Problem set 3 due October 27

1 Labor supply (from last year's exam)

Consider the savings and labor effort problem of an agent who lives for two periods and earns a wage of w_1 when young and w_2 when old. The interest rate is r and the agent maximizes

$$\max_{c_1,c_2,h_1,h_2} \left\{ u\left(c_1,h_1\right) + \beta u\left(c_2,h_2\right) \right\},$$

subject to

$$c_1 + \frac{c_2}{r} \leq w_1 h_1 + \frac{w_2 h_2}{r},$$

where c_t is consumption and h_t is labor supply. Assume that the utility function is given by

$$u\left(c,h\right) = \frac{c^{1-\gamma}}{1-\gamma} + \frac{\left(1-h\right)^{1-\sigma}}{1-\sigma}.$$

1. Suppose, first, that $w_2 = 0$, so that individuals work only when young. What is the effect of the wage rate on labor supply? Is it the workers with high wages or the workers with low wages who work more?

2. Over the last 100 years, wages have increased a lot, while labor effort has declined (on average, we have substantially more leisure than our grandparents). What does this tell us about the parameters of the utility function above?

3. Suppose now that individuals work in both periods of life. All workers earn the same wage when young ($w_1 = 1$). However, when old, some workers have a steep upwards age-profile of wages ($w_2 > 1$), while others have a decreasing age-profile of wages ($w_2 < 1$). What will their age-profile of labor supply look like?

From exam H07:

1 3 : Overlapping Generation Model (40%)

Consider an overlapping generations economy where agents live for two period for sure. The economy starts at time 1. Denote the generation born at time t as cohort t. A cohort t agent ($t \geq 1$) has the preference.

$$U_t = \log(c_{1t}) + \log(c_{2t+1})$$

Each individual has one unit of time to work when young. The individual does not work when old.

Population L_t grows at a rate n so that $L_t = (1+n)\, L_{t-1}$.

Assume also there is an initial old cohort at time 1 with size $L_0 = 1$.The initial old is endowed with capital stock K_0, which is given exogenously.

A representative firm hire labor and capital in a competitive market and pay wage rate w_t and interest rate r_t. The production function is

$$Y_t = e^{z_t} K_t^{\alpha} \left(\gamma^t L_t\right)^{1-\alpha}$$

where $\alpha \in (0,1)$, γ is the gross trend growth rate of this economy, and z_t is the deviation of aggregate productivity from the trend growth path. For simplicity, we assume that all agents have perfect foresight about the fluctuation of z_t. Finally, capital stock depreciates fully after one period so that

$$K_{t+1} + C_t = Y_t$$

where C_t is aggregate consumption.

1. Write down the problem for a young agent of cohort t $(t \geq 1)$. Define a competitive equilibrium for this economy. (7 points)

2. Show that this economy can be transform into a stationary sequential economy (no trend growth) by a change of variables. (7 points)

3. Given factor prices, solve for the consumption and saving allocation for a cohort t agent in this stationary economy. What is the level of saving at old age for cohort t, denoted as s_{2t+1}? (7 points)

4. Use the capital market clearing condition to derive the law of motion for capital in this stationary economy. (5 points)

5. Suppose the economy is originally at the balanced growth path, with $z = 0$. At time t, each agents in this economy receives news that at time $t+1$ there will be an increase in z_{t+1}, that is $z_t = 0, z_{t+1} > 0$. Show how aggregate saving rate at time t will respond to such news. Explain the intuition. (7 points)

6. Suppose now agents value leisure when young. More specifically, assume utility of an agent born at t is

$$U_t = \log(c_{1t}) + \psi \log(1 - l_t) + \log(c_{2t+1})$$

where l_t is the labor supply when young. Again, assume that at time t each agents in this economy receives news that at time $t+1$ there will be an increase in z_{t+1}, that is $z_t = 0, z_{t+1} > 0$. Verbally describe how labor supply (l_t) for a young agent at time t will respond to such news. Explain the intuition. (7 points)

3. Explain what is meant by the "Golden Rule of Accumulation". How do the levels of capital in the stationary states of Ramsey's and Dimond's growth models compare to the the golden rule level. Give an intuitive explanation. (No maths needed).