Final Exam
ECON 4310, Fall 2014

1. Do **not write with pencil**, please use a ball-pen instead.
2. Please answer in **English**. Solutions without traceable outlines, as well as those with unreadable outlines **do not earn points**.
3. Please start a **new page** for **every** short question and for every subquestion of the long questions.

Good Luck!

	Points	Max
Exercise A		**60**
Exercise B		**60**
Exercise C		**60**
Σ		**180**

Grade: __________

Exercise A:
Short Questions (60 Points)

Answer each of the following short questions on a seperate answer sheet by stating as a first answer True/False and then give a short but instructive explanation. You can write, calculate, or draw to explain your answer. You only get points if you have stated the correct True/False *and* provided a correct explanation to the question. We will not assign negative points for incorrect answers.

Exercise A.1: (10 Points) Finite horizon model of intertemporal consumption

Consider the optimal intertemporal consumption choice of a household in discrete and finite time $t = 0, 1, \ldots, T < \infty$. The optimal behvaior is characterized by the consumption Euler equation

$$\frac{c_{t+1}}{c_t} = [\beta(1 + r - \delta)]^{1/\theta},$$

and the private bugdet constraint

$$a_{t+1} + c_t = (1 + r - \delta)a_t, \quad a_0 = 0 \text{ given}, \quad a_{T+1} = 0,$$

where $r - \delta$ is the exogenous interest rate, c_t the individual consumption of the household, $\delta \in (0, 1)$ the depreciation rate of physical capital, $\beta \in (0, 1)$ is the subjective discount factor, and $1/\theta$ the intertemporal elasticity of substitution.

Suppose that $\beta(1 + r - \delta) > 1$, then the household will never borrow (have strictly negative asset holdings) over the life-cycle. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise A.2: (10 Points) OLG model, permanent increase in population growth

Consider the capital accumulation equation of the overlapping generations model with exogenous technology and population growth.

$$k_{t+1} = \frac{(1-\alpha)\beta}{(1+\beta)(1+g)(1+n)} k_t^{\alpha}, \quad k_t \equiv K_t/(A_t L_t),$$

where K_t is the aggregate capital stock, A_t is the state of technology, L_t the size of the population, $\beta \in (0,1)$ the discount factor, $\alpha \in (0,1)$ the capital income share in the economy, and $g \geq 0$ and $n \geq 0$ denote the net growth rate of technology and the population, respectively. The competitive wage rate is given by

$$w_t = (1-\alpha) A_t k_t^{\alpha}.$$

Let the economy be in the stable steady-state, $k > 0$. In response to a permanent increase in the population growth rate from n to $n' > n$ in period t_0 (the current level of population L_{t_0} is unaffected by this shock), the wage rate will jump down on impact and then increase as the economy adjusts gradually to the new steady-state. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise A.3: (10 Points) Optimal fiscal policy, Handlingsregelen

Suppose that wages in Norway will grow at a strictly positive net rate, $n > 0$ in the future, that the government chooses fiscal policy (take-out from the petroleum fund) optimally, and let the value of the oil fund be constant.

A government with a fiscal policy that keeps the value of the petroleum fund relative to wages constant puts a higher relative welfare weight on future generations compared to a government that keeps the absolute value of the oil fund constant. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise A.4: (10 Points) Optimal policy, Laffer curve

Suppose the aggregate labor supply, $h(\tau)$, of an economy as a function of the labor income tax rate, τ, is given by

$$h(\tau) = [(1-\tau)w]^{1/2}.$$

The top of the Laffer curve is given by $\bar{\tau} = 1/2$. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise A.5: (10 Points) Data, long-run trends

Suppose you download quarterly macroeconomic data for mainland Norway. You normalize the data by the corresponding mean of each series and plot the logarithm of each normalized macroeconomic series over time. You find the slope of the time trend for the Gross Domestic Product is the steepest, followed by that of Total Hours Worked, and the Hours Worked per Employee is more or less flat.

This pattern of time trends is in line with the steady-state equilibrium that we characterized for the Ramsey model with technology and population growth. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise A.6: (10 Points) Precautionary savings

Consider the simple two-period real business cycle model discussed in the seminar and in the lecture. With an asset supply of zero, $w_0 = \mathrm{E}[w(s_1)]$, and an optimal consumption profile, $c_0 = w_0$, $c_1(s_1) = w(s_1)$, the stochastic consumption Euler equation in this model is given by

$$\beta(1+r_1) = \frac{u'(c_0)}{\mathrm{E}[u'(c_1(s_1))]} = \frac{u'(\mathrm{E}[w(s_1)])}{\mathrm{E}[u'(w(s_1))]}.$$

The stochastic process for the wage in the second period, $w(s_1)$, takes the form

$$w(s_1) = \begin{cases} w(s_G) = 1 + \sigma/2, & \text{with prob. } 1/2, \\ w(s_B) = 1 - \sigma/2, & \text{with prob. } 1/2, \end{cases}$$

where $\sigma \in (0, 2)$ parametrizes the risk in this economy. Assume that the utility function is of the following form

$$u(c) = 1 - e^{\alpha c},\ \alpha > 0.$$

This utility function, $u(c)$, implies precautionary savings. True or false? (hint: the derivative of the exponential function e^x with respect to x is again the exponential function e^x)

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Exercise B:
Long Question (60 Points)

A comparison of the Solow model and the Ramsey growth model

Consider a closed economy without exogenous technology and population growth, where firms produce a generic good Y_t with the production function

$$Y_t = F(K_t, L) = K_t^{\alpha} L^{1-\alpha}, \quad 0 < \alpha < 1,$$

where K_t is aggregate capital and L is the number of workers in the economy. The law of motion for aggregate capital is given by

$$K_{t+1} = (1-\delta) K_t + I_t, \quad K_0 > 0, \tag{1}$$

where I_t denotes aggregate investment, and $0 < \delta < 1$ the depreciation rate. For simplicity, let aggregate labor supply (population) be equal to one, $L = 1$, such that consumption per worker, c_t, is the same as aggregate consumption, $C_t = c_t = c_t L$.

Consider now two different models. The Solow model where agents have constant savings rate, s, such that

$$I_t = sY_t.$$

And the Ramsey growth model where household utility is maximized

$$\sum_{t=0}^{\infty} \beta^t u(C_t), \tag{2}$$

such that aggregate investment (savings) is endogenous

$$I_t = Y_t - C_t.$$

In both models markets are competitive, thus input factors K_t and L are paid their marginal products.

(a) (10 Points) Compute the wage in the Solow model in a steady state. (hint: compute the aggregate steady state capital stock first.)

(b) (10 Points) In the Ramsey growth model households maximize lifetime utility in Equation (2) with respect to C_t and K_{t+1} and subject to the law of motion of capital stated in Equation in (1). Taking into account the functional form of output, Y_t, and investment, I_t, and that $L = 1$, write up the Lagrangian of this maximization problem and derive the following optimality conditions of the Ramsey growth model

$$\frac{u'(C_t)}{u'(C_{t+1})} = \beta \left[1 + \alpha (K_{t+1})^{\alpha-1} - \delta\right]$$
$$K_{t+1} - K_t = K_t^{\alpha} - \delta K_t - C_t.$$

The first optimality condition is the model's Euler equation and the second the resource constraint.

(c) (5 Points) Compute the wage in the Ramsey model in a steady state. (hint: compute the steady state capital stock first, you can solve this part even if you were not able to solve part (b).)

(d) (10 Points) Does the wage in the Solow model depend on the saving rate s in a steady state? Is the wage increasing or decreasing in s or independent from s. Why? (hint: you can try to answer this question even though you were not able solve previous parts.)

(e) (10 Points) Does the wage in the Ramsey model depend on the discount factor β in a steady-state? Is the wage increasing or decreasing in β or independent from β. Why? (hint: you can try to answer this question even though you were not able solve previous parts.)

(f) (5 Points) Compute the saving rate $\bar{s}$ which gives the same steady-state capital stock in the Solow model as in the Ramsey growth model. Is this saving rate, $\bar{s}$, increasing or decreasing in β?

(g) (5 Points) Compute the the saving rate $\hat{s}$ which gives the same wage in the Solow model as in the Ramsey growth model. Is this saving rate increasing or decreasing in β?

(h) (5 Points) Compare the saving rates $\bar{s}$ computed in part (f) the rate $\hat{s}$ computed in part (g). Explain your findings.

Exercise C:
Long Question (60 Points)

Labor Supply

Consider a representative consumer living for two periods, denoted by $t \in \{1,2\}$, in a small open economy. The consumer has preferences over consumption, c_t, and the hours of labor supplied, h_t, of the following form

$$U = \log(c_1) + \phi \log(1 - h_1) + \beta \left[\log(c_2) + \phi \log(1 - h_2)\right],$$

where log denotes as always the natural logarithm. The real wage is w_1 in period 1 and w_2 in period 2. The consumer has no capital income in the first period as she starts life without any assets, but the consumer may transfer income between periods (savings) at the exogenous world interest rate r.

(a) (15 Points) Set up the optimization problem of this consumer and derive the optimality conditions. Note that consumption, labor supply, and savings are the choice variables of this optimization problem. (hint: you can do the optimization subject to two period-by-period budget constraints or subject to a single net present value budget constraint.)

(b) (10 Points) Derive the optimal supply of labor, h_t, in each period as a function of only exogenous parameters. Note that savings are not necessarily zero. (hint: if you were not able to solve part (a), you can assume that optimal consumption and labor supply is characterized through the intratemporal optimality condition

$$\phi(1 - h_t)^{-1} = c_t^{-1} w_t,$$

the intertemporal optimality condition (Euler equation)

$$\frac{c_2}{c_1} = \beta(1 + r),$$

and the lifetime budget constraint

$$c_1 + \frac{c_2}{1 + r} = h_1 w_1 + \frac{h_2 w_2}{1 + r}.$$

and you will be able to solve the following parts of this exercise.)

(c) (5 Points) Suppose now that the considered economy were closed, so that the interest rate is endogenously determined within the country and assume that there is no capital and no bond supply in the economy (so, the asset supply of the economy is zero). The representative firm produces with the production function

$$Y_t = A_t H_t,$$

where H_t is the firm's labor demand in period t and input factor markets are competitive. What is the equilibrium wage rate in both periods, $t \in \{1,2\}$?

(d) (10 Points) Still, consider the closed economy described in part (c) where equilibrium savings must be zero. Furthermore, assume that the consumer anticipates in period $t = 1$ a recession in the second period $t = 2$, this means A_2 drops to $A_2' = A_2/2$. How do labor supply and consumption in both periods change compared to the scenario where the productivity was still A_2? How does the wage in the two periods change? Explain the intuition of your findings (hint: you will be able so solve this exercise even if you struggled before. Work with the intratemporal optimality condition stated in part (b) and the period-by-period constraints

$$c_1 = h_1 w_1 - s$$
$$c_2 = h_2 w_2 + (1 + r)s,$$

and anticipate the equilibrium savings behavior, s.)

(e) (10 Points) Consider the same economy and experiment as before that is a recession in the second period $t = 2$, this means A_2 drops to $A_2' = A_2/2$. But now assume that the economy is small open such that the consumer can save at the fixed world interest r (small open economy considered in parts (a) and (b)). How do labor supply and consumption in both periods change? Explain intuitively what changes relative to the closed economy case discussed in part (d).

(f) (10 Points) Consider again the closed economy setup where the asset supply is zero and the interest rate endogenous. Assume that in the second period the government decides to tax labor income at rate $\tau = 50\%$, that is for every NOK you earn the government takes half of it, so that the after-tax wage for the consumer in the second period is cut in half. How does labor supply and consumption in both periods change? How do the wages paid by the firm in the two periods change? Explain the intuition of your findings. (hint: the answer does not involve any additional math.)