Intertemporal macroeconomics

Econ 4310 Lecture 11

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The permanent income hypothesis

$$U = \sum_{t=1}^{T} u(C_t), \qquad u' > 0, u'' < 0 \tag{1}$$

$$\sum_{t=1}^{T} C_t = A_0 + \sum_{t=1}^{T} Y_t \tag{2}$$

$$u'(C_t) = u'(C_{t+1}) \tag{3}$$

Solution:

$$C_t = \frac{1}{T}\left(A_0 + \sum_{t=1}^{T} Y_t\right) = \bar{Y} \qquad t = 1, 2, \ldots, T \tag{4}$$

$\bar{Y}$ = permanent income, $Y_t - \bar{Y}$ = transitory income (Milton Friedman)
Assumptions: Zero interest rate, no discounting, no uncertainty.

Propensities to save

Keynes' "fundamental psychological law":

$$C_t = a + bY_t, \quad S_t = (1-b)Y_t - a, \quad 0 < b < 1, a > 0$$

Friedman's permanent income hypothesis:

$$C_t = \bar{Y}, \quad S_t = Y_t - \bar{Y}$$

Three apparent facts to explain:

- The savings rate declines in recessions, increases in booms.
- Savings rates are on average lower for low income households
- The savings rate has no long-run trend

Friedman versus Keynes 3 - 2

Our old workhorse

$$U = \sum_{t=1}^{T} \beta^t u(C_t), \beta = 1/(1+\rho) \tag{5}$$

$$\sum_{t=1}^{T}(1+r)^{-(t-1)}C_t = A_0(1+r) + \sum_{t=1}^{T}(1+r)^{-(t-1)}Y_t = W_1 \tag{6}$$

$$u'(C_t) = \beta(1+r)u'(C_{t+1}) \tag{7}$$

$$C_t = C_{t+1} \Leftrightarrow \beta(1+r) = 1 \Leftrightarrow r = \rho$$

Consumption grows when $r > \rho$, declines when $r < \rho$
Consumption growth rate is independent of income growth rate

$$C_t = f(r, W_t, T-t), \qquad t = 1, 2, \ldots, T \tag{8}$$

Consumption depends on total wealth $f_1' < 0$, $f_2' > 0$, dC_t/dr ambiguous

CES-utility

$$u(C) = (C^{1-\theta} - 1)/(1-\theta), \quad \theta = 1/\sigma$$

$$\frac{C_{t+1}}{C_t} = [\beta(1+r)]^\sigma = \left(\frac{1+r}{1+\rho}\right)^\sigma = 1 + g_C$$

Constant consumption growth rate when r is constant

$$C_t = f(r, T-t)W_t \tag{9}$$

Consumption *proportional* to total wealth

A digression on permanent income

Permanent income defined by $\sum_{t=1}^{T}(1+r)^{-(t-1)}\bar{Y} = W_1$ or

$$\bar{Y} = \frac{r}{1-(1+r)^{-T}} W_1/(1+r)$$

Infinite horizon ($T \to \infty$)

$$\bar{Y} = rW_1/(1+r) = r\left(A_0 + \sum_{t=1}^{T}(1+r)^{-t} Y_t\right)$$

If you want constant consumption to eternity, consume the real interest on your total wealth.

Constant growth rates, infinite horizon

Income growth rate $g_Y < r$, consumption growth rate $g_C < r$
Insertion in budget equation yields:

$$C_t = (r - g_C)A_t + \frac{r - g_C}{r - g_Y} Y_t = (r - g_C)W_t/(1 + r)$$

- Spend only the difference between the real interest rate and the consumption growth rate

Define $a(t) = A(t)/Y(t)$

- ▷ If $g_C < g_Y$, then a_t converges to $a_* = (g_C - g_Y)/(r - g_Y) < 0$
- ▷ If $g_C > g_Y$, then $a_t \to \infty$ when $t \to \infty$ if $a_0 > a_*$ ($a_t \to -\infty$ if $a_0 < a_*$)

Income uncertainty with quadratic utility

- ▷ Labor income uncertain.
- ▷ No trend in consumption, $r = 0$, $\rho = 0$
- ▷ Quadratic utility, $u(C) = C - aC^2/2$, $a > 0$

$$E[U] = \left[\sum_{t=1}^{T} \left(C_t - \frac{a}{2}C^2\right)\right] \tag{10}$$

Change in consumption is unpredictable!

Consumption Euler equation: $u'(C_t) = E_t[u'(C_{t+1})]$ In this case:

$$1 - aC_t = E_t[1 - aC_{t+1}], \qquad t = 2, 3, \ldots, T$$

$$C_t = E_t[C_{t+1}] \tag{11}$$

$$C_t = E_{t-1}[C_t] + e_t, \qquad E_{t-1}e_t = 0$$

$$C_t = C_{t-1} + e_t, \qquad t = 2, 3, \ldots, T \tag{12}$$

Hall's random walk hypothesis

Certainty equivalence

By law of iterated expectations

$$E_1[C_t] = C_1, \qquad t = 2, 3, \ldots, T$$

Plans have to satisfy budget constraint:

$$\sum_{t=1}^{T} E_1[C_t] = A_0 + \sum_{t=1}^{T} E_1[Y_t]$$

Consume as if expected income were certain:

$$C_1 = \frac{1}{T}\left(A_0 + \sum_{t=1}^{T} E_1[Y_t]\right) \tag{13}$$

Change in C determined by income surprise

$$C_{t+1} - C_t = \frac{1}{T-t}\left(\sum_{j=t+1}^{T} E_{t+1}[Y_j] - \sum_{j=t+1}^{T} E_t[Y_j]\right) \tag{14}$$

The effect of $Y_{t+1} - E_t[Y_{t+1}]$ on C_{t+1} depends on how the news about Y_{t+1} affect expectations in future periods.

Observations on the theory

- ▷ The purpose of saving is future consumption
- ▷ Consumption depends on total wealth, only indirectly on current income
- ▷ Consumption growth is independent of expected income growth
- ▷ Savings are high when incomes are temporarily high
- ▷ Except for the trend growth, changes in consumption are not predictable
- ▷ Savings depend on the interest rate, but the effect is ambiguous

Uncomfortable facts

- ▷ Consumption profiles are steeper for individuals in professions with steeper income profiles
- ▷ Consumption growth is higher in countries with high income growth
- ▷ Consumption has been growing rapidly over long period when the real interest rate was close to zero (risk-free rate puzzle)
- ▷ Changes in consumption (relative to the trend) can often be predicted
- ▷ Many consumers save little, a few save a lot

An advantage of life-cycle theory

- Saving when young, dissaving when old
- Population growth: More saving then dissaving
- Young earn more than old: More saving
- Aggregate saving positively correlated with growth

Precautionary saving

$$
\begin{aligned}
u'(C_t) &= E_t[(u'(C_{t+1})] \\
&\approx E_t\left[u'(C_t) + u''(C_t)(C_{t+1} - C_t) + \frac{1}{2}u'''(C_t)(C_{t+1} - Ct)^2\right]
\end{aligned}
$$

$$
\frac{E_t(C_{t+1}) - C_t}{C_t} \approx -\frac{1}{2}\frac{u'''(C_t)C_t}{u''(C_t)}E_t\left(\frac{C_{t+1} - C_t}{C_t}\right)^2 \qquad (15)
$$

With CES utility:

$$
\frac{E_t(C_{t+1}) - C_t}{C_t} \approx \frac{1}{2}(1+\theta)Var_t\left(\frac{C_{t+1} - C_t}{C_t}\right) \qquad (16)
$$

Don't sell the skin before ....!

$$g_{t+1} \approx \frac{1}{2}(1+\theta)Var_t(g_{t+1}), \qquad g_{t+1} = (E_t(C_{t+1}) - C_t)/C_t \tag{17}$$

More uncertainty about future income implies

- ▷ Steeper expected consumption growth
- ▷ More saving early in life
- ▷ Larger net assets

Precautionary motive may make age profiles of consumption and income closer.

Credit constraints

Some potential motives for borrowing:

- ▷ High expected future incomes
- ▷ Impatience
- ▷ Negative real interest rate (after tax)
- ▷ Investment i durable consumer goods
- ▷ Investment in education

Prudent lenders will limit borrowing when future ability to pay is uncertain.

High expectations, an example

Quadratic utility, no discounting, zero interest rate, two periods.
Optimum without credit constraints:

$$C_1 = (1/2)(A_0 + Y_1 + E_1(Y_2)) \tag{18}$$

Consumer wishes to borrow if $E_1(Y_2) > A_0 + Y_1$. Credit constraint $C_1 \leq A_O + Y_1 + \bar{L}$

$$C_1 = \min[(1/2)(A_0 + Y_1 + E_1(Y_2)), A_O + Y_1 + \bar{L}] \tag{19}$$

Constraint binds if

$$E_1[Y_2] > A_0 + Y_1 + 2\bar{L}$$

Potential future constraint reduces consumption now

Introduce period 0 (with 0 initial assets)

$$E_0[C_1] < \frac{1}{2}(A_0 + E_0[Y_1 + Y_2])$$

Since $C_0 = E_0[C_1]$

$$\begin{aligned} C_0 &< \frac{1}{2}(A_0 + E_0[Y_1 + Y_2]) \\ C_0 &< \frac{1}{2}(Y_0 - C_0) + E_0[Y_1 + Y_2]) \\ C_0 &< \frac{1}{3}E_0[Y_0 + Y_1 + Y_2] \end{aligned}$$

Progress report

- Precautionary motive and credit constraints can explain why consumption profiles are closer to income profiles early in life
- Increased credit availability can explain some consumption booms
- Precautionary motive and retirement motive can explain why saving is higher in fast-growing countries
- Changed perceptions of risk can affect consumption.

Why do US households save so little?

Why do households in some welfare states save so much?

Alternative theories and extensions

- Durable goods
- Rational habit formation and investment in good memories
- Lack of will-power or inability to plan
- Savings as residual: Peer comparisons, unconsciously acquired habits
- Saving to become wealthy: Status, power, business opportunities
- "Animal spirits". Changed perceptions of future as source of fluctuations