

ECON4310 Problem set 4

Due Nov. 3

Question 1

From exam Fall 2004

Consider a standard Real Business Cycle economy. *NOTE: you can answer the questions below without using details of the economy – the details are meant as a way to fix ideas.*

The economy consisting of a large number of identical, price taking firms and a large number of identical, price taking infinitely lived households. Output is given by a Cobb-Douglas function:

$$Y_t = e^{z_t} \cdot K_t^\alpha H_t^{1-\alpha} \quad (1)$$

where T_t is output, K_t is capital stock, and H_t is labor supply. The technology shock evolves according to

$$z_t = \rho z_{t-1} + \varepsilon_t \quad (2)$$

where ε_t is i.i.d. normally distributed disturbance, and capital evolves according to

$$K_{t+1} = (1 - \delta) K_t + I_t,$$

where $\delta < 1$ is the annual depreciation rate and I_t is investment. Output is used for investment and consumption C_t :

$$Y_t = C_t + I_t.$$

There are no frictions in the labor market and agents have standard preferences over consumption and leisure:

$$u(c, h) = \log c + \log(1 - h)$$

1. Explain *briefly* why in a competitive equilibrium prices (rental rates w_t and $r_t + \delta$) should equal marginal productivity of labor and capital.
2. What happens after a positive technology shock? Please describe and give intuition for the evolution of output Y_t , consumption C_t , investment I_t , capital stock K_t , labor supply H_t , wage rate w_t , and interest rate r_t .
3. In what sense is the propagation of business cycles in this model driven by the exogenous technology process and capital accumulation?
4. Can this model shed light on empirical data?

Question 2

Agents maximize utility, i.e. solve the following problem:

$$\max_{\{k_t\}\{h_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(c_t) - \psi \frac{h_t^{1+\sigma}}{1+\sigma} \right\} \quad (1)$$

subject to $\forall t$

$$(1+r_t)k_t + w_t h_t = c_t + k_{t+1}$$

Suppose that labor is “indivisible” in the sense that agents can work either h_0 hours or nothing at all (such assumption can be defended as a technological constraint). For simplicity, take prices as exogenous.

- Consider the problem of a social planner with equal weights on each individual. Show that the planner-problem can be represented as the problem of a representative agent with a different utility function, namely

$$\max_{\{k_t\}\{\alpha_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \{ \log(c_t) - B\alpha_t \}, \quad (2)$$

where α_t is the fraction of agents who work and B is a constant.

- Explain how the planner-solution can be decentralized as a competitive equilibrium outcome (with complete markets) using lotteries over whether to work or not.
- What is the elasticity of substitution of labor supply (i.e., the Frisch elasticity) in the specification of preferences in (1) and (2)?
- Are the unemployed are better or worse off than the employed? Is unemployment Pareto optimal in this model?

Question 3

Explain how researchers in the real business cycle tradition usually quantify their models and how they judge their empirical validity. Mention one or more objections that have been raised to these procedures and try to assess the objection(s).