Final Exam (Solutions)
ECON 4310, Fall 2014

1. Do **not write with pencil**, please use a ball-pen instead.
2. Please answer in **English**. Solutions without traceable outlines, as well as those with unreadable outlines **do not earn points**.
3. Please start a **new page** for **every** short question and for every subquestion of the long questions.

Good Luck!

	Points	**Max**
Exercise A		**60**
Exercise B		**60**
Exercise C		**60**
Σ		**180**

Grade: ___________

Exercise A:
Short Questions (60 Points)

Answer each of the following short questions on a separate answer sheet by stating as a first answer True/False and then give a short but instructive explanation. You can write, calculate, or draw to explain your answer. You only get points if you have stated the correct True/False *and* provided a correct explanation to the question. We will not assign negative points for incorrect answers.

XX Instruction for graders: in general short questions are rewared with either full (10) or no (0) points. Full points are allocated if the correct short answer True/False is provided AND accompanied by a correct explanation. On exception, if someone forgot to give the short answer, but provided an explanation that clearly indicates the right short question, or if some other borderline case occurs, then it is upon your judgement to allocate half (5) of the points. But this should not be the rule but the exception. XX

Exercise A.1: (10 Points) Finite horizon model of intertemporal consumption

Consider the optimal intertemporal consumption choice of a household in discrete and finite time $t = 0, 1, \ldots, T < \infty$. The optimal behavior is characterized by the consumption Euler equation

$$\frac{c_{t+1}}{c_t} = \left[\beta(1 + r - \delta)\right]^{1/\theta},$$

and the private bugdet constraint

$$a_{t+1} + c_t = (1 + r - \delta)a_t, \quad a_0 = 0 \text{ given}, \quad a_{T+1} = 0,$$

where $r - \delta$ is the exogenous interest rate, c_t the individual consumption of the household, $\delta \in (0, 1)$ the depreciation rate of physical capital, $\beta \in (0, 1)$ is the subjective discount factor, and $1/\theta$ the intertemporal elasticity of substitution.

Suppose that $\beta(1 + r - \delta) > 1$, then the household will never borrow (have strictly negative asset holdings) over the life-cycle. True or false?

Your Answer:

True ☒ False: ☐

XX Instruction for graders: there is no wage income in the budget constraint, so the agent could never pay back any debt. Thus, the agent will just keep the assets at the zero level. If students answer as if there was a wage income (in line with the explanation I provide below), then still allocate full points.XX

The parameter restriction $\beta(1 + r - \delta) > 1$ implies - through the consumption Euler equation - that there will be consumption growth over the life-cycle. As the household starts with zero assets, the only way to increase consumption over time is to safe in the asset in the early periods and to dissave at later periods. Thus, the household will never have strictly negative asset holdings.

Exercise A.2: (10 Points) OLG model, permanent increase in population growth

Consider the capital accumulation equation of the overlapping generations model with exogenous technology and population growth.

$$k_{t+1} = \frac{(1-\alpha)\beta}{(1+\beta)(1+g)(1+n)} k_t^{\alpha}, \quad k_t \equiv K_t/(A_t L_t),$$

where K_t is the aggregate capital stock, A_t is the state of technology, L_t the size of the population, $\beta \in (0,1)$ the discount factor, $\alpha \in (0,1)$ the capital income share in the economy, and $g \geq 0$ and $n \geq 0$ denote the net growth rate of technology and the population, respectively. The competitive wage rate is given by

$$w_t = (1-\alpha) A_t k_t^{\alpha}.$$

Let the economy be in the stable steady-state, $k > 0$. In response to a permanent increase in the population growth rate from n to $n' > n$ in period t_0 (the current level of population L_{t_0} is unaffected by this shock), the wage rate will jump down on impact and then increase as the economy adjusts gradually to the new steady-state. True or false?

Your Answer:

True: ☐ False: ☒

The change in n will not affect the current capital stock per efficiency unit, thus the wage rate will be unaffected too. However, the dilution of the capital stock due to population growth will be more pronounced after the shock, such that the new steady-state capital stock per efficiency unit is lower than before. As the capital stock adjusts to this lower steady-state level, also the wage rate will fall.

Exercise A.3: (10 Points) Optimal fiscal policy, Handlingsregelen

Suppose that wages in Norway will grow at a strictly positive net rate, $n > 0$ in the future, that the government chooses fiscal policy (take-out from the petroleum fund) optimally, and let the value of the oil fund be constant.

A government with a fiscal policy that keeps the value of the petroleum fund relative to wages constant puts a higher relative welfare weight on future generations compared to a government that keeps the absolute value of the oil fund constant. True or false?

Your Answer:

True: ☒ False: ☐

Let $r > n$ be the return of the petroleum fund, then the first government will only take out $r - n$ from the fund for each generation, while the latter government will take out r. Thus, implicitly, the first government puts a relatively higher weight on future generations, as they will be able to enjoy a larger fraction of the fund than under the latter policy.

Exercise A.4: (10 Points) Optimal policy, Laffer curve

Suppose the aggregate labor supply, $h(\tau)$, of an economy as a function of the labor income tax rate, τ, is given by

$$h(\tau) = [(1-\tau)w]^{1/2}.$$

The top of the Laffer curve is given by $\bar{\tau} = 1/2$. True or false?

Your Answer:

True: ☐ False: ☒

Answer:

The top of the Laffer curve is characterized by

$$\bar{\tau} = \arg\max_{0\leq\tau\leq 1} \tau h(\tau) w,$$

with the associated optimality condition

$$\begin{aligned} 0 &= [(1-\bar{\tau})w]^{1/2} w + \frac{1}{2}\bar{\tau}\,[(1-\bar{\tau})w]^{1/2-1} w(-w) \\ &= 1 - \frac{1}{2}\bar{\tau}\,[(1-\bar{\tau})w]^{-1} w, \end{aligned}$$

such that the top of the Laffer curve is given by

$$2 = \frac{\bar{\tau}}{1-\bar{\tau}} \quad \Leftrightarrow \quad \bar{\tau} = 2/3.$$

Alternatively, student's can answer along the notation in the seminar. Setting the Frisch elasticity of labor supply to $\varphi = 1/2$, the tax elasticity of the labor supply is given by $e(\tau) = \varphi\tau/(1-\tau)$. The top of the Laffer curve satisfies $e(\bar{\tau}) = 1$ which yields the same value, $\bar{\tau} = 2/3$.

Exercise A.5: (10 Points) Data, long-run trends

Suppose you download quarterly macroeconomic data for mainland Norway. You normalize the data by the corresponding mean of each series and plot the logarithm of each normalized macroeconomic series over time. You find the slope of the time trend for the Gross Domestic Product is the steepest, followed by that of Total Hours Worked, and the Hours Worked per Employee is more or less flat.

This pattern of time trends is in line with the steady-state equilibrium that we characterized for the Ramsey model with technology and population growth. True or false?

Your Answer:

True: ☒ False: ☐

In the steady-state of the Ramsey model aggregate output grows at the rate of technology plus the rate of population, total hours worked should grow with the population (labor force), and the hours worked per worker is constant. This pattern is consistent with the long-run trends observed in the data, as the slope of the log of each normalized time-series reflects the growth rate of the corresponding series.

Exercise A.6: (10 Points) Precautionary savings

Consider the simple two-period real business cycle model discussed in the seminar and in the lecture. With an asset supply of zero, $w_0 = \mathrm{E}[w(s_1)]$, and an optimal consumption profile, $c_0 = w_0$, $c_1(s_1) = w(s_1)$, the stochastic consumption Euler equation in this model is given by

$$\beta(1+r_1) = \frac{u'(c_0)}{\mathrm{E}[u'(c_1(s_1))]} = \frac{u'(\mathrm{E}[w(s_1)])}{\mathrm{E}[u'(w(s_1))]}.$$

The stochastic process for the wage in the second period, $w(s_1)$, takes the form

$$w(s_1) = \begin{cases} w(s_G) = 1 + \sigma/2, & \text{with prob. } 1/2, \\ w(s_B) = 1 - \sigma/2, & \text{with prob. } 1/2, \end{cases}$$

where $\sigma \in (0, 2)$ parametrizes the risk in this economy. Assume that the utility function is of the following form

$$u(c) = 1 - e^{\alpha c},\ \alpha > 0.$$

This utility function, $u(c)$, implies precautionary savings. True or false? (hint: the derivative of the exponential function e^x with respect to x is again the exponential function e^x)

Your Answer:

True: ☐ False: ☒

XX Instructions for graders: there is a minus missing in the exponent of the stated utility function, this implies that utility is decreasing in consumption which might have confused some students. In any case, there is no precautionary savings motive in the stated utility function because the marginal utility is concave and not strictly convex. XX

The marginal utility function is strictly concave, $u'''(c) = -\alpha^3 e^{\alpha c} < 0$. Thus, there is no precautionary savings motive.

Exercise B:
Long Question (60 Points)

A comparison of the Solow model and the Ramsey growth model

Consider a closed economy without exogenous technology and population growth, where firms produce a generic good Y_t with the production function

$$Y_t = F(K_t, L) = K_t^{\alpha} L^{1-\alpha}, \quad 0 < \alpha < 1,$$

where K_t is aggregate capital and L is the number of workers in the economy. The law of motion for aggregate capital is given by

$$K_{t+1} = (1-\delta) K_t + I_t, \quad K_0 > 0, \tag{1}$$

where I_t denotes aggregate investment, and $0 < \delta < 1$ the depreciation rate. For simplicity, let aggregate labor supply (population) be equal to one, $L = 1$, such that consumption per worker, c_t, is the same as aggregate consumption, $C_t = c_t = c_t L$.

Consider now two different models. The Solow model where agents have constant savings rate, s, such that

$$I_t = sY_t.$$

And the Ramsey growth model where household utility is maximized

$$\sum_{t=0}^{\infty} \beta^t u(C_t), \tag{2}$$

such that aggregate investment (savings) is endogenous

$$I_t = Y_t - C_t.$$

In both models markets are competitive, thus input factors K_t and L are paid their marginal products.

(a) (10 Points) Compute the wage in the Solow model in a steady state. (hint: compute the aggregate steady state capital stock first.)
Solution:

XX Allocation of points: (i) derivation of the wage as a function of the capital stock, (3 points), (ii) derivation of the steady-state capital stock, (5 points), (iii) derivation of the steady-state wage (combination of (i) and (ii) such that the wage is a function of parameters only), (2 points). PLEASE CHECK whether the wage is derived in later parts, probably in (b), (c), or (d), IF MISSING here and allocate points accordingly. XX

Input factors are priced according to the marginal product, thus the wage rate in any given period is given by

$$w_t = \frac{\partial Y_t}{\partial L} = (1-\alpha) K_t^{\alpha} L^{-\alpha} = (1-\alpha) K_t^{\alpha}.$$

As there is no exogenous growth in technology or population, aggregate capital will be constant such that the steady-state condition reads

$$K^\star = (1-\delta)K^\star + s(K^\star)^\alpha,$$

such that

$$\delta(K^\star)^{1-\alpha} = s \quad \Leftrightarrow \quad K^\star = \left(\frac{s}{\delta}\right)^{1/(1-\alpha)}.$$

Thus, the wage rate in the steady-state reads

$$w^\star = (1-\alpha)(K^\star)^\alpha = (1-\alpha)\left(\frac{s}{\delta}\right)^{\alpha/(1-\alpha)}.$$

(b) (10 Points) In the Ramsey growth model households maximize lifetime utility in Equation (2) with respect to C_t and K_{t+1} and subject to the law of motion of capital stated in Equation in (1). Taking into account the functional form of output, Y_t, and investment, I_t, and that $L = 1$, write up the Lagrangian of this maximization problem and derive the following optimality conditions of the Ramsey growth model

$$\begin{aligned}\frac{u'(C_t)}{u'(C_{t+1})} &= \beta\left[1 + \alpha(K_{t+1})^{\alpha-1} - \delta\right]\\ K_{t+1} - K_t &= K_t^\alpha - \delta K_t - C_t.\end{aligned}$$

The first optimality condition is the model's Euler equation and the second the resource constraint.

Solution:

XX Allocation of points: (i) Lagrangian, (4 points), (ii) optimality conditions in raw form, (3 points), (iii) derivation of the Euler equation, (3 points). XX

The Lagrangian reads

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(C_t) + \lambda_t \left[K_t^\alpha + (1-\delta) K_t - C_t - K_{t+1}\right],$$

with associated optimality conditions

$$\begin{aligned}0 &= \frac{\partial \mathcal{L}}{\partial C_t} = \beta^t u'(C_t) - \lambda_t\\ 0 &= \frac{\partial \mathcal{L}}{\partial K_{t+1}} = -\lambda_t + \lambda_{t+1}\left[\alpha K_{t+1}^{\alpha-1} + (1-\delta)\right]\\ 0 &= \frac{\partial \mathcal{L}}{\partial \lambda_t} = K_t^\alpha + (1-\delta) K_t - C_t - K_{t+1}.\end{aligned}$$

Eliminating the Lagrange multiplier the second optimality condition can be written as

$$u'(C_t) = \beta u'(C_{t+1})\left[\alpha K_{t+1}^{\alpha-1} + (1-\delta)\right]$$

Thus the optimality conditions can be summarized as

$$\frac{u'(C_t)}{u'(C_{t+1})} = \beta\left[1 + \alpha(K_{t+1})^{\alpha-1} - \delta\right]$$
$$K_{t+1} - K_t = K_t^{\alpha} - \delta K_t - C_t.$$

(c) (5 Points) Compute the wage in the Ramsey model in a steady state. (hint: compute the steady state capital stock first, you can solve this part even if you were not able to solve part (b).)
Solution:

XX Allocation of points: (i) derivation of the steady-state capital stock, (3 points), (ii) derivation of the steady-state wage as a function of parameters only), (2 points). XX

Since there is no exogenous growth, aggregate consumption will be constant in a steady-state. The Euler equation then implies that

$$1 = \beta\left[\alpha(K^{\star})^{\alpha-1} + (1-\delta)\right] \quad \Leftrightarrow \quad K^{\star} = \left(\frac{\alpha}{1/\beta - (1-\delta)}\right)^{1/(1-\alpha)},$$

such that the steady state wage rate is given by

$$w^{\star} = (1-\alpha)(K^{\star})^{\alpha} = (1-\alpha)\left(\frac{\alpha}{1/\beta - (1-\delta)}\right)^{\alpha/(1-\alpha)}.$$

(d) (10 Points) Does the wage in the Solow model depend on the saving rate s in a steady state? Is the wage increasing or decreasing in s or independent from s. Why? (hint: you can try to answer this question even though you were not able solve previous parts.)
Solution:

Yes, the wage in the steady state increases in the savings rate (*XX (right answer, 5 points) XX*). The reason is that the aggregate steady state capital stock is increasing in the savings rate because with higher savings a higher amount of depreciating capital can be offset to hold the aggregate capital stock constant. With the production function under consideration, the marginal product of the constant stock of labor increases with the amount of capital, thus a higher steady state capital stock will be reflected in a higher wage rate (*XX (correct economic reasoning, 5 points, if reasoning is rather technical and not very deep then allocate 2 out of 5 points) XX*).

(e) (10 Points) Does the wage in the Ramsey model depend on the discount factor β in a steady-state? Is the wage increasing or decreasing in β or independent from β. Why? (hint: you can try to answer this question even though you were not able

solve previous parts.)
Solution:

Yes, the wage in the steady state increases in the discount factor (*XX (right answer, 5 points) XX*). The reason is that the aggregate steady state capital stock is increasing in the discount factor as more patient households will want to shift resources into the future by saving more (in capital). With the production function under consideration, the marginal product of the constant stock of labor increases with the amount of capital, thus a higher steady state capital stock will be reflected in a higher wage rate (*XX (correct economics reasoning, 5 points, if reasoning is rather technical and not very deep then allocate 2 out of 5 points) XX*).

(f) (5 Points) Compute the saving rate $\bar{s}$ which gives the same steady-state capital stock in the Solow model as in the Ramsey growth model. Is this saving rate, $\bar{s}$, increasing or decreasing in β?
Solution:

The saving rate is characterized by (*XX (right calculation, 3 points) XX*)

$$\frac{\bar{s}}{\delta} = \frac{\alpha}{1/\beta - (1-\delta)} \quad \Leftrightarrow \quad \bar{s} = \frac{\delta\alpha}{1/\beta - (1-\delta)}.$$

According to the previous analysis (and also if you do the math), as the steady state capital stock is increasing in s in the Solow model and in β in the Ramsey growth model, $\bar{s}$ must be increasing in β (*XX (right answer, 2 points) XX*).

(g) (5 Points) Compute the the saving rate $\hat{s}$ which gives the same wage in the Solow model as in the Ramsey growth model. Is this saving rate increasing or decreasing in β?
Solution:

The savings rate is the same as before, $\hat{s} = \bar{s}$ (*XX (right insight, 3 points) XX*), as the wage rate is a function of the aggregate capital stock and the parameter α only. Thus, also $\hat{s}$ is increasing in the discount factor β (*XX (right answer, 2 points) XX*).

(h) (5 Points) Compare the saving rates $\bar{s}$ computed in part (f) to the rate $\hat{s}$ computed in part (g). Explain your findings.
Solution:

As derived above, the two saving rates are the same, $\bar{s} = \hat{s}$, as the wage rate is a function of the aggregate capital stock and the parameter α only (*XX allocate full points or no points XX*).

Exercise C:
Long Question (60 Points)

Labor Supply

Consider a representative consumer living for two periods, denoted by $t \in \{1, 2\}$, in a small open economy. The consumer has preferences over consumption, c_t, and the hours of labor supplied, h_t, of the following form

$$U = \log(c_1) + \phi \log(1 - h_1) + \beta \left[\log(c_2) + \phi \log(1 - h_2)\right],$$

where log denotes as always the natural logarithm. The real wage is w_1 in period 1 and w_2 in period 2. The consumer has no capital income in the first period as she starts life without any assets, but the consumer may transfer income between periods (savings) at the exogenous world interest rate r.

(a) (15 Points) Set up the optimization problem of this consumer and derive the optimality conditions. Note that consumption, labor supply, and savings are the choice variables of this optimization problem. (hint: you can do the optimization subject to two period-by-period budget constraints or subject to a single net present value budget constraint.)

Solution:

XX Allocation of points: (i) Lagrangian, (6 points), (ii) optimality conditions in raw form, (5 points), (iii) derivation of the intratempotal efficiency condition and the Euler equation, (4 points). PLEASE CHECK whether (iii) is DERIVED in one of the later parts, in that case allocate the corresponding points here. XX

The period-by-period budget constraints of the consumer read

$$\begin{aligned} c_1 + s &= h_1 w_1 \\ c_2 &= h_2 w_2 + (1 + r)s. \end{aligned}$$

As a lifetime constraint, the two can be combined to yield

$$c_1 + \frac{c_2}{1 + r} = h_1 w_1 + \frac{h_2 w_2}{1 + r}.$$

The Lagrangian of the optimization problem reads (sequential formulation)

$$\begin{aligned} \mathcal{L} = &\log(c_1) + \phi \log(1 - h_1) + \beta \left[\log(c_2) + \phi \log(1 - h_2)\right] \\ &+ \lambda_1 \left[h_1 w_1 - c_1 - s\right] + \lambda_2 \left[h_2 w_2 + (1 + r)s - c_2\right], \end{aligned}$$

or (lifetime formulation)

$$\begin{aligned} \mathcal{L} = &\log(c_1) + \phi \log(1 - h_1) + \beta \left[\log(c_2) + \phi \log(1 - h_2)\right] \\ &+ \lambda \left[h_1 w_1 + \frac{h_2 w_2}{1 + r} - c_1 - \frac{c_2}{1 + r}\right]. \end{aligned}$$

The optimality conditions are (sequential formulation)

$$
\begin{aligned}
0 &= \frac{\partial \mathcal{L}}{\partial c_t} = \beta^{t-1} c_t^{-1} - \lambda_t \\
0 &= \frac{\partial \mathcal{L}}{\partial h_t} = \beta^{t-1} \phi (1-h_t)^{-1}(-1) + \lambda_t w_t \\
0 &= \frac{\partial \mathcal{L}}{\partial s} = -\lambda_1 + \lambda_2 (1+r) \\
0 &= \frac{\partial \mathcal{L}}{\partial \lambda_1} = h_1 w_1 - c_1 - s \\
0 &= \frac{\partial \mathcal{L}}{\partial \lambda_2} = h_2 w_2 + (1+r)s - c_2,
\end{aligned}
$$

or (lifetime formulation)

$$
\begin{aligned}
0 &= \frac{\partial \mathcal{L}}{\partial c_t} = \beta^{t-1} c_t^{-1} - \frac{\lambda}{(1+r)^{t-1}} \\
0 &= \frac{\partial \mathcal{L}}{\partial h_t} = \beta^{t-1} \phi (1-h_t)^{-1}(-1) + \frac{\lambda w_t}{(1+r)^{t-1}} \\
0 &= \frac{\partial \mathcal{L}}{\partial \lambda} = h_1 w_1 + \frac{h_2 w_2}{1+r} - c_1 - \frac{c_2}{1+r}.
\end{aligned}
$$

In any case, the optimality conditions can be summarized by the intratemporal optimality condition

$$\phi (1-h_t)^{-1} = c_t^{-1} w_t,$$

the intertemporal optimality condition (Euler equation)

$$\frac{c_2}{c_1} = \beta(1+r),$$

and the period-by-period constraints

$$
\begin{aligned}
c_1 &= h_1 w_1 - s \\
c_2 &= h_2 w_2 + (1+r)s,
\end{aligned}
$$

or the lifetime budget constraint

$$c_1 + \frac{c_2}{1+r} = h_1 w_1 + \frac{h_2 w_2}{1+r}.$$

(b) (10 Points) Derive the optimal supply of labor, h_t, in each period as a function of only exogenous parameters. Note that savings are not necessarily zero. (hint: if you were not able to solve part (a), you can assume that optimal consumption and labor supply is characterized through the intratemporal optimality condition

$$\phi (1-h_t)^{-1} = c_t^{-1} w_t,$$

the intertemporal optimality condition (Euler equation)

$$\frac{c_2}{c_1} = \beta(1+r),$$

and the lifetime budget constraint

$$c_1 + \frac{c_2}{1+r} = h_1 w_1 + \frac{h_2 w_2}{1+r}.$$

and you will be able to solve the following parts of this exercise.)

Solution:

XX Allocation of points: (i) derivation of optimal consumption levels, or some other intermediate steps (some students directly went after the labor supply in the lifetime budget constraint), (5 points), (ii) derivation of the optimal labor supply, (5 points). PLEASE also allocate points (2 instead of 5) in each step for derivations that are wrong, but go in the right direction and show some understanding of the model. XX

The intratemporal optimality conditions can be written as

$$1 - h_t = \frac{\phi}{w_t} c_t \quad \Leftrightarrow \quad h_t = 1 - \frac{\phi}{w_t} c_t.$$

Combining the Euler equation, $c_2 = \beta(1+r)c_1$, with the lifetime budget constraint yields

$$\begin{aligned} c_1 + \frac{\beta(1+r)c_1}{1+r} &= h_1 w_1 + \frac{h_2 w_2}{1+r} \\ &= \left(1 - \frac{\phi}{w_1} c_1\right) w_1 + \frac{\left[1 - \frac{\phi}{w_2}\beta(1+r)c_1\right] w_2}{1+r} \\ &= w_1 - \phi c_1 + \frac{w_2}{1+r} - \phi\beta c_1, \end{aligned}$$

such that first-period consumption can be written as

$$c_1\left(1 + \beta + \phi(1+\beta)\right) = w_1 + \frac{w_2}{1+r} \quad \Leftrightarrow \quad c_1 = \frac{1}{(1+\phi)(1+\beta)}\left(w_1 + \frac{w_2}{1+r}\right), \tag{3}$$

such that

$$c_2 = \frac{\beta(1+r)}{(1+\phi)(1+\beta)}\left(w_1 + \frac{w_2}{1+r}\right), \tag{4}$$

and the optimal labor supply is given by

$$h_1 = 1 - \frac{\phi}{(1+\phi)(1+\beta)}\left(1 + \frac{w_2}{w_1(1+r)}\right) \tag{5}$$

$$h_2 = 1 - \frac{\phi\beta}{(1+\phi)(1+\beta)}\left(\frac{1 + w_1(1+r)}{w_2}\right). \tag{6}$$

(c) (5 Points) Suppose now that the considered economy were closed, so that the interest rate is endogenously determined within the country and assume that there is no capital and no bond supply in the economy (so, the asset supply of the economy is zero). The representative firm produces with the production function

$$Y_t = A_t H_t,$$

where H_t is the firm's labor demand in period t and input factor markets are competitive. What is the equilibrium wage rate in both periods, $t \in \{1, 2\}$?
Solution:

Equilibrium wages are given by the marginal product of labor

$$w_t = \frac{\partial Y_t}{\partial H_t} = A_t,$$

which is a function of labor productivity, A_t, only (*XX allocate full or no points XX*).

(d) (10 Points) Still, consider the closed economy described in part (c) where equilibrium savings must be zero. Furthermore, assume that the consumer anticipates in period $t = 1$ a recession in the second period $t = 2$, this means A_2 drops to $A_2' = A_2/2$. How do labor supply and consumption in both periods change compared to the scenario where the productivity was still A_2? How does the wage in the two periods change? Explain the intuition of your findings (hint: you will be able so solve this exercise even if you struggled before. Work with the intratemporal optimality condition stated in part (b) and the period-by-period constraints

$$\begin{aligned} c_1 &= h_1 w_1 - s \\ c_2 &= h_2 w_2 + (1 + r)s, \end{aligned}$$

and anticipate the equilibrium savings behavior, s.)
Solution:

Relative wages across periods are given by the relative productivity

$$\frac{w_2}{w_1} = \frac{A_2}{A_1},$$

thus if the productivity in the second period drops, also the wage in the second period will drop. The relative wage will be half of what it has been before (*XX 2 points. XX*). With a zero asset supply, the period-by-period constraints imply

$$c_t = h_t w_t = w_t - \phi c_t \quad \Leftrightarrow \quad c_t = \frac{w_t}{1 + \phi},$$

such that the labor supply is given by

$$h_t = 1 - \frac{\phi}{w_t} c_t = 1 - \frac{\phi}{1 + \phi} = \frac{1}{1 + \phi}.$$

Thus the labor supply does not respond to relative price changes across periods (*XX derivation and interpretation labor supply, 4 points XX*). And as the equilibrium savings have to be zero (the equilibrium interest rate will adjust), the only margin of adjustment for the consumer will be to adjust relative consumption across periods in the same fashion as relative productivity. Thus, second period consumption must fall (*XX correct adjustment and interpretation of consumption, 4 points XX*).

(e) (10 Points) Consider the same economy and experiment as before that is a recession in the second period $t = 2$, this means A_2 drops to $A_2' = A_2/2$. But now assume that the economy is small open such that the consumer can save at the fixed world interest r (small open economy considered in parts (a) and (b)). How do labor supply and consumption in both periods change? Explain intuitively what changes relative to the closed economy case discussed in part (d).
Solution:

According to Equations (3) and (4) consumption in both periods will fall when the second period wage falls (*XX correct adjustment, 4 points XX*) as the agent has less lifetime resources available but can smooth consumption across periods through savings. Equations (5) and (6) imply that labor supply will be increasing in period 1 to finance more savings, and falling in period 2 as the low wage distorts the incentive to provide labor (*XX correct adjustment, 4 points XX*).

The main difference to the closed economy case is that the agent can shift resources across time by saving resources in the period with a high productivity. In the closed economy however, this not possible and the interest rate will instead adjust to bring the economy into an equilibrium (*XX correct comparison, 2 points XX*).

(f) (10 Points) Consider again the closed economy setup where the asset supply is zero and the interest rate endogenous. Assume that in the second period the government decides to tax labor income at rate $\tau = 50\%$, that is for every NOK you earn the government takes half of it, so that the after-tax wage for the consumer in the second period is cut in half. How does labor supply and consumption in both periods change? How do the wages paid by the firm in the two periods change? Explain the intuition of your findings. (hint: the answer does not involve any additional math.)
Solution:

The wages paid by the firm are unchanged, they are still equal to the marginal product of labor, $w_t = A_t$ (*XX 2 points XX*). However, the lifetime income of the agent is affected in the same way as if her productivity dropped to $A_2/2$. Thus, the results are the same as in part (d). The labor supply does not respond the labor income tax (*XX right adjustment, 4 points XX*), and the consumption is only reduced in the second period (*XX right adjustment, 4 points XX*).
