

Problem Set 9

ECON 4310, Fall 2015

1. Do **not write with pencil**, please use a ball-pen instead.
2. Please answer in **English**. Solutions without traceable outlines, as well as those with unreadable outlines **do not earn points**.
3. Please start a **new page** for **every** short question and for every subquestion of the long questions.

Good Luck!

	Points	Max
Exercise A		XX
Exercise B		XX
⋮		⋮
Σ		180

Grade: _____

Exercise A: Short Questions (XX Points)

Answer each of the following short questions on a separate answer sheet by stating as a first answer True/False and then give a short but instructive explanation. You can write, calculate, or draw to reason your answer. You only get points if you have stated the correct True/False *and* provided a correct explanation to the question. We will not assign negative points for incorrect answers.

(XX Points) Exercise A.1: Precautionary savings

Consider the simple two-period real business cycle model discussed in the seminar and in the lecture. With an asset supply of zero, $w_1 = E[w(s_2)]$, and an optimal consumption profile, $c_1 = w_1$, $c_2(s_2) = w(s_2)$, the stochastic consumption Euler equation in this model is given by

$$\beta(1 + r_2) = \frac{u'(c_1)}{E[u'(c_2(s_2))]} = \frac{u'(E[w(s_2)])}{E[u'(w(s_2))]}.$$

The stochastic process for the wage in the second period, $w(s_2)$, takes the form

$$w(s_2) = \begin{cases} w(s_G) = 1 + \sigma/2, & \text{with prob. } 1/2, \\ w(s_B) = 1 - \sigma/2, & \text{with prob. } 1/2, \end{cases}$$

where $\sigma \in (0, 2)$ parametrizes the risk in this economy. Assume that the utility function is of the following form

$$u(c) = c - \frac{1}{4}c^2, \quad \forall c \in [0, 2].$$

Does this utility function, $u(c)$, imply precautionary savings?

Your Answer:

Yes:

No:

(XX Points) Exercise A.2: Asset pricing

Let m_{t+1} and x_{t+1} be two random scalars. Then

$$E_t[m_{t+1}x_{t+1}] = E_t[m_{t+1}]E_t[x_{t+1}] + 2Cov_t[m_{t+1}, x_{t+1}],$$

where

$$Cov_t(m_{t+1}, x_{t+1}) \equiv E_t[(m_{t+1} - E_t[m_{t+1}])(x_{t+1} - E_t[x_{t+1}])].$$

Your Answer:

True:

False:

(XX Points) Exercise A.X: Some more short questions follow ...

Exercise B: Long Question (XX Points)

Consumption-based asset pricing

Consider a representative household of an economy that maximizes expected lifetime utility

$$U(c_t, c_{t+1}(s_{t+1})) = \frac{c_t^{1-\theta}}{1-\theta} + \beta E_t \left[\frac{c_{t+1}(s_{t+1})^{1-\theta}}{1-\theta} \right], \quad 0 < \beta < 1,$$

by choosing consumption c_t and c_{t+1} in both periods. The agent is endowed with the income e_t and e_{t+1} in the corresponding periods (let e_{t+1} be stochastic), and can shift resources across periods by borrowing or lending in a risk free bond, b_{t+1} , and in a risky asset, a_{t+1} . The risk free bond pays 1 unit of consumption for sure in the future period and has a price q_t in terms of today's consumption, while the risky asset's payoff is the realization of a random variable $x_{t+1} \equiv p_{t+1} + d_{t+1}$ and is of price p_t in terms of today's consumption. The agent initially owns neither assets nor bonds. Expectations are with respect to the future state, $s_{t+1} = (e_{t+1}, x_{t+1}) \in S$, where the set S contains the possible realizations of the state, s_{t+1} .

- (XX Points) Derive the agent's budget constraints for both periods (hint: the second period constraint is state-by-state).
- (XX Points) Reduce consumption in the utility function using the agent's budget constraints and derive the optimality conditions with respect to bond holdings, b_{t+1} , and asset holdings, a_{t+1} , taking as given prices and returns. (hint: the expectation operator E_t is linear (think of it as a probability weighted sum), so you can just go ahead and take derivatives inside the expectation operator).
- (XX Points) Use the optimality conditions to show that the price of the risky asset is a function of the price of the risk free bond, q_t , and the covariance of the stochastic discount factor, m_{t+1} , with the risky return on the asset,

$$p_t = \beta E_t[m_{t+1}]E_t[x_{t+1}] + \beta \text{Cov}_t(m_{t+1}, x_{t+1}), \quad m_{t+1} \equiv \left(\frac{c_{t+1}}{c_t} \right)^{-\theta}.$$

What factors drive the agent's valuation of the asset?

**Exercise X:
Some More Long Questions ... (XX Points)**