ECON4310 Fall 2009: Problem set 2

Due October 12

1 Bequests

(Note that in this question some thinking may reduce tedious calculations)

Consider a household that lives for four periods and which has utility function

$$\ln(c1) + \ln(c2) + \ln(c3) + \ln(c4) \tag{1.1}$$

Income in the four periods is: $y1 = 10; y2 = 40; y3 = 20$ and $y4 = 10$. Assume the interest rate is exogenously given and constant equal to 0.

(a) Write down the intertemporal budget constraint.

(b) Compute the optimal consumption choices $(c1; c2; c3; c4)$

(c) Suppose the household cannot borrow. Now what are the optimal consumption choices?

Now consider two members of the same dynasty that both live for two periods. Children have utility function

$$\ln(c3) + \ln(c4) \tag{1.2}$$

and parents have the utility function

$$\ln(c1) + \ln(c2) + v(b) \tag{1.3}$$

where b are the bequests left to the children and v (b) is the maximal utility children can obtain when given bequests b: In- come of parents is $(y1, y2) = (10; 40)$ and that of children is $(y3, y4) = (20, 10)$

(e) Solve the maximization problem of the children to obtain v(b); that is,solve

$$v(b) = \max\{\ln(c3) + \ln(c4)\} \tag{1.4}$$

subject to

$$c3 + c4 = y3 + y4 + b. \tag{1.5}$$

(f) Use your answer from the previous question to solve the parents maximization problem. Allow bequests to be negative.

(g) Now suppose the government introduces taxes in period 2 of 30 and distributes 30 in lump-sum transfers in period 3. What is the optimal bequest level and consumption allocation now?

2 The Diamond model

In this exercise we will be looking at the steady-state of the Diamond-model. We assume zero growth in population and in productivity.

The consumers have utility functions

$$U = u(c_1) + \frac{1}{1+\rho} u(c_2) \tag{2.1}$$

where c_1 and c_2 are consumption when young and old respectively, and

$$u(c) = \begin{cases} \frac{1}{1-(1/\sigma)}[c^{1-(1/\sigma)} - 1] & \text{if } \sigma \neq 1 \\ \\ \ln(c) & \text{if } \sigma = 1 \end{cases} \tag{2.2}$$

The per-capita production function is

$$y = f(k) = k^{\alpha} \tag{2.3}$$

(a) The consumers supply one unit of labor when young and receive a wage w. They do not work when they are old. Show that the savings of the young can be expressed as

$$s_1 = s(r)w \equiv \frac{(1+r)^{\sigma-1}}{(1+\rho)^{\sigma} + (1+r)^{\sigma-1}} w \tag{2.4}$$

Discuss briefly how s_1 depends on r for a given w.

(b) Indirectly there will also be a connection between r and w. The first-order condition for capital (with zero depreciation)

$$r = \alpha k^{\alpha-1} \tag{2.5}$$

can be inverted to yield $k = (r/\alpha)^{-1/(1-\alpha)}$. The first order condition for labor is

$$w = (1-\alpha)k^{\alpha} \tag{2.6}$$

Hence

$$w = (1-\alpha)(r/\alpha)^{-\alpha/(1-\alpha)}$$

and

$$s_1 = s(r)(1-\alpha)(r/\alpha)^{-\alpha/(1-\alpha)} \tag{2.7}$$

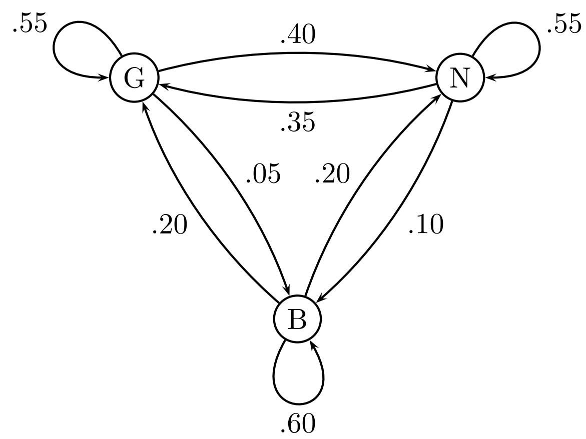
Discuss the total effect of r on s_1.

(c) In the capital market (5) describes the demand for capital by firms, while (7) describes the supply of capital from consumers. Sketch the equilibrium in the capital market in the usual demand-supply diagram. How would an increase in ρ affect the equilibrium?

(d) Use Matlab to draw the deamand-supply diagram for the capital market for $\alpha = 0.2$, $\rho = 0.2$ and four different values for σ: 0.1, 0.5, 1 and 2. Comment.

3 Computation

You are set to compute the net present value of all future cash flows from a traded company. You know that the company can be in a good state (G), a normal state (N) or a bad state (B). If it is in a good state the free cash flow is NOK 20M, if it is in a normal state the free cash flow is NOK 10M, and if it is in a bad state it loses money and the free cash flow is NOK -5M. The transition probabilities can be measured from historical data. The results can be represented by the following figure:



(a) Write up the Markov transition matrix, i.e. the matrix where a typical element P_{ij} gives the probability of going from state i to state j.

(b) Define V_G, V_N, V_B as the present value of starting in state G, N, B today, and changing states according to the above Markov matrix. Use a recursive set up to find a set of three equations to determine the values.

(c) The period-by-period discount rate is 4%. Compute the value of the company (i.e. the net present value of the free cash flows) in each realization of the state. Comment on the difference in price in the different realizations of the state.

(d) The period-by-period discount rate changes to 3%. Recompute the value of the company. Comment briefly on the results.