

Problem Set 1: Introduction

Exercise 1.1: A static competitive equilibrium

Consider a static economy with a representative consumer that has the following preferences over consumption, c , leisure, l , and a public good, g ,

$$u(c, l, g) = \log(c - \psi(1 - l)^\theta) + \log(g), \quad \theta > 1,$$

and is subject to the budget constraint

$$c = (1 - \tau^n)(1 - l)w + rk_0.$$

where τ^n is a proportional tax rate on labor income, and k_0 denotes the consumers' initial endowment of physical capital. The representative firm produces consumable output, y , with the following technology

$$y = k^\alpha n^{1-\alpha}, \quad 0 < \alpha < 1,$$

by renting physical capital, k , from consumers at the rental rate r , and labor, n , at the wage rate w . The government spends a fixed fraction $\gamma = g/y$ of output on public goods by setting labor income taxes to balance the government's budget

$$\tau^n n w = g.$$

Remember the definition of a competitive equilibrium with a public sector:

A **competitive equilibrium** is an allocation $\{c, l, k, n\}$ and a set of prices and taxes $\{r, w, \tau^n\}$ such that

- (1) The representative consumer chooses c and l to maximize utility subject to the private budget constraint, taking as given prices and taxes.
- (2) The representative firm chooses physical capital k and labor input n to maximize profits, taking as given prices.
- (3) The government chooses tax policy τ^n to balance the government budget.
- (4) The markets for goods, capital, and labor clear.

In what follows, we will compute the competitive equilibrium of this economy step by step.

- (a) Solve the consumers maximization problem.
- (b) Derive the optimality conditions of the firm's maximization problem and show that it will make zero profit.
- (c) Find the equilibrium prices $\{r, w\}$ that clear the labor and the capital market for a given tax rate τ^n .

- (d) Show that the wage income of the consumer is equal to a constant share of output, $wn = (1 - \alpha)y$. What is the equilibrium tax rate τ^n of this economy that balances the government budget?
- (e) Verify that Walras' law holds in the competitive equilibrium derived above, i.e., check whether the goods market clears at equilibrium prices and taxes

$$y(\tau^n) = c(\tau^n) + g.$$

Exercise 1.2: Immigration in the Solow model

Consider a closed economy with a neoclassical production function, exogenous technological progress, A_t , a fixed saving rate, s , and a constant labor force, L , as described by the following equations (the Solow model)

$$K_{t+1} - K_t = sY_t - \delta K_t \quad (1)$$

$$Y_t = K_t^\alpha (A_t L)^{1-\alpha}, \quad 0 < \alpha < 1, \quad (2)$$

$$A_{t+1} = (1 + g)A_t, \quad A_0 > 0,$$

where $0 \leq \delta \leq 1$ is the depreciation rate of physical capital (in the lecture we have abstracted from depreciation for simplicity).

- (a) Remove the trend from Equations (1) and (2) by writing all endogenous variables X_t in terms of efficiency units $x_t \equiv X_t / (A_t L)$.
- (b) Compute the wage rate, w_t , and the rental rate of capital, r_t , (the interest rate of this economy will be $r_t - \delta$) in this economy.
- (c) Compute the stable steady-state capital stock per efficiency unit, $k^* > 0$, of this economy.
- (d) Show that the capital stock per efficiency unit, k_t , is increasing over time as long as $0 < k_t < k^*$ (hint: look at the ratio k_{t+1}/k_t in the capital accumulation equation to answer this question).
- (e) Suppose the economy is in a steady-state. What happens to the aggregate output, the wage rate, and the rental rate on impact if the number of workers in the economy increases by ∂L due to immigration?
- (f) What happens to aggregate output, the wage rate, and the rental rate over time after the immigration wave?
- (g) Would the wage rate be higher in the long-run if there had been no immigration?

Additional Exercises (available on the course website):

- Exam 2015: B.(a), B.(d).
- Retake Exam 2015: A.1, A.2, A.3.