UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Term paper in: **ECON4310 – Consumption, investment and pensions**

Handed out: Wednesday, October 20, 2004

To be delivered by: Wednesday, October 27, 2004

Place of delivery: Department office, 12^{th} floor

Further instructions:

- This term paper is **compulsory**.
- You must use a printed front page, which will be found at http://www.oekonomi.uio.no/info/EMNER/Forside_obl_eng.doc
- It is of importance that the term paper is delivered by the deadline (see above). Term papers delivered after the deadline, **will not be corrected**.*)
- All term papers must be delivered to the place given above. You must not deliver your term paper to the course teacher or send it by e-mail. If you want to hand in your term paper **before** the deadline, please contact the department office on 12^{th} floor.
- If the term paper is not accepted, you will be given a new attempt. If you still not succeed, you will not be permitted to take the exam in this course. You will then be withdrawn from the exam, so that this will not be an attempt.

*) If a student believes that she or he has a good cause not to meet the deadline (e.g. illness) she or he should discuss the matter with the course teacher and seek a formal extension. Normally extension will only be granted when there is a good reason backed by supporting evidence (e.g. medical certificate).

Econ 4310, Problem Set 4
SEMESTEROPPGAVE, 2004

Due Thursday October 27 at 16:00, 2004
in Storesletten's mailbox (12th floor, Eilert Sundt-huset).

Note: It is mandatory to hand in this problem set.

1 Consumption-based CAPM

1. Consider the following economy: $N(t) = 100$, $\omega_t^h [2, 0]$,

$$u_t^h = E_t \left[v\left(c_t^h(t)\right) + \beta v\left(c_t^h(t+1)\right) \right],$$

for all t, where $\beta = 0.9$ is the subjective discount factor and the instantaneous utility is logarithmic $v\left(c_t^h(t)\right) = \log\left(c_t^h(t)\right)$. Assume that we have 25 units of land and that each unit of land yields the following stochastic crop:

$$d(t) = 2 + \varepsilon_t$$

$$\varepsilon_t = \left\{ \begin{array}{c} 1 \text{ with probability } \frac{1}{2} \\ -1 \text{ with probability } \frac{1}{2} \end{array} \right\}$$

(a) Find the interest rate, the price of land and the expected equity premium in a stationary equilibrium.

(b) Compare them with the equilibrium prices in the case of no uncertainty: $\varepsilon_t = 0$. How are the equilibrium prices affected by uncertainty.

2. Assume that the utility function is

$$u_t^h\left(c_t^h(t+1)\right) = \log\left(c_t^h(t)\right) + \beta \cdot \log\left(c_t^h(t+1)\right).$$

and that endowments are constant over time; $\omega_t^h = [w_1, w_2]$. The population is constant at $N(t) = N$, and there exists two assets, private lending (which pays an interest rate $r(t)$) and A units of land, which pays a stochastic crop $d(t) = 1 + \varepsilon(t+1)$ per unit of land in period t. The crop has a variance $var(d(t)) = \sigma^2$ and the price of land is denoted $p(t)$.

(a) Write down equilibrium conditions for this economy.

(b) The "risk premium" is defined as the expected return on land minus the interest rate. **Discuss how the risk premium depends on three factors**: *(i)* the riskiness of the crop (i.e. on σ^2), *(ii)* the variability of consumption of agents who hold the risky assets (i.e. the old), and *(iii)* the correlation between the return on land and the consumption of the old agents. *[Hint: for this and the following questions it is helpful to consider the formula expressing expected return on a risky asset as a function of consumption, the safe interest rate and the standard deviation of risky return]*

(c) Suppose there are two identical countries situated next to each other. Rain is critical for getting a good crop, but each period it rains only in one of the countries. Therefore, when country 1 gets a crop of $d_t^1 = 1 + \varepsilon_t$, the other country gets $d_t^2 = 1 - \varepsilon_t$. Thus, the stochastic crop is negatively correlated (i.e. $corr\left(d_t^1, d_t^2\right) = -1$). Suppose that the countries initially are closed, i.e. that it is initially illegal to own land in a foreign country or to borrow or lend across the border. Thus, initially the equilibrium is as above (in 2 *(a)-(b)*), and the prices of land are the same in both countries (although the returns are negatively correlated). **Explain why agents in country 1 (or country 2) are interested in buying land in the other country, given the prices (of the closed economies). In particular, explain why, in equilibrium, the young agents in each country would like to invest half of their savings in country 1 land and the other half in country 2 land**.

(d) Given the portfolio allocation above, **what will the new risk premium be**?

(e) Suppose instead that the countries liberalize cross-border borrowing and lending, but prohibit agents from owning land abroad. **Explain why the equilibrium will be the same as when the countries remained completely closed**.

2 Savings

Consider a storage economy with return on storage of λ and endowments $\omega_t = [\omega_1, \omega_2]$. The population is constant and agents have preferences

$$u_t^h = u\left(c_t^h(t)\right) + \beta u\left(c_t^h(t+1)\right)$$

and their budget constraints are

$$\begin{aligned} c_t^h(t) &\leq \omega_1 - l(t) - k(t+1) \\ c_t^h(t) &\leq \omega_2 + r(t)\, l(t) + \lambda k(t+1) \end{aligned}$$

1. Start by assuming that $u(c) = \log(c)$ and that there is positive storage in equilibrium (so $r(t) = \lambda$). Compute individual savings (as a function of the parameters).

2. Explain what happens to savings as (1) old-age endowment falls (lower ω_2), and (2) the interest rate increases (higher λ). Relate these findings to the motives driving individual savings.

3. Suppose that endowments when old are risky, i.e. that $\omega_t = [1, \tilde{\omega}]$, where $\tilde{\omega}$ is a stochastic variable with $E(\tilde{\omega}) = 1$. Describe in words what will happen to savings as the riskiness of $\tilde{\omega}$ increases. Can you prove it?

4. Consider now a different utility function, namely $u(c) = ac - \frac{1}{2}c^2$, and assume for simplicity that $\lambda = 1/\beta$. Show that there will be no precautionary savings. Can you give the intuition for this result?

3 Land

Suppose the utility function is given by

$$u_t^h = \log c_t^h(t) + \frac{1}{2}\log\left(c_t^h(t+1)\right)$$

and that endowments are $\omega_t^h = [12, 2]$. The population is constant at $N = 10$ and there exists two assets, private lending (which pays an interest rate $r(t)$) and $A = 10$ units of land, which pays a dividend of $d = 1$ per unit of land.

1. (a) Write down the budget constraints for this problem and show that the optimal (individual) savings function is

$$s_t^h = \frac{w}{3} - \frac{4}{3r(t)}$$

(b) Write down the equilibrium conditions and show that the competitive equilibrium is unique and that the interest rate is given by $r(t) = 4/3$. [Hint: it is convenient to use diagrammatic analysis in order to show uniqueness.]

(c) Suppose that the dividend on land became risky (and i.i.d. over time). Explain why the return on land would be positively correlated with consumption of the old in the competitive equilibrium.

(d) Based on the answer in (c) and stuff covered in class, what would happen to the (expected) return on land in equilibrium, relative to the return on private lending? Please motivate your answer.

(e) Suppose an environmental catastrophe strikes land in this economy, such that dividends fall to $d = 0$ for all future. Show that now there are two stationary equilibria, one with price of land $p = 0$ and one with $p = \frac{8}{3}$. What is the associated equilibrium equilibrium interest rates? Are there any other equilibria?

(f) Show that the economy with a positive value of land Pareto dominates the equilibrium with $p = 0$. What is the intuition for this result?