

Problem Set 6: Optimal Fiscal Policy

Exercise 6.1: The Norwegian Handlingsregelen

Consider a small open economy populated with non-overlapping generations of households that live for one period. The size of each generation is one, and the generation living in period t earns an exogenously given wage w_t . The government of the economy is endowed with initial resources (due to an oil windfall, for example) of value

$$B = -b_0$$

where b_0 denotes the initial debt position of the government as in previous problem sets (negative debt can be interpreted as assets). The government can impose transfers T_t on each generation to redistribute resources across generations, such that the period-by-period budget constraint of the generation living in period t reads

$$c_t = w_t + T_t, \quad (1)$$

where c_t denotes the consumption level of each generation. The period-by-period budget constraint of the infinitely-lived government reads

$$b_{t+1} = (1+r)b_t + T_t, \quad (2)$$

where r denotes the exogenous interest rate on the international capital market (which is assumed to be constant for the ease of exposition). Without imposing any further restrictions on fiscal policy (except a no-Ponzi condition of course), the net present value budget constraint of the government reads

$$\sum_{t=0}^{\infty} \frac{T_t}{(1+r)^{t+1}} = B, \quad (3)$$

such that the present value of all transfers cannot exceed the value of initial assets, B . The government is benevolent towards present and future generations and maximizes a welfare function equal to a weighted sum of each generation's utility

$$\sum_{t=0}^{\infty} \beta_t u(c_t), \quad \beta_0 = 1, \quad (4)$$

where β_t (not to be confused with the discount factor β^t , where t denotes the power of β) denotes the welfare weight that the government puts on each generation t .

- (a) State the optimality conditions of the government's decision problem (hint: reduce consumption from the problem before maximizing the objective)

$$W_t = \max_{\{c_t, T_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta_t u(c_t) \text{ s.t. } (1), (3).$$

Why does the Ricardian equivalence proposition not apply to this economy?

- (b) Assume that marginal utility is given by $u'(c) = c^{-\theta}$, $\theta > 0$. Derive the government's Euler equation, by combining the optimality conditions of two subsequent generations, t and $t + 1$, respectively.
- (c) Solve for c_t as a function of c_0 using the government's Euler equation. Then, only for this subquestion, set the parameter $\theta = 1$ and derive the optimal level of consumption c_0 from Equations (1) and (3).
- (d) Consider the Norwegian Handlingsregelen which roughly state that fiscal policy is restricted to be

$$-b_{t+1} = B,$$

for all generations t . Or in words, the government is only allowed to take out the returns on the stock of assets, B . What transfer and private consumption pattern does this imply for each generation? What sequence of welfare weights $\{\beta_{t+1}\}_{t=0}^{\infty}$ would correspond to this fiscal policy rule?

- (e) Let the wage growth be given by $w_{t+1}/w_t = (1 + g)$. Suppose that the government followed instead the fiscal rule

$$-b_t/w_t = B/w_0,$$

for each generation t . Or in words, the government wants to keep the stock of assets as a fraction of wages constant. What sequence of welfare weights $\{\beta_{t+1}\}_{t=0}^{\infty}$ would correspond to this fiscal policy rule?

- (f) Calculate the relative welfare weight β_{t+1}/β_t under both fiscal policy rules considered in parts (d) and (e). What policy rule puts a higher relative welfare weight on future generations?

Exercise 6.2: The Laffer Curve

Consider a representative household of a static economy with the following preferences over private consumption, c , labor supply, h , and public goods, g

$$U = \max \left[\frac{\left(c - \frac{h^{1+1/\varphi}}{1+1/\varphi} \right)^{1-\theta} - 1}{1-\theta} + \sigma \log(g) \right], \quad \theta > 0, \quad (5)$$

where $0 < \varphi < \infty$ denotes the Frisch elasticity of labor supply, and $\sigma > 0$ is a parameter that measures the household's relative preference for public over private goods. The household faces an exogenously given wage rate, w , and interest rate, r , and labor income is taxed at the proportional rate τ^n yielding the private budget constraint (the household is born without assets)

$$c = (1 - \tau^n)wh. \quad (6)$$

- (a) Write down the household's optimality conditions with respect to consumption, c and labor supply, h (the public good provision by the government is taken as given), and derive the optimal labor supply which we will denote by $h(\tau^n)$.

- (b) Compute the elasticity of the labor supply with respect to the tax rate

$$e(\tau^n) \equiv -\frac{\partial h(\tau^n)}{\partial \tau^n} \frac{\tau^n}{h(\tau^n)}.$$

Show that this elasticity is increasing in the tax rate, τ^n , i.e., the higher the tax rate the more distorted is the labor supply in this economy.

- (c) Derive the government's labor income tax revenue as a function of the tax rate - the so called Laffer curve. What tax rate $\bar{\tau}$ is associated with the top of the Laffer curve (the maximum tax revenue)? What value takes the elasticity $e(\tau)$ at the top of the Laffer curve? What was the tax rate at the top of the Laffer curve if the labor supply is completely inelastic, $\varphi \rightarrow 0$, or inelastic, $\varphi \rightarrow \infty$?
- (d) Suppose the government wants to finance the specific level of government expenditure g^* that is located within the bounds

$$0 < g^* < \bar{\tau}wh(\bar{\tau}).$$

Assume that $\varphi = 1$. Find the optimal tax rate, τ^* , to finance the government expenditure level, g^* , with a balanced government budget. Would a benevolent government ever choose a tax rate above $\bar{\tau}$?

Additional Exercises (available on the course website):

- Exam 2015: A.3, A.4.