Intertemporal macroeconomics

Econ 4310 Lecture 1

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Some practical information

- Reading list with net addresses
- Lecture plan
- No lectures next week
- Extra lecture the week after
- Matlab course Wednesday and Thursday, Espen Henriksen, 14.15-16 and 16.15-18
- Seminars start later (not week 36)

Introduction

- Explaining the behavior of economic aggregates over time
- Micro-based macro theory
- Starting from static general equilibrium
- Extending it with a time dimension and uncertainty
- Intertemporal choice (saving and investment decisions)
- Flows accumulating to stocks
  - Investment to capital
  - Saving to wealth
- Trade period by period
- Financial assets

ECON4310 Some substantial issues

- Economic growth
- Government deficits and debts
- Public pension schemes
- Petroleum funds
- Business cycles
- Asset pricing

Basic questions about research strategy

- Equilibrium or disequilibrium?
- Always start with individual agents optimizing?
- Rational opinions or animal spirits?
- Money - important or not?

Less emphasis in ECON4310

- Money, monetary policy (ECON4325, ECON4330)
- Open economy issues (ECON4330)

This lecture

1. A simple static general equilibrium model
2. Solow's growth model

A simple, static general equilibrium model

- Source: Williamson 1.1
- Example of micro approach to macro
- Static starting point
- Need to refresh micro theory?

Model structure

- N identical households
- Two goods: consumption (c) and leisure ($0 \leq \ell \leq 1$)
- M identical firms, constant returns to scale in each
- Two factors of production: labor n and capital k
- The aggregate capital stock K is exogenously given
- Three markets: output / consumption, labor, capital
- Two relative prices: real wage (w) and real rental price of capital (r). (The consumption good is the numeraire with price 1)
- All agents are price-takers

Competitive equilibrium

1. Each household chooses c and ℓ optimally given w and r.
2. Each firm chooses n and k optimally given w and r.
3. All markets clear.The choices of consumers and producers are mutually consistent.

Households

Maximize:

$$U = u(c, \ell) \tag{1}$$

with respect to c and ℓ given budget

$$c = w(1 - \ell) + rK/N \tag{2}$$

$0 \leq \ell \leq 1$, $1 - \ell$ being labor supply.
Each household owns the same share of K.
First order condition:

$$\frac{u_2(c, \ell)}{u_1(c, \ell)} = w \tag{3}$$

(2) and (3) can be solved for c and ℓ as functions of w, r and K/L.
Regularity conditions on u to ensure that there is one and only one solution (e.g. u strictly increasing and quasi-concave u).

Firms

Production function:

$$y = zf(k, n) \tag{4}$$

$z =$ exogenous productivity shock
Constant returns to scale:

$$f(\lambda k, \lambda n) = \lambda f(k, n)$$

Marginal cost constant and equal to average cost.

"First-order conditions for maximum profit":

$$zf_1(k, n) = r \tag{5}$$
$$zf_2(k, n) = w \tag{6}$$

Is there a (unique) profit maximum when returns to scale are constant?

Firm behavior with constant returns to scale (CRS)

First order condition for minimum cost:

$$\frac{zf_1(k,n)}{zf_2(k,n)} = \frac{r}{w} \tag{7}$$

Zero profit condition:

$$zf(k,n) - rk - wn = 0 \tag{8}$$

Necessary for equilibrium with finite and strictly positive output.

The marginal-productivity conditions, (5) and (6), are equivalent to the minimum cost and the zero profit condition, (7) and (8).

- No profits to distribute to owners / consumers
- Distribution of output on firms indeterminate (assume equal)

Proof of equivalence 1

1) (5) and (6) $\Longrightarrow$ (7) and (8)
(5) and (6) obviously imply (7). To prove that zero profits is also implied, we need the Euler equation

$$f(k,n) = f_1(k,n)k + f_2(k,n)n \tag{9}$$

(This is a general property of functions that are homogenous of degree one, easily proved by differentiating with respect to λ).
Insertion from (5) and (6) in Euler equation yields

$$f(k,n) = rk/z + wn/z$$

or after rearrangement

$$zf(k,n) - rk - wn = 0$$

which is the zero profit condition.

Proof of equivalence 2

2) (7) and (8) $\Longrightarrow$ (5) and (6)

Combining the Euler equation and the zero profit condition yields:

$$zf_1(k,n)k + zf_2(k,n)n = rk + wn$$

Use the condition for cost-minimization to eliminate f_2 and solve for f_1. You then end up with (5) and (6)follows accordingly.

Market clearing

Labor market:

$$N(1-\ell) = Mn \tag{10}$$

Capital market:

$$K = Mk \tag{11}$$

Goods market:

$$Nc = My \tag{12}$$

Competitive equilibrium

1. Each household chooses c and ℓ optimally given w and r. Eq: (2) (3)
2. Each firm chooses n and k optimally given w and r. Eq: (4) (5) (6)
3. All markets clear.The choices of consumers and producers are mutually consistent. (10) (11) (12)

8 equations, 7 unknowns: w, r, n, k, c, ℓ, y

Walras' law: When n-1 markets are in equilibrium, the n'th market will also be in equilibrium.

Representative consumers and producers: Let $M = N = 1$

Properties of equilibrium

$$\frac{u_2(c,\ell)}{u_1(c,\ell)} = zf_2(k, 1-\ell) = w \tag{13}$$

$$c = zf(k, 1-\ell) \tag{14}$$

$$r = zf_1(k, 1-\ell), \;\; w = zf_2(k, 1-\ell) \tag{15}$$

Output per capita determined by:

- Capital stock
- Productivity
- Preference for leisure

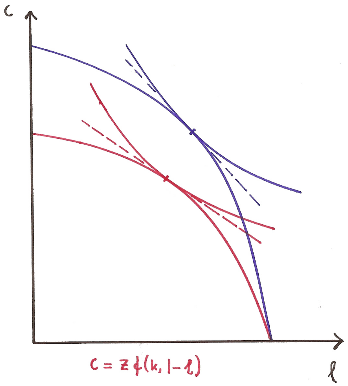
k given at the macro level

What if..?

- Productivity increases?
- Preferences for leisure increases?
- More capital is available?
- A government starts taxing and spending?

The effect of increased productivity

Income effect favors leisure, substitution effect work



Some properties of competitive equilibria

Existence, uniqueness

- Competitive equilibrium exists provided preferences and production possibilities satisfy some regularity conditions and markets are complete
- With sufficient regularity conditions we also get unique solutions for prices and individual utilities, but not necessarily a unique distribution of production among firms (c.f. constant returns case).

Welfare theorems

1. A competitive equilibrium is Pareto optimal
2. All Pareto optimal allocation can be achieved by appropriate redistributions of the initial endowments.

Caveats: No external effects, no collective goods, no increasing returns, complete markets, no asymmetric, information, no distorting taxes, etc

Social optimum

- Requires a welfare function that makes interpersonal comparisons
- Pareto optimum necessary, but not sufficient for social optimum.
- With a single consumer Social optimum, Pareto optimum and competitive equilibrium coincide

Social planner's problem

$$\max\ U = u(c, \ell) \tag{16}$$

$$\text{given } c = zf(k, 1-\ell) \tag{17}$$

First-order condition

$$\frac{u_2(c, \ell)}{u_1(c, \ell)} = zf_2(k, 1-\ell) \tag{18}$$

Solution to planning problem is also market solution.

Solow's growth model

- Source: Romer Ch. 1
- Discrete versus continuous time
- Read Ch. 1 on your own

Solow's growth model

$$Y_t = F(K_t, A_t L_t) \qquad (1)$$

$$I_t = sY_t \qquad (2)$$

$$K_{t+1} = K_t + I_t \qquad (3)$$

$$L_t = L_0(1+n)^t \qquad (4)$$

$$A_t = A_0(1+g)^t \qquad (5)$$

Y_t = output
K_t = capital,
L_t = labor
I_t = investment
A_t = labor productivity

F = production function
F: constant returns to scale,
$F_1, F_2 > 0, F_{11}, F_{22} < 0$
K_0, L_0 given

Solving the model

$$K_{t+1} - K_t = I_t = sF(K_t, A_t L_t) \tag{6}$$

$A_t L_t$ = Labor input in efficiency units.

$$A_t L_t = A_0 L_0 [(1+g)(1+n)]^t = A_0 L_0 (1+\gamma)^t$$

Divide through (6) by $A_t L_t$:

$$\frac{K_{t+1}}{A_t L_t} - \frac{K_t}{A_t L_t} = sF\left(\frac{K_t}{A_t L_t}, \frac{A_t L_t}{A_t L_t}\right)$$

Define $k = K/AL$, $f(k) = F(k, 1)$

$$k_{t+1}(1+\gamma) - k_t = sf(k_t) \tag{7}$$

Assumed properties of f:

$$f'(k) > 0, \;\; f''(k) < 0$$

$$f(0) = 0, \;\; f'(0) = \infty, \;\; f'(\infty) = 0$$

Wages and rental price of capital:

$$r_t = F_1(K_t, A_t L_t) = f'(k_t)$$

$$w_t = A_t F_2(K_t, A_t L_t) = A_t[f(k_t) - k_t f'(k_t)]$$

(Use the Euler equation and that when $F(X_1, X_2)$ is homogeneous of degree 1, then $F_i'(X_1, X_2)$ is homogenous of degree 0).

The steady state

$$k_{t+1}(1+\gamma) - k_t = sf(k_t)$$

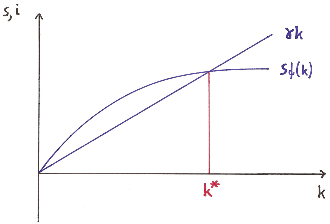
Steady state k^* defined by $k_{t+1} = k_t = k^*$ or

$$k^*(1+\gamma) - k^* = sf(k^*)$$

$$sf(k^*) = \gamma k^* \qquad (8)$$

γk^* = investment needed to keep up with growth in AL.

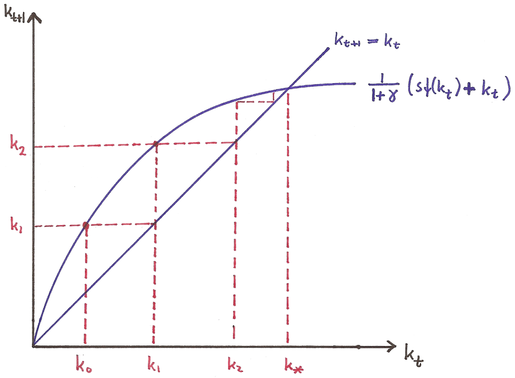
Two steady states, one with $k^* = 0$, one with $k^* > 0$



More about the steady state

- Growth rate of output per capita is equal to productivity growth rate g
- Capital intensity, k^* depends positively on s, negatively on n and g.
- Level of output depends positively on s, negatively on n
- Real interest rate, $r^* = f'(k^*)$ depends negatively on s, positively on n and g
- Real wage, $w^* = A[f(k^*) - k^* f'(k^*)]$, grows with rate of productivity growth
- The share of wage income in total output is constant.
- Level of real wage depends positively on s, negatively on n and g

Transitional dynamics



The Golden Rule of Accumulation

Consumption per efficiency unit of labor in steady state is:

$$c = f(k) - \gamma k \tag{9}$$

First order condition for maximum is $f'(k) - \gamma = 0$. Golden rule level of k, k^{**} is determined by

$$f'(k^{**}) = \gamma \tag{10}$$

$$r^{**} = \gamma$$

Interest rate equal to natural growth rate
Savings rate required to reach k^{**}:

$$s^{**} = \gamma k^{**}/f(k^{**}) = r^{**}k^{**}/f(k^{**})$$

Along the Golden rule path the savings rate equals the income share of capital.
If s is increased beyond s^{**}, consumption is reduced both now and in all future!

Some questions

1. Is it conceivable that rational agents will save too much for society's long-run good?
2. Should society aim at the golden rule level of capital in the long run?
3. How much private saving should we expect in a market equilibrium?

Discrete versus continuous time

$$k_{t+1} - k_t = sf(k_t) - \gamma k_{t+1}$$

$$sf(k^*) = \gamma k^*$$

$$\gamma = n + g + ng$$

$$\dot{k}(t) = sf(k(t)) - \gamma k(t)$$

$$sf(k^*) = (n + g)k^*$$

$$\gamma = n + g$$