

- Expected pledgeable income must exceed lenders' initial expenses
- Other names
  - breakeven constraint
  - participation constraint
- A necessary and sufficient condition for financing of the entrepreneur's project
- Minimum level of own funds in order to get outside financing

$$A \ge \overline{A} = p_H \frac{B}{\Delta p} - (p_H R - I)$$

• Assumption:

$$\overline{A} > 0 \Leftrightarrow (0 <) p_H R - I < p_H \frac{B}{\Delta p}$$
 (\*)

- Otherwise, even a borrower without any wealth of her own would get credit
- NPV of project is less than the minimum that must be left to the borrower in order to ensure incentives.
- A project may have NPV > 0, and still not get funded
  - This happens in cases where  $A < \overline{A}$ .
  - o "One only lends to the rich".

• The *agency rent*: what must be left to the borrower to ensure incentives

$$p_{\scriptscriptstyle H} \frac{B}{\Delta p}$$

• The condition  $A \ge \overline{A}$  says that agency rent net of borrower's own input must be less than the project's NPV

$$p_{H} \frac{B}{\Delta p} - A \le p_{H} R - I$$

• The borrower's net utility

$$U_b = 0,$$
 if  $A < \overline{A}$ ;  
=  $p_H R_b - A = p_H (R - R_l) - A = p_H R - I$ , if  $A \ge \overline{A}$ .

- The borrower gets the entire net present value, if only she can get the project funded.
- Determinants of credit rationing
  - Little cash on hand (low *A*)
  - High agency costs (high  $p_H \frac{B}{\Delta p}$ ).
- Moral hazard determined by two factors
  - The extent of private benefits from misbehavior: B
  - The extent to which the verifiable final outcome reveals misbehavior
    - Informativeness measured by the likelihood ratio

$$\frac{\Delta p}{p_{\scriptscriptstyle H}} = \frac{p_{\scriptscriptstyle H} - p_{\scriptscriptstyle L}}{p_{\scriptscriptstyle H}}$$

- Is this debt or equity?
  - O Debt: Entrepreneur owes  $R_l$  and must pay this or go bankrupt
  - Equity: Entrepreneur and investor own  $R_b/R$  and  $R_l/R$  each in the firm.
- A few "dynamic" considerations
  - A second investment (sec. 3.2.4)
    - Dilution of initial lenders' claim
    - Overinvestment
    - Argument for a negative debt covenant prohibiting further debt
  - o Reputational capital (sec. 3.2.5)
    - The borrower would gain by a lowering of private benefits *B*.

$$b < B \Rightarrow \overline{A}(b) < \overline{A}(B)$$

- A more reliable borrower is more likely to get loan
- Two benefits of successful projects today
  - Increased retained earnings: A higher
  - Improved reputation: (lenders' perception of) *B* lower

### Relative performance evaluation

- Making agents accountable for events they have no control over weakens incentives in general
- One should always try to make use of the most precise measurement of the agent's performance the *sufficient statistic* (Holmström, 1979).
- Benchmarking
- Reinterpreting the model in terms of benchmarking
  - Three states of nature
    - Favorable state (probability  $p_L$ ): Project will succeed whatever the entrepreneur does.
    - Unfavorable state (probability  $1 p_H$ ): Project will fail whatever the entrepreneur does.
    - Intermediate state (probability  $\Delta p = p_H p_L$ ): Success not guaranteed but will result if entrepreneur behaves.
  - No-one knows the true state. But lenders can say, by looking at other firms in the same industry learn whether or not the state is favorable.
  - $\circ$  Contract: Entrepreneur receives nothing in the favorable state; otherwise, she receives  $R_b$  if success.
  - Incentive compatibility constraint is the same:  $R_b \ge \frac{B}{\Delta p}$
  - But *pledgeable income* is increased, since entrepreneur is not paid for being lucky:  $p_H R \Delta p \frac{B}{\Delta p} = p_H R B$ .

## Debt overhang

- Project is profitable, but entrepreneur is unable to raise funds because of previously incurred debt
- Two interpretations
  - $\circ$  Previous investors have collateral claims that reduce net worth A to below the threshold level  $\overline{A}$ .
  - Previous debt needs to be renegotiated in order to enable new investments.

#### Previous debt reduces net worth

- Suppose the entrepreneur has A in cash but owes D to the initial investors.
- Initial investors insisted on a covenant specifying that further loans require their consent.
- The assets *A* are pledged as collateral to initial investors in case of default.
- Let  $A > \overline{A} > A D \ge 0$ .
- The new project would have been undertaken in absence of previous debt but is not undertaken, because the investors (old and new together) cannot recoup their expenses (I-A) plus the previous debt (D), since  $A-D<\overline{A}$ , but they can get D by seizing the collateral, since  $A \ge D$ .

# Lack of renegotiation with previous lenders

- Suppose the borrower has no cash: A = 0
- But  $\overline{A}$  < 0: the project would be able to attract funds even without any net worth for the borrower.
- The borrower has already a long-term debt *D*, which is due later.
- The problem cannot be overcome by the (expected) profitability of the new project: The slack in pledgeable income,  $-\bar{A}$ , is smaller than what has to be paid back to previous investors,  $p_HD$ , if the project is funded:

$$p_H D > -\overline{A} \iff \overline{A} + p_H D > 0$$

- Initial investors may want to put in more funds, since they get nothing in case of bankruptcy now (A = 0).
- But what if initial investors have no funds available? Are new investors willing? The problem is that old debt is senior, and that the borrower needs to keep a minimum stake in the project to ensure incentives; so expected pledgeable income is

$$p_H \left( R - \frac{B}{\Delta p} - D \right)$$

New investors are willing to fund if and only if:

$$p_H \left( R - \frac{B}{\Delta p} - D \right) \ge I \iff \overline{A} + p_H D \le 0$$

- This contradicts the assumption above.
- It is impossible to raise funds from new investors unless some debt forgiveness is renegotiated with old investors.

## Borrowing capacity: a variable-investment model

Contract: Moral hazard Outcome 0 or RIInvestment I. Sharing rule Private Pr  $R_b + R_\ell = RI$ benefit (success) 0 Behaves  $p_H$ Misbehaves  $\overline{BI}$  $p_L$ 

- Constant returns to scale in investment: Investing  $I \ge 0$  yields a return RI if success, 0 if failure, with R > 0.
- Borrower's private benefit from misbehaving: BI, with B > 0.
- Borrower can choose to behave or not.
- Borrower's cash: A; must borrow I A to invest I.
- Loan contract:  $\{R_b, R_l\}$ , where  $R_b + R_l = RI$ .
- Assume project is profitable if and only if borrower behaves

$$p_H R > 1 > p_L R + B$$

• ... but that NPV per unit of investment is less than agency costs per unit

$$p_H R - 1 < \frac{p_H B}{\Delta p}$$

- Equivalent to the  $\overline{A} > 0$  assumption in the fixed-investment model
- Needed here to ensure equilibrium investment being finite, because of the constant-returns-to-scale technology.

- Lenders behave competitively
- Lenders' credit analysis
  - o Incentive compatibility:  $R_b \ge \frac{BI}{\Delta p}$
  - Breakeven:  $p_H(RI R_b) \ge I A$
  - o Borrower's net utility:  $U_b = (p_H R 1)I$ 
    - The borrower would like as much funding as possible.
- The equity multiplier
  - Determined by incentive compatibility and breakeven constraints. Combining them, we get

$$I \leq kA$$
, where

$$k = \frac{1}{1 - p_H \left( R - \frac{B}{\Delta p} \right)} > 1.$$

- The borrower can lever her wealth, with the *equity multiplier k*.
- The *equity multiplier* is smaller, the higher is the private benefit B, and the lower is the likelihood ratio  $\Delta p/p_H$  our two measures of agency cost.

- The entrepreneur's borrowing capacity.
  - Outside financing capacity; debt capacity
  - $\circ$  It is possible for the borrower to invest k times her cash A, that is, to borrow d = k 1 times her cash, where

$$d = \frac{p_H \left( R - \frac{B}{\Delta p} \right)}{1 - p_H \left( R - \frac{B}{\Delta p} \right)}.$$

- $\circ$  The maximum loan, dA, is the borrowing capacity.
- The borrowing capacity
  - increases with per-unit return *R*
  - decreases with the extent of the agency problem
- The shadow value of equity
  - o Borrower's gross utility:  $U_b^s = A + U_b$
  - Combine  $U_b = (p_H R 1)I$  and I = kA to get:

$$U_b^g = vA$$
, where  $v = \frac{p_H \frac{B}{\Delta p}}{1 - p_H \left(R - \frac{B}{\Delta p}\right)} > 1$ 

- $\circ$  The shadow value of equity  $\nu$ 
  - increases in the per-unit return R
  - decreases in the extent of the agency problem

- Useful notation
  - Expected payoff per unit of investment:  $\rho_1 = p_H R$
  - Expected pledgeable income per unit of investment:

$$\rho_0 = p_H \left( R - \frac{B}{\Delta p} \right)$$

- Earlier assumptions imply:  $\rho_1 > 1 > \rho_0$ .
- The equity multiplier:  $k = \frac{1}{1 \rho_0}$
- The borrowing capacity per unit of net worth:  $d = \frac{\rho_0}{1 \rho_0}$
- The shadow value of equity:  $v = \frac{\rho_1 \rho_0}{1 \rho_0}$
- o Borrower's net utility:  $U_b = (\nu 1)A = (\rho_1 1)I$ .
- Note: Firms with a low agency cost has a greater *sensitivity of investment to cash flow*.

$$\frac{\partial^2 I}{\partial A \partial \rho_0} = \frac{\partial k}{\partial \rho_0} = \frac{1}{\left(1 - \rho_0\right)^2} > 0.$$

## The maximal incentives principle

- Resolving the debt vs equity question.
- Salvage value of assets
  - Investing  $I \ge 0$  yields a return  $R^SI$  if success,  $R^FI$  if failure, with  $R^S > R^F > 0$ .
  - O Define  $RI = (R^S R^F)I$  as the profit increase following success.
  - $\circ$  When secondary asset markets perform better, we should expect  $R^F$  to be higher.
- Generalizing  $\rho_1 > 1 > \rho_0$ :

$$p_{\scriptscriptstyle H}R + R^{\scriptscriptstyle F} > 1 > p_{\scriptscriptstyle H} \left( R - \frac{B}{\Delta p} \right) + R^{\scriptscriptstyle F}$$

- Contract:  $\{R_b^S, R_b^F, I\}$  how much to invest, and how much of the returns generated that the borrower should have following success and failure.
- The optimal contract maximizes the entrepreneur's net utility,

$$p_H R_b^S + (1 - p_H) R_b^F - A,$$

subject to two constraints:

o the entrepreneur's incentive compatibility constraint:

$$R_b^S - R_b^F \ge \frac{BI}{\Delta p}$$

o the investors' breakeven constraint:

$$p_H(R^SI - R_b^S) + (1 - p_H)(R^FI - R_b^F) \ge I - A$$

• In equilibrium, both constraints will be binding.

• As before, the entrepreneur receives all the NPV:

$$U_b = (p_H R + R^F - 1)I$$

- In equilibrium, the entrepreneur receives nothing following failure:  $R_b^F = 0$ .
  - O Suppose instead  $R_b^F > 0$ . Then one can reduce it, and increase  $R_b^S$ , at a rate  $\frac{\Delta R_b^F}{\Delta R_b^S} = -\frac{p_H}{1-p_H}$ , keeping the breakeven constraint binding and the entrepreneur's utility unchanged; but this would make the incentive compatibility constraint slack a contradiction.
- An all-equity firm is not optimal
  - $\circ$  With no debt, the entrepreneur would, after a failure, receive her share of  $R^FI$  corresponding to her share of the firm's stocks.
- Outside investors must hold debt  $D \ge R^F I$ .
- Borrowing capacity: I = kA, and so D = I A = dA = (k 1)A, where now

$$k = \frac{1}{1 - \left[ p_H \left( R - \frac{B}{\Delta p} \right) + R^F \right]}.$$

- Firms borrow more
  - o the lower agency costs are;
  - o the more liquid assets are.
- Incentives are maximized when outside investors hold a combination of debt and equity.

## Extensions of the analysis

- Supplementary sections to chapter 3
- A *continuum of effort levels*, disutility of effort *g*(*e*)
- A *continuum of outcomes*, probability of outcome R with effort level e is p(R|e).
- Linking effort and outcome: higher effort tends to increase income *the monotone likelihood ratio property* (MLRP)

$$\left| \frac{\partial}{\partial R} \left[ \frac{\partial p(R \mid e)}{\partial e} \right] > 0$$

- Essentially same result: A *standard debt contract* making entrepreneur a residual claimant for the marginal income above the debt repayment level
- <u>Risk aversion</u> brings in another problem: the *insurance/incentives tradeoff*.
  - Providing incentives means making the risk averse entrepreneur take part in the lottery.
  - A solution exists if effort can be verified after contracts are signed, but before outcome is realized, so that contracts can be *renegotiated*. This makes it possible to separate the insurance and incentives problems.

### • Semi-verifiable outcome

- Outcome from investment not verifiable, unless outside investors incur an *audit cost*.
- The incentive problem is related to hiding income, rather than to enjoying private benefits or reducing effort.
- Outcome is reported by entrepreneur:  $\hat{R}$ .
- o The problem for outsiders is to induce truthful reporting.
- Contract now includes a probability  $y(\hat{R})$  of no audit for each report  $\hat{R}$ .
- o Again, a standard debt contract.

### • Non-verifiable outcome

- o Not even an audit can verify outcome.
- Repayment is the result of threats of termination or nonfinancing of future projects.