Problem Set 6: Optimal Fiscal Policy (Solution 6.1 (c) and 6.2 (d))

Exercise 6.1: The Norwegian Handlingsregelen

Consider a small open economy populated with non-overlapping generations of households that live for one period. The size of each generation is one, and the generation living in period t earns an exogenously given wage w_t. The government of the economy is endowed with initial resources (due to an oil windfall, for example) of value

$$B = -b_0$$

where b_0 denotes the initial debt position of the government as in previous problem sets (negative debt can be interpreted as assets). The government can impose transfers T_t on each generation to redistribute resources across generations, such that the period-by-period budget constraint of the generation living in period t reads

$$c_t = w_t + T_t, \tag{1}$$

where c_t denotes the consumption level of each generation. The period-by-period budget constraint of the infinitely-lived government reads

$$b_{t+1} = (1 + r)b_t + T_t, \tag{2}$$

where r denotes the exogenous interest rate on the international capital market (which is assumed to be constant for the ease of exposition). Without imposing any further restrictions on fiscal policy (except a no-Ponzi condition of course), the net present value budget constraint of the government reads

$$\sum_{t=0}^{\infty} \frac{T_t}{(1 + r)^{t+1}} = B, \tag{3}$$

such that the present value of all transfers cannot exceed the value of initial assets, B. The government is benevolent towards present and future generations and maximizes a welfare function equal to a weighted sum of each generation's utility

$$\sum_{t=0}^{\infty} \beta_t u(c_t), \quad \beta_0 = 1, \tag{4}$$

where β_t (not to be confused with the discount factor β^t, where t denotes the power of β) denotes the welfare weight that the government puts on each generation t.

(a) State the optimality conditions of the government's decision problem (hint: reduce consumption from the problem before maximizing the objective)

$$W_t = \max_{\{c_t, T_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta_t u(c_t) \text{ s.t. } (1), (3).$$

Why does the Ricardian equivalence proposition not apply to this economy?

(b) Assume that marginal utility is given by $u'(c) = c^{-\theta}$, $\theta > 0$. Derive the government's Euler equation, by combining the optimality conditions of two subsequent generations, t and $t+1$, respectively.

(c) Solve for c_t as a function of c_0 using the government's Euler equation. Then, only for this subquestion, set the parameter $\theta = 1$ and derive the optimal level of consumption c_0 from Equations (1) and (3).

Solution:

Make use repeatedly of the government's Euler equation to yield

$$\begin{aligned} c_t &= [\beta_t/\beta_{t-1}(1+r)]^{1/\theta} c_{t-1} \\ &= [\beta_t/\beta_{t-1}(1+r)]^{1/\theta} [\beta_{t-1}/\beta_{t-2}(1+r)]^{1/\theta} c_{t-2} \\ &= [\beta_t/\beta_{t-2}(1+r)(1+r)]^{1/\theta} c_{t-2} \\ &\vdots \\ &= \left[\beta_t/\beta_{t-t}(1+r)^t\right]^{1/\theta} c_{t-t} \\ &= \left[\beta_t/\beta_0(1+r)^t\right]^{1/\theta} c_0 \end{aligned}$$

Then rewrite the net present value budget constraint

$$\sum_{t=0}^{\infty} \frac{T_t}{(1+r)^{t+1}} = \sum_{t=0}^{\infty} \frac{c_t - w_t}{(1+r)^{t+1}} = B,$$

and substitute out consumption to yield (henceforth we set $\theta = 1$)

$$\begin{aligned} \sum_{t=0}^{\infty} \frac{c_t}{(1+r)^{t+1}} &= \sum_{t=0}^{\infty} \frac{\beta_t/\beta_0(1+r)^t c_0}{(1+r)^{t+1}} \\ &= \frac{c_0}{1+r}\left(\sum_{t=0}^{\infty} \beta_t/\beta_0\right) = B + \sum_{t=0}^{\infty} \frac{w_t}{(1+r)^{t+1}}. \end{aligned}$$

The initial consumption level can then be written as

$$c_0 = (1+r)\left(\sum_{t=0}^{\infty} \beta_t/\beta_0\right)^{-1}\left(B + \sum_{t=0}^{\infty} \frac{w_t}{(1+r)^{t+1}}\right),$$

and the consumption level for any generation t is given by

$$\begin{aligned} c_t &= \beta_t/\beta_0(1+r)^t c_0 \\ &= (1+r)^{t+1} \frac{\beta_t/\beta_0}{\sum_{t=0}^{\infty} \beta_t/\beta_0}\left(B + \sum_{t=0}^{\infty} \frac{w_t}{(1+r)^{t+1}}\right). \end{aligned}$$

Note that the optimal consumption level only depends on the relative welfare weights (we chose β_0 as the basis, but you could choose any other β_t). By normalizing the basis $\beta_0 = 1$ we can simplify the expressions without loosing generality in the results.

(d) Consider the Norwegian Handlingsregelen which roughly state that fiscal policy is restricted to be

$$-b_{t+1} = B,$$

for all generations t. Or in words, the government is only allowed to take out the returns on the stock of assets, B. What transfer and private consumption pattern does this imply for each generation? What sequence of welfare weights $\{\beta_{t+1}\}_{t=0}^{\infty}$ would correspond to this fiscal policy rule?

(e) Let the wage growth be given by $w_{t+1}/w_t = (1+g)$. Suppose that the government followed instead the fiscal rule

$$-b_t/w_t = B/w_0,$$

for each generation t. Or in words, the government wants to keep the stock of assets as a fraction of wages constant. What sequence of welfare weights $\{\beta_{t+1}\}_{t=0}^{\infty}$ would correspond to this fiscal policy rule?

(f) Calculate the relative welfare weight β_{t+1}/β_t under both fiscal policy rules considered in parts (d) and (e). What policy rule puts a higher relative welfare weight on future generations?

Exercise 6.2: The Laffer Curve

Consider a representative household of a static economy with the following preferences over private consumption, c, labor supply, h, and public goods, g

$$U = \max \left[\frac{\left(c - \frac{h^{1+1/\varphi}}{1+1/\varphi} \right)^{1-\theta} - 1}{1-\theta} + \sigma \log(g) \right], \quad \theta > 0, \tag{5}$$

where $0 < \varphi < \infty$ denotes the Frisch elasticity of labor supply, and $\sigma > 0$ is a parameter that measures the household's relative preference for public over private goods. The household faces an exogenously given wage rate, w, and interest rate, r, and labor income is taxed at the proportional rate τ^n yielding the private budget constraint (the household is born without assets)

$$c = (1 - \tau^n) w h. \tag{6}$$

(a) Write down the household's optimality conditions with respect to consumption, c and labor supply, h (the public good provision by the government is taken as given), and derive the optimal labor supply which we will denote by $h(\tau^n)$.

(b) Compute the elasticity of the labor supply with respect to the tax rate

$$e(\tau^n) \equiv -\frac{\partial h(\tau^n)}{\partial \tau^n} \frac{\tau^n}{h(\tau^n)}.$$

Show that this elasticity is increasing in the tax rate, τ^n, i.e., the higher the tax rate the more distorted is the labor supply in this economy.

(c) Derive the government's labor income tax revenue as a function of the tax rate - the so called Laffer curve. What tax rate $\bar{\tau}$ is associated with the top of the Laffer curve (the maximum tax revenue)? What value takes the elasticity $e(\tau)$ at the top of the Laffer curve? What was the tax rate at the top of the Laffer curve if the labor supply is completely inelastic, $\varphi \to 0$, or inelastic, $\varphi \to \infty$?

(d) Suppose the government wants to finance the specific level of government expenditure $g^\star$ that is located within the bounds

$$0 < g^\star < \bar{\tau} w h(\bar{\tau}).$$

Assume that $\varphi = 1$. Find the optimal tax rate, $\tau^\star$, to finance the government expenditure level, $g^\star$, with a balanced government budget. Would a benevolent government ever choose a tax rate above $\bar{\tau}$?

Solution:

When $\varphi = 1$ then the top of the Laffere curve is given by $\bar{\tau} = 1/2$ and the maximum tax revenue is

$$\bar{\tau} w \left[(1-\bar{\tau})w\right] = 1/4w^2 > g^\star.$$

The government budget constraint reads

$$\begin{aligned} g^\star &= \tau^\star w h(\tau^\star) \\ &= \tau^\star w \left[(1-\tau^\star)w\right] = \tau^\star w^2 - (\tau^\star)^2 w^2, \end{aligned}$$

which can be written as the quadratic equation

$$\begin{aligned} 0 &= w^2(\tau^\star)^2 - w^2\tau^\star + g^\star \\ &\equiv ax^2 + bx + c. \end{aligned}$$

The two solutions are characterized by

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{w^2 \pm \sqrt{w^4 - 4w^2 g^\star}}{2w^2} = 1/2 \pm \frac{\sqrt{1 - 4w^{-2}g^\star}}{2}. \end{aligned}$$

Since it can never be optimal to raise the same tax revenue $g^\star$ with a higher tax rate than necessary (because it would increase the labor supply distortion and reduce the available budget of the household unnecessarily), the optimal tax rate must be given by

$$\tau^\star = 1/2 - \frac{\sqrt{1 - 4w^{-2}g^\star}}{2} < \bar{\tau} = 1/2.$$

Note that the term under the square root is strictly positive since $g^\star < 1/4w^2$.

Additional Exercises (available on the course website):

- Exam 2015: A.3, A.4.